Birkhoff's earlier work in the asymptotic theory of ordinary differential equations, as well as in consequence of certain considerations of mathematical physics, the truth of this conjecture would offhand appear as rather likely. This fact explains the purpose of the present work—taking a purely mathematical point of view, the author considers linear partial differential equations containing a parameter λ and first establishes existence of formal solutions containing those of Birkhoff as a special case. He then establishes (for second order equations) some general existence theorems, asserting existence of 'actual' solutions, which are functions asymptotic to the formal series. This theory is naturally divided into two parts—one relating to equations of elliptic type, the other referring to those of hyperbolic type. Equations of parabolic type have not been considered in the present work. (Received November 24, 1941.)

61. S. M. Ulam: *A geometrical approach to the theory of representations of topological groups.* Preliminary report.

The theorem of von Neumann on the representations of compact groups by sequences of finite matrices is proved by consideration of geometrical properties of compact convex bodies in the space of continuous (real-valued) functions on a compact space. (The use of Haar measure is avoided.) A number of related results are derived. (Received November 25, 1941.)

62. František Wolf: *On majorants of analytic functions.*

If $f(z)$ is analytic in $|x| < a, |y| < b$ and $|f(z)| \leqq M(x)$ where $\int_{-a}^{a} \log^+ \log^+ M(x)dx < \infty$, then to an arbitrary $\delta > 0$ corresponds a ϕ, dependent only on $M(x)$ and δ, but independent on the particular $f(z)$, such that $|f(z)| \leqq \phi$ for $|x| < a - \delta$, $|y| < b - \delta$. This is a generalization of a result of N. Levinson (*Gap and Density Theorems*, p. 127, Theorem XLIII). The theorem is proved by a method used by the author to prove a generalization of Phragmén-Lindelöf theorem (Journal of the London Mathematical Society, vol. 14 (1939), p. 208) which becomes a corollary of the above theorem. Another corollary is Nils Sjögren's result (Congrès des Mathématiques Scandinaves à Helsinfors, 1938): If $f(z)$ is analytic in the unit circle and such that $|f(z)| \leqq M(\arg z)$ for $1 - \epsilon < r < 1$ and $\int_0^{2\pi} \log^+ \log^+ M(\theta) \cdot d\theta < \infty$, then $|f(z)| \leqq \phi$ in $|z| \leqq 1 - \delta < 1$. ϕ does not depend on the particular $f(z)$, but only on $M(\theta)$ and δ. (Received October 25, 1941.)

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63. A. E. Engelbrecht: *Circular plates with large deflections.*

The nonlinear system of equations derived by von Kármán is used to obtain a solution for a family of thin circular plates involving radial symmetry and having a uniform moment applied at the periphery. The edge of the plate to which the external moment is applied suffers no displacement normal to the plane of the plate, but is free to move laterally. The solution is effected by expanding the deflection w and the stress function ϕ in terms of a small parameter $\epsilon = h/a$ where h is the plate thickness and a the radius of the plate. By this expansion the nonlinear system reduces to an iterative process for determining the successive terms. Satisfactory numerical results are obtained for the deflection, bending moments and direct planar stresses for plates whose maximum deflection is twice the order of the plate thickness. (Received October 21, 1941.)

64. R. E. Gaskell: *On longitudinal vibrations of a bar.*

A uniform cylindrical bar, built-in at the end $x=0$, has attached to the free end, $x=1$, a weight, W. Each section of the bar has an initial displacement, $f(x)$, and the system is released from rest at $t=0$. The longitudinal vibrations of the bar are found by use of the Laplace transformation in both integral and series form. The solution obtained can be verified, except on characteristic lines, when $f(x)$ is a continuous function with its first three derivatives piecewise continuous and $f^{iv}(x)$ bounded and integrable. (Received November 21, 1941.)

65. A. E. Heins: *On the transformation theory of the solution of partial differential equations.* II. Preliminary report.

A complete transformation theory of the solution of the partial differential equations of mathematical physics in two dimensions was recently considered by the author (abstract 47-5-254). With the aid of the finite Fourier transform, solutions were obtained to the two-dimensional equation of heat conduction under prescribed initial and boundary conditions, when the boundaries were either rectangles or circles. This present paper treats the solution of the same equations when the boundaries are conveniently represented in polar coordinates. Instead of a finite Fourier transform, a finite Hankel transform is employed. The regularity of the finite Hankel transform serves as a condition to eliminate any superfluous boundary elements which do not enter directly into the problem. Some special examples are considered. (Received November 21, 1941.)

66. F. B. Hildebrand: *Note on the integro-differential equation of a problem in the theory of plane stress.*

It has been shown by E. Reissner (Proceedings of the National Academy of Sciences, vol. 26 (1940), pp. 300–305) that the stress σ in a stiffener of length L and cross-sectional area A, attached to a semi-infinite elastic sheet in a direction normal to the edge of the sheet, and subjected to a concentrated axial force F, satisfies the Cauchy integro-differential equation $\sigma(x)+A(L)\sigma(L)\{(L-x)^{-1}+K(L,x)\} = \lambda \mathcal{J}_0^L[A(\xi)\sigma(\xi)]'\{(\xi-x)^{-1}+K(\xi,x)\}d\xi$, where $K(\xi,x) = B_1x\xi/(\xi+x)^3 +(B_2x+B_3\xi)/(\xi+x)^2+B_4/(\xi+x)$ and λ and the B_i are certain constants depending upon the dimensions and properties of the stiffener and sheet. In the present note, use is made of a classical theorem of Plessner to prove, under certain physically reasonable restrictions on the function $A(x)\sigma(x)$, that a solution of this equation must satisfy the condition $A(L)\sigma(L)=0$, so that the stress is determined by the abbreviated equation $\sigma(x)=\lambda\mathcal{J}_0^L[A(\xi)\sigma(\xi)]'\{(\xi-x)^{-1}+K(\xi,x)\}d\xi$ and the boundary conditions $A(0)\sigma(0) = F$, $A(L)\sigma(L)=0$. (Received November 21, 1941.)

67. J. J. L. Hinrichsen: *Bounds for the libration points in a restricted problem of n bodies.*

Let $n-1$ bodies of equal mass be fixed in the vertices of a regular $(n-1)$-sided polygon and rotate with uniform velocity about their common center of gravity. Consider the motion of an infinitesimal mass, moving in the same plane, attracted by the $n-1$ bodies according to the Newtonian law of attraction. If $\Omega(x, y)$ represents the potential function of the system, bounds are found for the points whose coordinates (x, y) are solutions of $\partial\Omega/\partial x=\partial\Omega/\partial y=0$. For $n>3$, there will exist $3n-2$ such points, one at the center and $n-1$ equally distributed along each of three concentric circles.

They lie on axes of symmetry of the polygon and cannot lie further from the center than $(2n-1)/(n-1)$ of the distance from one of the $n-1$ bodies of equal mass to the center of mass of the other $n-2$ bodies. Approximate positions of the points are obtained for $n=3, 4, 5$. (Received October 25, 1941.)

68. D. L. Holl: *Consolidation of elastic earth layers.*

Biot (Journal of Applied Physics, vol. 12 (1941), p. 155) has established the equations (1) $\nabla^2\overline{U}+k_1$ grad div $\overline{U}-k_2$ grad $\sigma=0$ and (2) $\nabla^2\epsilon=k_3\,\partial\epsilon/\partial t$. For the consolidation of an elastic earth containing pores filled with moisture. The vector $\overline{U}$ is the displacement vector whose div $U=\epsilon$, and σ is the excess hydrostatic water pressure, and k_1, k_2, and k_3 are physical constants. Solutions of (1) and (2) are obtained for a layer infinite in horizontal extent but of finite depth and supported on a rigid surface which is either porous or is impervious to the passage of the imprisoned fluid. Specific solutions are given when the surface loads which cover a rectangular loading area and for axially symmetrical loads are known. (Received October 21, 1941.)

69. Isaac Opatowski: *On the theory of lethal irradiation of microörganisms.* II.

By means of the Laplace transformation three sets of formulas are obtained which give, each one independently of the other one, the vulnerability constants k_i (abstract 47-9-414) and the number of quanta $n+1$ necessary to kill the organism, when the number of killed organisms N_{n+1} is known as a function of the time t. However, the k_i's are uniquely determined only if their relative order of magnitude is known a priori. Two sets of these formulas give the homogeneous product sums $h_r(k_1, \cdots, k_{n+1})$ (D. E. Littlewood, *Theory of Group Characters*, p. 82) whereas a third set gives $h_r(k_1^{-1}, \cdots, k_{n+1}^{-1})$ in terms of the moments of dN_{n+1}/dt. Therefore, the evaluation of k_i's is reduced to the solution of an algebraic equation of degree $n+1$. If $\{k_i\}$ is an arithmetic progression, $N_{n+1}(t)$ reduces to the incomplete beta-function. Biophysically this case corresponds to a progressive destruction of the "sensitive volume" of the microörganism (B. M. Duggar, *Biological Effects of Radiation*, vol. II, p. 1321) by the quanta. The problem here treated is equivalent mathematically to that of finding a differential system of the type stated in the abstract referred to above, which is satisfied by a known function $N_{n+1}(t)$. (Received October 6, 1941.)

70. Willy Prager: *Fundamental theorems of a new mathematical theory of plasticity.*

The main difficulty all attempts at a mathematical theory of plasticity have, so far, had to cope with arises from the assumption that the material will not behave in a plastic manner unless a certain invariant of the stress tensor has reached a given critical value. Two different sets of equations will, therefore, be valid in the plastified and the not yet plastified regions. The problem becomes all the more involved by the fact that the boundary between these regions is not known beforehand but has to be determined so as to secure continuity of stresses. In order to avoid this difficulty, the author has proposed stress-strain relations giving a gradual transition from the elastic to the plastic state (Proceedings of the Fifth International Congress of Applied Mechanics, Cambridge, Massachusetts, 1938, p. 234). These relations have been applied to various problems of plane strain (Revue de la Faculté des Sciences de l'Université d'Istanbul, (A), vol. 5 (1941), p. 215). The present paper contains two variational principles which, in this new mathematical theory of plasticity, play the same role as the principle of least work and Castigliano's principle do in elasticity. (Received November 24, 1941.)

71. J. L. Synge: *On the theory of the airfoil of finite span.*

The problem of the potential flow past an airfoil of finite span is formulated, and then linearized by the usual approximation based on the assumptions that the thickness and curvature of the airfoil are small. The problem is reduced to the solution of an integro-differential equation, which agrees with that obtained by Mattioli in 1939 for a flat plate of no thickness. (Received November 21, 1941.)

72. Feodor Theilheimer: *The potential of curvilinear distributions.*

This paper deals with potentials which are induced by charges distributed along curve segments. It is assumed that the curves and the charge distributions are rational functions of a suitable parameter. Then it can be shown that there exists for every curve a finite number of transcendental functions $p_k(x, y, z)$ such that the potential $h(x, y, z)$ can be represented in closed form by means of these functions $p_k(x, y, z)$ together with algebraic and theta functions. (Cf. Bergmann, Mathematische Annalen, vol. 101 (1929), pp. 534–558.) If the curves can be represented as rational functions of second degree the functions $p_k(x, y, z)$ are connected in a simple way with the elliptic functions, and tables can be used for actual computation. These results supply a method for approximate solution of certain boundary value problems. This method is here applied to an aerodynamic problem taken from the theory of the finite air wing. (Received November 22, 1941.)

73. R. H. Tripp: *Bending of a thin plate having the form of a parallelogram.*

The Lagrange plate equation $N\nabla^4 w = p(x, y)$ is solved for a thin plate whose boundary is a parallelogram with opposite angles $\pi/4$ and $3\pi/4$. Two types of loading are considered: (1) a concentrated load at the center of symmetry and (2) a uniformly distributed load. When all the edges are simply supported the problem becomes that of the solution of two linear infinite systems of equations in infinitely many unknowns of the type $x_i + \sum_j a_{ij} x_j = b_i$. The properties of the determinant $|\partial_{ij} + a_{ij}|$ and b_i are such as to insure the existence and convergence of the solution. Derived results for the moments are discussed. (Received October 21, 1941.)

74. Alexander Weinstein: *On the flexural center and the center of twist.*

These concepts have been treated simultaneously by several authors by Saint-Venant's theory of beams. These authors introduced two essentially different definitions of the flexural center and three essentially different definitions of the center of twist. On the other hand, Sothwell has given a descriptive definition of both centers and has outlined a proof of their coincidence provided that one end of the beam is held by rigid constraints, a situation which cannot be met by Saint-Venant's theory. It is shown in the present paper that the coincidence of both centers can be established by using definitions previously employed by the authors who worked with Saint-Venant's methods. (Received November 21, 1941.)