ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

199. A. A. Albert: *Quasigroups*. I.

Associate with every quasigroup $\mathfrak{G}$ the group $\mathfrak{G}_T$ of nonsingular transformations on $\mathfrak{G}$ generated by the multiplications of $\mathfrak{G}$, and say that a multiplicative system $\mathfrak{G}'$ is isotopic to $\mathfrak{G}$ if there exist nonsingular mappings A, B, C on $\mathfrak{G}$ to $\mathfrak{G}'$ such that $xA \cdot yB = (x \cdot y)C$ for every x and y of $\mathfrak{G}$. Every quasigroup is isotopic to a loop (that is, a quasigroup with a two-sided identity element e). The normal divisors of a loop $\mathfrak{G}$ are then shown to be the subsets eW defined for normal divisors W of $\mathfrak{G}_T$, $\mathfrak{G}$ is simple if and only if the only intransitive normal divisor of $\mathfrak{G}_T$ is the identity group. All loops isotopic to simple loops are simple. A system $\mathfrak{G}$ is homotopic to $\mathfrak{G}'$ if $xA \cdot yB = (x \cdot y)C$ for equivalent mappings A, B, C on $\mathfrak{G}$ to $\mathfrak{G}'$ which may be singular. Then a loop $\mathfrak{G}$ is *homotopic* to a loop $\mathfrak{G}'$ if and only if $\mathfrak{G}$ is homomorphic to an isotope of $\mathfrak{G}'$. (Received June 7, 1943.)

200. William H. Durfee: *Congruence of quadratic forms over valuation rings*.

Let R be a complete valuation ring whose associated residue-class field has characteristic not two. An equivalent diagonal form for an arbitrary quadratic form over R is obtained, and it is shown that two such nonsingular diagonal forms are equivalent if and only if their corresponding subforms composed of those terms having the same value are equivalent. Using this the author proves for forms over R a theorem stated by Witt for fields and extended by Jones to the p-adic integers, namely, if f, g, and h are nonsingular quadratic forms such that g and h each has no variables in common with f, then $f+g$ and $f+h$ are equivalent if and only if g and h are equivalent. (Received August 2, 1943.)

201. H. L. Lee: *The sum of the kth power of polynomials of degree m in a Galois field*. Preliminary report.

Let $M = c_0x^m + c_1x^{m-1} + \cdots + c_{m-1}x + c_m$ be a polynomial in the Galois field $GF(p^n)$. If $c_0 = 1$, M is called primary and if $c_0 \neq 0$, write deg $M = m$. Let S_m^k and R_m^k denote respectively the sum of the kth power of polynomials M of degree m, and of all M of degree less than m. By the use of two functions $\psi_m(t) - F_m$, $\psi_m(t)$, which vanish when M is primary and of degree m in one case and when deg $M < m$ in the other, the sum may be made to depend on an exponent less than k. Then S_m^k and R_m^k

depend on the remainder in the division of t^k by $\psi_m(t)-F_m$ and $\psi_m(t)$ respectively. (Cf. H. L. Lee, Duke Math. J. vol. 9 (1943) pp. 277–292.) (Received July 19, 1943.)

202. W. V. Parker: *Limits to the characteristic roots of a matrix.*

Let $A=(a_{ij})$ be a square matrix of order n with elements in the field of complex numbers; and define $S_i=\sum_{j=1}^{n}|a_{ij}|$, $T_j=\sum_{i=1}^{n}|a_{ij}|$, $U_i=2|a_{ii}|-S_i$, and $V_j=2|a_{jj}|-T_j$. Let S, T be the greatest of the S_i, T_j, respectively; and let U, V be the least of the U_i, V_j, respectively. It is shown that the absolute value of each characteristic root of A is not less than the greater of the numbers U and V and is not greater than the smaller of the numbers S and T. Similar bounds are also found for the real and imaginary parts of the characteristic roots. (Received July 23, 1943.)

203. H. E. Salzer: *Table of first two hundred squares expressed as a sum of four tetrahedral numbers.*

The following empirical theorem is conjectured: Every square integer is expressible as the sum of four positive (including zero) tetrahedral numbers $(n^3-n)/6$. It has been verified by a table prepared for the first 200 squares. This empirical theorem is a partial improvement of the statement that five non-negative tetrahedrals suffice for any integer. (See F. Pollock, Proc. Roy. Soc. London Ser. A. vol. 5 (1850).) (Received June 4, 1943.)

ANALYSIS

204. R. H. Cameron and W. T. Martin: *Transformations of Wiener integrals under translations.*

Let $F[y]$ be a functional defined and Wiener summable over the space C consisting of all functions $x(t)$ continuous in $0\leqq t\leqq 1$ and vanishing at $t=0$. In addition, let F be continuous and let it be bounded over every bounded set $x(\cdot)$ of C. (F is called continuous if $F[y^{(n)}]\to F[y^{(0)}]$ whenever $y^{(n)}(t)\to y^{(0)}(t)$ uniformly in $0\leqq t\leqq 1$, and F is bounded over every bounded set $x(\cdot)$ of C if for every positive constant B there exists a constant $K=K_B$ such that $|F[y]|\leqq K$ for all $y(\cdot)$ of C for which $|y(t)|\leqq B$, $0\leqq t\leqq 1$.) Under these conditions on the functional F the authors obtain a transformation formula for Wiener integrals under translations of the form $y(t)=x(t)+x_0(t)$ where $x_0(t)$ is a given function of C with a first derivative $x_0'(t)$ of bounded variation in $0\leqq t\leqq 1$. The transformation formula is $\int_C^w F[y]d_wy=\int_C^w F[x+x_0]\exp\{-\int_0^1[x_0'(t)]^2dt-2\int_0^1x_0'(t)dx(t)\}d_wx$. The formula forms a basis for the calculation of various types of Wiener integrals. (Received July 30, 1943.)

205. M. M. Day: *Uniform convexity.* IV.

In this paper relationships between uniform convexity, factor spaces, and conjugate spaces are discussed. Theorem 1: A normed vector space B is uniformly convex if and only if all the two dimensional factor spaces of B are uniformly convex with a common modulus of convexity. The concept of uniform flattening is suggested by a description of a "sharp edge" on the unit sphere in terms of the norm of the space. It is shown [Theorem 2] that this is dual to uniform convexity; that is, $B[B^*]$ is uniformly flattened if and only if $B^*[B]$ is uniformly convex. It follows that a complete uniformly flattened B is reflexive. The proof of Theorem 2 uses a computation for