ranges of the variables. To avoid the contradictions, Quine has a metalogical axiom to the effect that only "stratified" formulas determine classes. The purpose of the present paper is the presentation of nine formal axioms and a demonstration of their equivalence with Quine's metalogical axiom in the presence of the restricted predicate calculus and the axiom for identity. (Received July 29, 1943.)

Numerical Computation

237. H. E. Salzer: *Coefficients for numerical integration with central differences.*

The numbers $M_{2s} \equiv B_{2s}^{(2s)}(s)/(2s)!$, where $B_{2s}^{(2s)}(s)$ denotes the $(2s)$th Bernoulli polynomial of order $2s$ for argument equal to s, were computed for $s=1, 2, \cdots, 10$, the result being exact values in lowest terms. Then these quantities were all checked by a cumulative recursion formula. (Previous calculations went only as far as $s=4$.) The numbers M_{2s} are coefficients of central differences of order $2s-1$ in the well known formula for numerical integration, sometimes called the Gauss-Encke formula or the second Gaussian summation formula. All important formulas for and references to these coefficients are included. This calculation was performed in the course of work done for the Mathematical Tables Project, National Bureau of Standards. (Received July 7, 1943.)

Statistics and Probability

238. E. J. Gumbel: *On the plotting of statistical observations.*

It is well known that there exist two stepfunctions corresponding to a continuous variate. We may attribute to the mth observation the ranks m or $m-1$. To obtain one and only one serial number m, which will, in general, not be integer, we attribute to x_m an adjusted frequency $m-\Delta$, namely the probability of the most probable mth value. The correction Δ for the rank is unlimited and possesses a mode, Δ increases for increasing value of the variate from zero up to unity. The correction is important for small numbers of observations. For large numbers of observations and for the ogive it is sufficient to choose $\Delta=1/2$. The calculation of Δ allows a correct plotting of all observations (including the first and last) on probability paper (equiprobability test). For the return periods, the ranks m and $m-1$ correspond to the observed exceedance and recurrence intervals. Generally the corrected return periods pass for increasing values of the variate from the exceedance to the recurrence intervals, provided the variate is unlimited and possesses a single mode. The asymptotic standard error of the partition values may be used to construct confidence bands for the ogive, the equiprobability test, and the return periods. This control for the fit between theory and observation may be applied to all observations which are not extreme. (Received July 30, 1943.)

239. Henry Scheffé: *On a measure-theoretic problem arising in the theory of non-parametric tests.*

Let μ be any measure on the real line, such that the measure of the whole line is unity, and form the "power" measure μ^k in Euclidean k-space—that is, the product