ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

243. Angeline J. Brandt: *The free Lie ring and Lie representations of the full linear group.*

The present paper is a continuation and amplification of a paper of Thrall (Amer. J. Math. vol. 64 (1942) pp. 371–388). In the latter paper a recursion formula was developed from which the irreducible constituents of the mth Lie representation for $m \leqq 10$ were obtained. The main result of the present paper is a direct formula for the character of the mth Lie representation, namely $[m] = [\sum \mu(d) s_d^{d'}]/m$ where $dd' = m$, the sum is over all divisors d of m, $\mu(d)$ is the familiar Möbius function and s_d is the trace of A^d, A being an arbitrary element of the full linear group. The nature of the irreducible invariant subspaces of L^m (all forms of degree m in the free Lie ring) is determined by this formula for $m \leqq 14$. Certain necessary and certain sufficient conditions are determined in order that a given ideal I be characteristic. Since the intersection of I with the module L^m is an invariant subspace of L^m relative to the representation of the full linear group afforded by L^m, it becomes important to know what elements constitute such a subspace and a method for determining this is developed. (Received October 1, 1943.)

244. R. H. Bruck: *Some results in the theory of quasigroups.*

This paper is primarily intended as an illustration of the usefulness of isotopy in quasigroup theory and as ground work for a later paper on linear non-associative algebras. It is largely devoted to the theory and construction of quasigroups with the inverse property. A quasigroup Q, finite or infinite, has the inverse property if there exist two one-to-one reversible mappings L, R, not necessarily distinct, of Q on itself, such that $b^L \cdot ba = ab \cdot b^R = a$ for all a, b of Q. Also contained in the paper are several new theorems on Moufang quasigroups, an explicit construction of all (Murdoch) abelian quasigroups, and necessary and sufficient conditions that the direct product of two finite quasigroups should contain no sub-quasigroup except itself. (Received August 13, 1943.)

245. Nathan Jacobson: *An extension of Galois theory to non-separable and non-normal fields.*

The extension of Galois theory obtained in this paper is based on two important concepts: self-representation of a field P, that is, a representation of P by matrices with elements in P, and composite (ring) of P with itself. The latter is a generalization of the usual concept of a composite field. With any two self-representations, associate a product self-representation obtained by substituting for the elements of the matrices of the first representation the matrices that represent these elements in the second representation. This leads to a definition of the product of composites. The latter concept is used to define Galois composites. The fundamental theorem then establishes a (1-1) correspondence between the Galois composites of P and the subfields Φ of P over which P is finite. This result gives a theorem recently announced by Kalujnine and specializes still further to the classical theorem for subfields Φ over which P is finite, separable and normal. In addition to the main result a general study of self-representations of fields is given in this paper. (Received August 9, 1943.)

246. R. E. Johnson: *On the equation $\chi\alpha=\gamma\chi+\beta$ over an algebraic division ring.*

The main purpose of this paper is to give necessary and sufficient conditions in order that the equation $\chi\alpha=\gamma\chi+\beta$ have a solution over an algebraic division ring. If α and γ are not transforms of each other, this equation always has a unique solution. In case γ equals α, this equation has a solution if and only if $\beta\alpha\beta^{-1}$ is a unilateral solution of a particular equation over the division ring. For the special case in which the division ring is a quaternion algebra, $\chi\alpha=\alpha\chi+\beta$ has a solution if and only if $\beta\alpha=\bar{\alpha}\beta$, where $\bar{\alpha}$ is the quaternion conjugate to α. (Received September 20, 1943.)

247. R. E. O'Connor and Gordon Pall: *On integral quadratic forms in many variables.*

A difficult problem proposed by Ko and Erdös, to find an integral quadratic form of determinant 1 and minimum greater than 2, is solved by the construction of a form in 24 variables of minimum 3, and a similar form in 40 variables. Improvements are found on the limits within which quadratic forms in genera of one class may exist. (Received August 6, 1943.)

248. R. R. Stoll: *Representations of completely simple semigroups.*

Every finite semigroup (that is, a finite system closed under an associative multiplication) can be represented by a set of correspondences of a suitable finite set to itself. This suggests the problem of the determination of all such representations of a given semigroup. Towards this end semigroups of correspondences are first studied. It is shown that if P is a semigroup on the set N then N can be written as the class sum of mutually disjoint subsets which form "weakly transitive" systems for P. Next, each weakly transitive system has a unique decomposition as the union of "transitive" and "quasi-transitive" systems. This reduces the representation problem to the determination of all transitive and quasi-transitive representations and methods for composing such representations. The above results are used to determine all representations of a type of finite semigroup called a "Kerngruppe" by Suschkewitsch (Communications de la Société Mathématique de Kharkow vol. 6 (1933) pp. 27–38) and a "completely simple semigroup without zero" by Rees (Proc. Cambridge Philos. Soc. vol. 36 (1940) pp. 387–400). (Received October 1, 1943.)

249. Dorothy M. Stone: *An algebraic characterization of measure algebras.* Preliminary report.

Let E be a non-atomic Boolean σ-algebra. This paper gives purely algebraic conditions on E which are necessary and sufficient for the existence of a countably additive, real-valued, non-negative, finite measure function on E, vanishing only for ϕ, the zero of E. E is given the star-topology of G. Birkhoff, and it is shown that E will have such a measure if and only if it is metrizable in a certain way in this topology. It is next shown that E is a topological space if and only if it satisfies a certain distributive law. A basis for the neighborhoods of ϕ can then be characterized algebraically, making it possible to state simple algebraic equivalents for the various separation axioms. If E satisfies the countable chain condition, T_3 implies metrizability, which gives an outer measure on E. A measure is obtained by means of an additional algebraic requirement. Thus, if E satisfies the distributive law referred to, the countable chain condition, the algebraic equivalent of T_3, and the additional requirement, there exists a measure-function on E. These conditions are easily seen to be necessary. It is not known whether they are independent. (Received October 2, 1943.)

250. R. M. Thrall: *On the decomposition of modular tensors.* II.

Let G be the n-rowed full linear group over a field k of characteristic p. A representation of G is called a tensor representation if its space is a direct sum of subspaces and factor spaces of tensor spaces. A main result of the present paper is that for a finite field k, the k-group ring of G has a faithful tensor representation. In paper I the representations afforded by all tensors of rank $m<2p$ were determined subject to the condition that k has more than p elements. In this paper the same is done for the field k with p elements. A main tool in this investigation is the construction of a representaton of G from each irreducible representation of the non-modular full linear group, and a corresponding extension of the Brauer-Nesbitt modular character theory to this case. The presence of zero divisors in the ring of polynomial functions over a finite field enters into the treatment of the case of tensors of rank $2p-1$ over a two-dimensional vector space, and the situation in that case should help point the way to the general decomposition theory. (Received October 1, 1943.)

ANALYSIS

251. Stefan Bergman: *Fundamental solutions of partial differential equations of the second order.*

As was previously shown, for every differential equation $L(U)=U_{Z\bar{Z}}+H(Z,\bar{Z})U=0$, $Z=X+iY$, $\bar{Z}=X-iY$, there exists a function $\mathrm{E}(Z,\bar{Z},t)=1+Z\bar{Z}t^2\mathrm{E}^*(Z,\bar{Z},t)$ such that $U=\mathrm{P}(f)\equiv\int_{-1}^{+1}\mathrm{E}(Z,\bar{Z},t)f(Z(1-t^2)/2)dt/(1-t^2)^{1/2}$, where f is an arbitrary analytic function, is a solution of $L(U)=0$ (see Duke Math. J. vol. 6 (1940) p. 537). The author shows that a fundamental solution $\Gamma(z,\bar{z},\zeta,\bar{\zeta})$ of the equation $S(v)=v_{z\bar{z}}+F(z,\bar{z})v=0$ is given by $\mathrm{P}(1/2\pi)\log|Z|+G(Z,\bar{Z})$. Here $Z=z-\zeta$, $\bar{Z}=\bar{z}-\zeta$, and P is the operator introduced above for the equation $L(U)=0$ with $H(Z,\bar{Z})=F(Z+\bar{\zeta},\bar{Z}+\bar{\zeta})$. $G(Z,\bar{Z})=-\int_0^Z\int_0^{\bar{Z}}D(Z,\bar{Z})dZd\bar{Z}+\int_0^Z\int_0^{\bar{Z}}H(Z,\bar{Z})(\int_0^Z\int_0^{\bar{Z}}D(Z,\bar{Z})dZd\bar{Z})dZd\bar{Z}+\cdots$ where $D(Z,\bar{Z})=(1/2)\int_{-1}^{+1}t^2[2\mathrm{E}^*+\bar{Z}\mathrm{E}^*_{\bar{Z}}+Z\mathrm{E}^*_Z]dt/(1-t^2)^{1/2}$. Using the representations of functions $v(z,\bar{z})$, $S(v)=0$, in the form of a line integral over a closed curve in terms of Γ, $\partial\Gamma/\partial n$, v and $\partial v/\partial n$ the author studies the growth of v and $\partial v/\partial n$ along circles $|z|=r$, $r\rightarrow\infty$. The existence of an analogous function $\mathrm{E}(X,Y,t)$ for every equation $H(U)=U_{XY}+H(X,Y)U=0$ (of hyperbolic type) has been established (see above reference). $(1/2\pi)\int_{-1}^{+1}\mathrm{E}(X,Y,t)dt/(1-t^2)^{1/2}$ is now shown to be the Riemann function of the equation $v_{xy}+F(x,y)v=0$, where $X=x-\xi$, $Y=y-\eta$ and $H(X,Y)=F(X+\xi,Y+\eta)$. Analogous relations hold for more general equations. (Received September 11, 1943.)