ANALYSIS

179. R. P. Agnew: *A family of bounded sequences summable M.*

A simple example is given of a family of bounded sequences, summable M, which is not a separable subset of the space B of bounded sequences. (Received May 2, 1944.)

180. R. P. Agnew: *Convergence fields of methods of summability.*

Let A be a multiplicative matrix method of summatibility with multiplier ρ. If $\rho=0$, each sequence x_n which oscillates sufficiently slowly is summable A to 0. If $\rho\neq 0$ and x_n is summable A to L, then the sequence $(x_n-L/\rho)\xi_n$ is summable A to 0 if the sequence ξ_n oscillates sufficiently slowly. (Received May 16, 1944.)

181. I. S. Reed: *On the solution of a general transform.*

The purpose of this paper is to give a brief extension to the solution of a Watson transform with an unsymmetrical kernel. Use has been made of the work that has been done by Hardy, Watson, Titchmarsh, Goodspeed, and others. (Received April 26, 1944.)

182. Tibor Rado: *On the Geöcze area of Frêchet surfaces.* Preliminary report.

The Geöcze area of a surface S is defined in terms of projections upon the coordinate planes. Perfecting previous theories originating with the work of Geöcze, Reichelderfer (Trans. Amer. Math. Soc. vol. 53 (1943) pp. 251–291) introduced and studied an *essential area*. In his definition (loc. cit. p. 274) we replace *simple Jordan regions* by *domains of any connectivity*, and obtain an area to be denoted by $G(S)$, where S stands for a Fréchet surface of the type of the sphere or the disc. In this paper we develop a comprehensive theory of $G(S)$ as a foundation for the theory of the area. One of the results states that if the Lebesgue area $L(S)$ is finite, then $G(S)=L(S)$. Essential use is made of methods and results developed in recent years by Reichelderfer, Morrey, Youngs, and the author. (Received April 19, 1944.)

183. A. R. Schweitzer: *On functional equations with solutions containing arbitrary functions.* VI.

The author constructs equations in iterative compositions satisfied by specific functions of variables and derived from conditions of invariance of iterative compositions of these functions under substitution groups on the variables of the latter. For example, functional equations satisfied by arbitrary functions of the type $\gamma(x_1+x_2+\cdots+x_n)$ are obtained by assuming that the function $\beta\{\alpha(x_1+x_2+\cdots+x_m)+\alpha(y_1+y_2+\cdots+y_m)+\cdots+\alpha(t_1+t_2+\cdots+t_m)\}$ corresponds to a set of imprimitive systems of a given imprimitive substitution group G_S on the variables $x_i, y_i, \cdots, t_i$ $(i=1, 2, \cdots, m)$. These equations express invariance (in the sense indicated) under G_S of the corresponding functional composition $\phi\{f(x_1, x_2, \cdots, x_m), f(y_1, y_2, \cdots, y_m), \cdots, f(t_1, t_2, \cdots, t_m)\}$. A set of functional equations of the preceding type corresponds to any abstract group G with subgroup H, since G can be represented as a (regular) substitution group G_S simply isomorphic to G and such that G_S is imprimitive with set of imprimitive systems corresponding to H. Functional equations are also constructed when the arbitrary function β has as arguments functions whose variables are not necessarily equal in number. (Received May 29, 1944.)

184. A. R. Schweitzer: *On functional equations representing abstract groups.*

Let $f(x_1, x_2, \cdots, x_{n+1}) = a_1x_1 + a_2x_2 + \cdots + a_{n+1}x_{n+1}$ and assume $a_i \cdot (a_1x_1 + a_2x_2 + \cdots + a_{n+1}x_{n+1}) = (a_ia_1)x_1 + (a_ia_2)x_2 + \cdots + (a_ia_{n+1})x_{n+1}$; then the equation $f(u_1, u_2, \cdots, u_{n+1}) = f(L_1, L_2, \cdots, L_{n+1})$, where $u_i = f(x_{i1}, x_{i2}, \cdots, x_{in+1})$ and L_i are certain formal sums of the x's, represents a class of multiplication tables for the a's. Group representation by a single functional equation follows if the associated multiplication table defines an abstract group G (with a_1 as the identical element). In this case the formal sums $L_1, L_2, \cdots, L_{n+1}$ are, in effect, derived from L_1 by applying to the variables of L_1 substitutions of a regular group simply isomorphic to G. The preceding representation is of formal additive, that is, special type. Group representation of general type is obtained by subjecting the variables x_{ij} in the composition $f(u_1, u_2, \cdots, u_{n+1})$ to conditions of invariance of the latter suggested by the invariance of the formal sums serving as arguments of the function $f(L_1, L_2, \cdots, L_{n+1})$. An analogous representation of linear algebras is discussed. (Received May 29, 1944.)

185. A. R. Schweitzer: *Functional relations valid in the domains of abstract groups and Grassmann's space analysis.*

The function of group elements $f(x_1, x_2, \cdots, x_{n+1}) = x_1 \cdot x_2^{-1} \cdot x_3, \cdots, x_{n+1}^{\pm 1}$ when n is odd satisfies the author's postulates generalizing the group concept to $(n+1)$-ary composition (abstract 50-3-73). When n is even, the preceding function satisfies the postulates: 1. $f(u_1, u_2, \cdots, u_{n+1}) = f(u, t_1, t_2, \cdots, t_n)$ where $u_i = f(x_i, t_1, t_2, \cdots, t_n)$ and $u = f(x_1, x_2, \cdots, x_{n+1})$. 2. Given $f(x_1, x_2, \cdots, x_{n+1})$ there exists $\phi(x_1, x_2, \cdots, x_{n+1})$ such that $f\{\phi(x, t_1, t_2, \cdots, t_n), t_1, t_2, \cdots, t_n\} = x$ and $\phi\{f(x, t_1, t_2, \cdots, t_n), t_1, t_2, \cdots, t_n\} = x$. 3. $f(x, x, \cdots, x) = x$. 4. The set S of elements x_i is closed under the compositions f and ϕ. When $n=1$ the postulates are satisfied by $f(x_1, x_2) = x_2 \cdot x_1 \cdot x_2^{-1}$ and $f(x_1, x_2) = x_2 \cdot x_1^{-1} \cdot x_2$. A solution containing an arbitrary function is $f(x_1, x_2, \cdots, x_{n+1}) = x_1 \cdot x_{n+1}^{-1} \cdot \alpha(w_2, w_3, \cdots, w_n) \cdot x_{n+1}$ where $w_i = x_i \cdot x_{n+1}^{-1}$. In Grassmann's space the postulates are satisfied if $f(x_1, x_2, \cdots, x_{n+1}) = a_1x_1 + a_2x_2 + \cdots + a_{n+1}x_{n+1}$ where the x_i are points and the a_i belong to a field with sum equal to unity. In the above postulates the functions f and ϕ are dual to one another; if $f\{f(x, t_1, t_2, \cdots, t_n), t_1, t_2, \cdots, t_n\} = x$ then $f = \phi$. Also f is distributive over ϕ. For $n=1$ the restricted associative properties, $f\{x_1, f(x_2, x_1)\} = f\{f(x_1, x_2), x_1\}$ and $f\{x_1, \phi(x_2, x_1)\} = \phi\{f(x_1, x_2), x_1\}$ hold. (Received May 29, 1944.)

186. W. J. Thron: *A family of simple convergence regions for continued fractions.*

In this paper a new two parameter family of simple convergence regions for continued fractions (1) $1 + K(-c_n^2/1)$ is obtained. It is further shown that the regions cannot be improved except possibly by the removal of an epsilon. A simpler though less inclusive convergence criterion is provided by the following corollary of the main theorem: the continued fraction (1) converges if for all $n \geqq 1$ its elements $c_n = re^{i\theta}$ satisfy the conditions (a) $r \leqq (d - \epsilon + 1/2) \cos \alpha \sec (\theta - \alpha)$, $-\pi/2 + \alpha < \theta \leqq \beta_1$, (b) $r \leqq (d - 1/2) \cos \alpha \sec (\theta - \alpha)$, $0 \leqq \theta < \pi/2 + \alpha$, (c) $r = 0$ otherwise, (d) $\sum |b_n| = \infty$, where $b_1 = 1/c_1^2$, $b_n = 1/c_n^2 b_{n-1}$. Here $-\pi/2 < \alpha < \pi/2$, $0 \leqq d \leqq 1/2$, $0 < \epsilon < d/2$ and $\cot \beta_1 = -(\tan \alpha + ((d+1/2)/d)^{1/2} \sec \alpha)$. For $d=0$ this result reduces to the well known parabola theorem, an isolated result of Paydon (abstract 47-11-473) is also included in this theorem. (Received June 1, 1944.)