Applied Mathematics

187. A. N. Lowan and H. E. Salzer: *Formulas for complex interpolation.*

Whenever an analytic function is tabulated for arguments z_ν located at equidistant points along any straight line in the z-plane, the application of the Lagrange-Hermite interpolation formula, which approximates the function by a complex polynomial of degree n passing through any $n+1$ points, leads to the sum from $\nu=-[n/2]$ to $\nu=[(n+1)/2]$ of $f(z)\sim\sum A_\nu^{(n)}(p)f(z_\nu)$ where $A_\nu^{(n)}(P)$ are polynomials in $P=(z-z_0)/h$ and h is the complex tabular interval and where $[x]$ stands for the largest integer in x. Since in general z is not on the straight line containing the point z_ν, P is complex $(=p+iq)$. Expressions for the real and imaginary parts of $A_\nu^{(n)}(P)$ as functions of p and q were obtained for the cases of three, four, five and six point interpolation and arranged in a form especially suited for computational purposes. These expressions are particularly applicable to the cases of functions tabulated along rays through the origin, as for instance: the table of the Bessel functions $J_0(z)$ and $J_1(z)$ computed by the Mathematical Tables Project (Columbia Press, 1943), the forthcoming table of $Y_0(z)$ and $Y_1(z)$, H. T. Davis' table of $1/\Gamma(z)$ and Kennelly's tables of complex circular and hyperbolic functions for Cartesian and polar arguments. (Received April 12, 1944.)

188. H. E. Salzer: *Coefficients for mid-interval numerical integration with central differences.*

Coefficients in the formula for numerical integration from mid-interval to mid-interval using central differences were calculated to the extent where one can employ central differences up to the forty-ninth order. (Previous calculations have gone only as far as coefficients of the seventh central difference.) The coefficients K_{2s} occur in the following formula, which is also known as the first Gaussian summation-formula: $(1/h)\int_{a-h/2}^{a+(n-1/2)h} f(x)dx=[f(a)+f(a+h)+\cdots+f(a+(n-2)h)+f(a+(n-1)h)]+\sum_{s=1}^{m-1}K_{2s}[\delta^{2s-1}f(a+(n-1/2)h)-\delta^{2s-1}f(a-h/2)]+nh^{2m}K_{2m}f^{(2m)}(\xi)$. Due to the extreme rapidity with which the coefficients K_{2s} decrease with increasing s, the calculated table can be used with high accuracy in either integration or summation even though successive differences might not show the slightest tendency to decrease for the given interval. The quantities K_2 to K_{20} are given exactly and K_{22} to K_{50} are given to 18 decimals, accurate to within 0.6 units in the 18th place. These coefficients were checked by two cumulative recursion formulas, by differencing of the ratios K_{2s}/K_{2s+2}, and by a numerical example. (Received May 13, 1944.)

189. H. E. Salzer: *Table of coefficients for differences in terms of the derivatives.*

A table was prepared which lists the exact values of the coefficients $B_{m,s}$ for $m=1, 2, \cdots, 20$ and $s=m, \cdots, 20$ in the formula of Markoff which expresses the mth advancing difference in terms of the derivatives according to the equation $\Delta_h^m f(a)=\sum_{s=m}^{n-1}B_{m,s}h^sD^sf(a)+B_{m,n}h^nD^nf(\eta)$ where h denotes the tabular interval. The quantity $B_{m,s}$ in Milne-Thomson's notation is equal to the $(s-m)$th Bernoulli number of order $-m$, divided by $(s-m)!$ and in Jordan's notation $B_{m,s}$ is equal to $m!/s!$ multiplied by $\mathfrak{S}_s^m$ where $\mathfrak{S}_s^m$ is the Stirling number of the second kind. The coefficients $B_{m,s}$ were calculated by first obtaining the Stirling numbers $\mathfrak{S}_s^m$ (using their well known recursion relationships, and checking by a neat formula given in Jordan's *Calculus of finite differences*) and then multiplying by $m!/s!$. The numbers $B_{m,s}$ are expressed in lowest terms. (Received April 20, 1944.)

190. Seymour Sherman, J. DiPaola, and H. F. Frissel: *Routh's discrimant, flutter, and ground resonance.* Preliminary report.

Routh's criterion for the stability of the solutions of system of linear differential equations with constant coefficients is extended to cover cases arising in airplane flutter and helicopter ground resonance calculations. With this new tool, the stability of the flutter "polynomial" at a given reduced frequency for more than two degrees of freedom can be determined in one-fifth of the time hitherto required. (Received May 29, 1944.)

GEOMETRY

191. Edward Kasner and John DeCicco: *Isothermal families in general curvilinear coordinates, and loxodromes.*

If the square of the linear element of a surface Σ is given in isothermal coordinates (x, y) by $ds^2=E(x, y)(dx^2+dy^2)$, then the family of curves $g(x, y)=\text{const.}$ on Σ is isothermal if and only if $(\partial^2/\partial x^2+\partial^2/\partial y^2)$ arc tan $g_y/g_x=0$. In the present paper, the authors obtain the necessary and sufficient condition that $g(x, y)=\text{const.}$ represent an isothermal family when (x, y) are general curvilinear coordinates. This gives a large extension of Lie's theorem. The condition is simpler when the parametric curves form an orthogonal net. As an application, the condition is obtained that $g(x, y)=\text{const.}$ represent an isothermal family upon the Cartesian surface $z=f(x, y)$. Finally the condition is found that the level curves of the surface be an isothermal family. This is applied to the mapping of loxodromes, showing that they can be represented by straight lines for a sphere (Mercator) and spheroid (Lambert), but not for an ellipsoid of three unequal axes. Use is made of Kasner's theorem in Math. Ann. (1904). (Received April 20, 1944.)

192. Abraham Seidenberg: *Valuation ideals in polynomial rings.*

A constructive study of the valuation ideals in a polynomial ring $\mathfrak{O}=K[x, y]$ in two indeterminates, where K is an algebraically closed (ground-) field, is made. Let $\mathfrak{q}_1, \mathfrak{q}_2, \cdots$ be the Jordan sequence of v-ideals belonging to a valuation B of Σ/K, where Σ is the quotient field of $\mathfrak{O}$, and let $\mathfrak{q}_{i_j}$ be the jth ideal such that $v(\mathfrak{q}_{i_j})$ is not in the additive group generated by $v(\mathfrak{q}_1), \cdots, v(\mathfrak{q}_{i_j-1})$. A tool corresponding to the Puiseux series expansion for a valuation, which is available if K is of characteristic 0 but not in general, is found in introducing certain polynomials f_{i_j} such that $v(f_{i_j}) =v(\mathfrak{q}_{i_j})$. Considerations are reduced to valuations of rational rank 2. If B is of rational rank 2, place $v(\mathfrak{q}_1)=1$ and let τ be the least irrational value assumed by elements of $\mathfrak{O}$. The description of the v-ideals in $\mathfrak{O}$ for B is intimately connected with the approximants and quasiapproximants to a certain integral multiple of τ. In particular, a simple 0-dimensional v-ideal $\mathfrak{q}_i$ is characterized in terms of the values of $\mathfrak{q}_i$ and $\mathfrak{q}_{i+1}$. This characterization yields a proof that the transform of a simple v-ideal under a quadratic transformation is simple. If $\mathfrak{q}_i$ is not simple, an explicit factorization of $\mathfrak{q}_i$ in terms of the mentioned values is given. (Received May 22, 1944.)

193. A. H. Wheeler: *One-sided polyhedra from the five regular solids.*