A GENERALIZATION OF MOORE'S THEOREM ON SIMPLE TRIODS

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R. L. Moore has proved that the plane does not contain uncountably many mutually exclusive simple triods.[1] The generalization to spaces of higher dimension given below appears to have escaped notice, and to be of some interest.

DEFINITION. *If n is a non-negative integer, by a T_n-set we shall mean a continuum which is the sum of an n-cell, g, and an arc, t, such that $g \cdot t$ is a point which is an end point of t and a relatively interior point of g. The point $g \cdot t$ will be called a junction point.*

Obviously, a T_1-set is a simple triod.

THEOREM. *Euclidean n-space does not contain uncountably many mutually exclusive T_{n-1}-sets.*

PROOF. Supose that the theorem is false. Then for some positive number ϵ there exists an uncountable collection, G, of mutually exclusive T_{n-1}-sets such that the junction point of each is at distance greater than ϵ from the boundary of its $(n-1)$-cell. There is a point, P, which is a point of condensation of the set of all junction points of elements of G; let U be a spherical domain of n-space with center P and radius less than $\epsilon/2$. If the point X of U is a junction point of an element T of G, let $g(X)$ denote the component that contains X of the intersection of U and the $(n-1)$-cell of T. It is an easy consequence of the Alexander duality theorem that $g(X)$ separates U. Hence the collection, G', of all sets $g(X)$ is an uncountable collection of cuttings of U, and it is clearly non-separated. By a theorem due to Whyburn,[2] G' contains an uncountable saturated subcollection, G''. But if $g(X)$ is an element of G'', U contains an arc, t, which is in the T_{n-1}-set of G that contains X and which has only X in common with $g(X)$. Since the elements of G are mutually exclusive, no element of G'' separates a point of t from $g(X)$ in U, which is a contradiction.

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[1] In fact, Moore has proved that the plane does not contain uncountably many mutually exclusive triodic continua. See *Concerning triods in the plane and the junction points of plane continua*, Proc. Nat. Acad. Sci. U.S.A. vol. 14 (1928) pp. 85–88, and *Concerning triodic continua in the plane*, Fund. Math. vol. 13 (1929) pp. 261–263.

[2] Theorem 2.2 of chap. 3 of his book, *Analytic topology*, Amer. Math. Soc. Colloquium Publications, vol. 28.