ence group $G-F(G)$ which contains properly the difference group $G'-F'(G)$ since $F'(G)=F(G)$. Now $I'(G)$ is isomorphic to $G'-F'(G)$ and to $I(G)$. Hence $G-F(G)$ is an I-group and it follows that $I(G)$ which is isomorphic to $G-F(G)$ is also an I-group.

It follows from Theorem 7 that the theorems and corollaries of §§2 and 3 survey completely all completely reducible groups, G, which are I-groups.

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AN EXISTENCE THEOREM FOR LATIN SQUARES

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1. **Introduction.** A latin square may be interpreted as a representation of a 3-web or as the multiplication table of a quasi-group. Hence the following theorem has application both in the theory of projective planes and in the theory of quasi-groups. It is derived from a very interesting result of P. Hall.

2. **The existence theorem.** Is there any combinatorial restriction which prevents us from constructing a latin square by adding a row at a time? The following theorem shows that such a procedure is permissible.

THEOREM. *Given a rectangle of $n-r$ rows and n columns such that each of the numbers $1, 2, \cdots, n$ occurs once in every row and no number occurs twice in any column, then there exist r rows which may be added to the given rectangle to form a latin square.*

PROOF. Let C_i, $i=1, 2, \cdots, n$ be the subset of the numbers $1, 2, \cdots, n$ which do not occur in the ith column of the given rectangle. Then each C_i contains r numbers and each number occurs r times in all the C's. For there are $n-r$ numbers in the ith column and each number has appeared in $n-r$ columns. It will be shown that the subsets satisfy the requirements of P. Hall's theorem:[1]

In order that a complete system of distinct representatives of subsets

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[1] P. Hall, *On representatives of subsets*, J. London Math. Soc. vol. 10 (1935) pp. 26–30.

$T_1, \cdots, T_m$ of a set S shall exist, it is sufficient that for each $k=1, \cdots, m$ any selection of k of the subsets shall contain between them at least k elements of S.

The necessity of these requirements is evident. Let us apply thi theorem to the subsets C_i. Any selection of k C's will contain kr numbers and at least k of these must be distinct since each number is contained in only r C's. The distinct representatives $c_1, \cdots, c_n$ of the subsets $C_1, \cdots, C_n$ may be added as a row to the given rectangle. For $c_1, \cdots, c_n$ must contain each of the numbers $1, \cdots, n$ once and no c_i has appeared in the ith column of the given $n-r$ rows. Repeatedly applying this process, we continue adding rows to the rectangle until it becomes a complete latin square.

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