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NOTE ON THE ZEROS OF $P_n^m(\cos\theta)$ AND $dP_n^m(\cos\theta)/d\theta$ CONSIDERED AS FUNCTIONS OF n

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In many physical problems in which the boundary conditions are specified over the surface of a cone, it is necessary to know the roots of the equations

$$P_n^m(\cos\theta) = 0 \tag{1}$$

and

$$dP_n^m(\cos\theta)/d\theta = 0 \tag{2}$$

considered as functions of n. This problem has been solved by Bholanath Pal.[1] In these papers he develops infinite series for the roots n which converge rapidly and are very suitable for numerical computation. In deriving his solution Pal introduced a parameter k which takes on successive integer values and thereby yields successive roots of the equations.

It is the purpose of this note to point out that the value $k=1$ with which Pal commenced the series does not always give the first root of the equation, and sometimes it gives a number which is not a root of the equation. For example, in treating the equation $P_n^2(\cos\theta)=0$, Pal gives three roots: $n=4.77$, 2.26, 1.52, corresponding to values of θ equal to $15°$, $30°$, $45°$, respectively. That these values are not roots

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[1] Bull. Calcutta Math. Soc. vol. 9 (1917–1918) p. 85; vol. 10 (1918–1919) p. 187.

of the equation can be established by examining the *Tables of associated legendre functions.*[2]

In numerous instances the value $k=1$ does not give the first root of the equation. In several of these cases the series for n will converge when k is set equal to zero, although in one or two instances the series apparently does not converge.

The tables given by Pal for the roots of $dP_n^1(\cos\theta)/d\theta=0$ start with the second root, and apparently he was not aware of the fact that a smaller root existed which could be obtained from his formulae by setting $k=0$. This occurs also in his table of roots for the equation $dP_n^2(\cos\theta)/d\theta=0$ for $\theta=15°$ and $30°$, although in the case of $\theta=45°$, $k=1$ does give the first root.

Thus, although Pal's formulae give satisfactory methods of computing the roots of equations (1) and (2), they must be used with discretion, since it is difficult to ascertain if the formulae yield more or less than the correct number of roots. Amended lists of the roots of equations (1) and (2) are given in Tables I and II respectively.

TABLE I

Roots of $P_n^m(\cos\theta)=0$ treated as a function of n

$\theta=15°$		$\theta=30°$		$\theta=45°$	
m	n	m	n	m	n
0	8.45[3]	0	4.13[3]	0	2.59[3]
	20.60		10.03		6.52
	32.55		16.02		10.51
	44.53		22.01		14.51
	56.53		28.01		18.50
	68.52		34.01		22.50
±1	14.14	±1	6.83	±1	4.40
	26.30		12.91		8.44
	38.36		18.93		12.46
	50.39		24.95		16.47
	62.41		30.96		20.47
±2	19.21[4]	±2	9.39[4]	±2	6.15[4]
	31.67		15.62		10.28
	43.90		21.72		14.34
	56.03		27.78		18.37

[2] A. N. Lowan, Columbia University Press, 1945.

[3] Values overlooked by Pal.

[4] In these sequences Pal gives a smaller value that does not exist.

Table II

Roots of $dP_n^m(\cos\theta)/d\theta=0$ treated as a function of n

$\theta=15°$		$\theta=30°$		$\theta=45°$	
m	n	m	n	m	n
0	14.12	0	6.83	0	4.42[3]
	26.29		12.89		8.33
	38.35		18.93		12.34
	50.39		24.95		16.34
	62.41		30.96		20.35
					24.35
±1	6.80[3]	±1	3.19[3]	±1	2.00[3]
	19.88		9.71		6.32
	32.11		15.82		10.33
	44.22		21.87		14.34
	56.28		27.89		18.34
	68.32		33.91		22.34
±2	11.80[3]	±2	5.75[3]	±2	3.73
	25.06		12.38		8.15
	37.58		18.58		12.24
	49.83		24.68		16.26
	61.96		30.74		20.28
	74.03		36.78		

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