

Schlaefli's theorem that every Riemannian space of  $n$  dimensions can be immersed in Euclidean space of  $n(n+1)/2$  dimensions is discussed in great detail (pp. 199–210) for the particular case  $n=3$  and reference is made to the proofs of Janet and Cartan for the general  $n$ . In this connection, it might be remarked that the detailed discussion given by Janet (*Annales de la Société Polonaise de Mathématique* vol. 5 (1926) pp. 39–40) for the case  $n=2$  is convincing, whereas the counter-example given for the same case by Forsyth (*Intrinsic geometry of ideal space*, vol. 1, pp. 231–233) is not. A relatively simple proof of the theorem would be highly desirable. Cartan's discussion of the case  $n=3$  may help in that direction.

J. M. THOMAS

*Tables of fractional powers.* Prepared by Mathematical Tables Project, National Bureau of Standards. New York, Columbia University Press, 1946. 489+30 pp. \$7.50.

The tables here printed yield the values of  $A^x$  and  $x^a$ . For example, there are tables of  $A^x$  for  $A=10, \pi, 10^{-3}P$  (where  $P$  is a prime between 100 and 1000), as well as for other values. Thus  $10^x$  is given to 15 decimals for  $0.001 \leq x \leq 1.000$  with  $x$  advancing in intervals of .001. The function  $x^a$  is computed for the values  $a = \pm 1/2, \pm 1/3, \pm 2/3, \pm 1/4$  with  $0 \leq x \leq 9.99$  in intervals of .01. There is a bibliography with 76 titles and an introduction by Dr. Lowan in which the method of computation of the tables is explained and the accuracy of interpolation is illustrated by examples.

E. R. LORCH

*Tables of the modified Hankel functions of order one-third and of their derivatives.* Cambridge, Harvard University Press; London, Oxford University Press, 1945. 36+235 pp. \$10.00.

This set of tables is the first to be published by the Computation Laboratory of Harvard University. The functions here considered are solutions of Stokes' differential equation  $d^2u/dz^2 + zu = 0$  and were needed in connection with the work of the Radiation Laboratory on diffraction and refraction of waves. Solutions to Stokes' equation are  $h_1(z) = (k/i\pi) \int_{L_1} e^{zt+t^3/3} dt$  (where  $k$  is a constant and  $L_1$  is an infinite broken line in the complex plane) and  $h_2(z)$ , which has a similar expression. It is the functions  $h_i(z)$  and their derivatives  $h_i'(z)$  which are tabulated. The tables give the real and imaginary parts to eight decimal places for  $z = x + iy$  with  $|x + iy| \leq 6$  and  $x, y$  progressing in intervals of 0.1. The functions  $h_i(z)$  are related to the Hankel functions of order  $1/3$  by the equations  $h_i(z) = ((2/3)z^{3/2})^{1/3} H_{1/3}^{(i)}((2/3)z^{3/2})$ ,

$i=1, 2$ . Although the  $H_{1/8}^{(i)}(x)$  are triple-valued functions, the  $h_i(z)$  are single-valued.

There is a thorough discussion of the properties of the functions. The computation time was forty-five days. It is stated that without the calculator years would have been necessary for this task. This fact gives some indication of the capacity of the machine.

E. R. LORCH

*A manual of operation for the automatic sequence controlled calculator.*  
Cambridge, Harvard University Press; London, Oxford University Press, 1946. 6+561 pp. \$10.00.

The "analytical engine" of Babbage has finally been realized in the Harvard Calculator described in this book. This device, a brilliant engineering achievement, was constructed and donated by the International Business Machines Corporation.

The Calculator is designed to carry out any sequence of calculations. A tape is punched in accordance with a certain code and this is inserted in the sequence control mechanism, a complicated arrangement of relays. Under the control of the sequence mechanism are a large number of storage counters which also function as adders, a "multiply-divide" unit, and various function units for  $\log_{10} x$ ,  $10^x$ , and  $\sin x$ . The sequence mechanism arranges for the transfer of a number in one unit to another, directs the operation of the various units and appropriately stores the result. The machine will print the tables it calculates.

The present volume is intended as a manual of operation for this device. Accordingly, we find a complete set of coding and plugging instructions and a chapter devoted to the solution of examples.

There are however a number of aspects of the present volume which are of more general interest. For instance, the descriptions of the relay controls, the multiply-divide unit, the storage counters and the function units would certainly appeal to many who are not concerned with the operation of the calculator.

In addition there is a long bibliography on numerical methods. The historical introduction in Chapter 1 is quite interesting and undoubtedly serves the present purpose, but it is not a complete history of even the direct ancestors of the present device.

Thanks are due to Professor Aiken and his associates for the preparation of this manual in a form which will permit a wide circulation.

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