ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

107. Iacobo Barsotti: *Structure theorems for algebraic algebras without a finite basis.*

A few elementary properties are established, and a complete structure theorem for division algebras of type 2 (locally finite) and a countable basis is given (Theorem 17). A few properties connecting arithmetic with the structure of algebras over p-adic fields are found. Theorem 22 reduces the main unsolved problem to a simpler one, when the underlying field is of a particular kind, including algebraic and local fields. Theorem 24 analyzes the structure of algebras of type 1 with a noncountable basis. Such a structure will be treated in another paper. (Received December 26, 1946.)

108. Iacobo Barsotti: *Valuations in division algebras without a finite basis.*

The author studies arithmetic in infinite division algebras, based on nonarchimedean valuations. Algebras satisfying a certain condition are called valuable, and a relation is found between this condition and the algebraic character of the algebra. Ideal theory in valuable algebraic algebras is studied. Algebraic algebras over local fields are shown to be valuable under a very general assumption. Three *types* of algebras are introduced, according to the existence of particular finite subalgebras. A principal theorem on the structure of unramified infinite local fields is found, and another one on the structure of local algebras of type 2. At the end of the paper an unsolved problem is stated together with some clues about its tentative solution. A new proof is given of a particular case of a known theorem about algebras of finite degree. (Received December 26, 1946.)

109. P. T. Bateman: *Modular subgroups of finite index.* Preliminary report.

The modular group (Modulgruppe) is the group of linear fractional transformations $\tau'=(a\tau+b)/(c\tau+d)$, where a, b, c, d are integers such that $ad-bc=1$. The subgroups of the modular group most frequently encountered are those which can be defined by means of certain arithmetical congruence relations on a, b, c, d with respect to some positive integer, called the Stufe of the subgroup. By using the geometry of the transformations (cf. Ford, *Automorphic functions*, New York, 1929, chap. 3, in particular Theorem 22) it is proved here that these congruence subgroups are the only subgroups of finite index. (Received February 1, 1947.)

110. Richard Brauer: *Applications of induced characters.*

By application of a theorem on induced characters proved in an earlier paper it is shown that if G is a group of finite order and if q is the least common multiple of the orders of the elements of G, then every representation of G can be written in the field of the qth roots of unity. This improves a result given by the author (Amer. J. Math. vol. 67 (1945) pp. 461–471). As a further application of the theorem on induced characters it is shown that the characters of a group G of finite order are uniquely determined provided that the following information is given: (I) The linear characters of those subgroups H of G which are direct products of cyclic groups and groups of prime power order. (II) The manner in which the classes of conjugate elements of G break up into classes of H, if only elements of H are considered (H denoting the subgroups of the same type as in (I)). (Received January 27, 1947.)

111. Richard Brauer: *On Artin's L-series with general group characters.*

It is shown that if G is a group of finite order, every character of G can be expressed as a linear combination with integral rational coefficients of characters ω^* such that every ω^* is a character of G induced by a linear character ω of a subgroup of G. This result had been conjectured by E. Artin (Abh. Math. Sem. Hamburgischen Univ. vol. 3 (1924) pp. 89–108; vol. 8 (1931) pp. 292–306). It now follows that Artin's L-series with general group characters are meromorphic functions. (Received January 27, 1947.)

112. Richard Brauer: *On the zeta-functions of algebraic number fields.*

It is shown that if the algebraic number field K is normal over the subfield k, the quotient $\zeta(s, K)/\zeta(s, k)$ of the corresponding zeta-functions is an integral function of the complex variable s. A proof is given for the following theorem, conjectured by C. L. Siegel and proved by him in special cases: Consider all algebraic number fields k of a fixed degree n. Let d be the discriminant of k, let h be the number of classes of ideals in k, and let R be the regulator of k. Then $\log (hR) \sim \log(|d|)^{1/2}$ for $|d| \to \infty$. (Received January 27, 1947.)

113. Leonard Carlitz and E. L. Cohen: *Divisor functions of polynomials in a Galois field.*

The main result given in this paper is concerned with the number of solutions of the equation $(\alpha_1 + \cdots + \alpha_t)F = \alpha_1 X_1 Y_1^e + \cdots + \alpha_t X_t^e Y_t$ in primary polynomials of $GF[p^n, x]$ of degree k, ek respectively, where $e \geqq 1$ is fixed. For $e = 1$, the result reduces to a known one. The proof is considerably simpler than that used previously. (Received January 28, 1947.)

114. Dr. P. W. Carruth: *Generalized power series fields.*

Kaplansky (Duke Math. J. vol. 9 (1942) pp. 303–321 and vol. 12 (1945) pp. 243–248) has shown that if the characteristic of a field that is maximal with respect to a valuation is the same as that of its residue class field, under certain conditions the maximal field is analytically isomorphic to a power series field. In this paper, a generalized power series field is constructed. Criteria not dependent on the characteristics of a maximal field and its residue class field are stated for a maximal field to be analytically isomorphic to this generalized power series field. (Received January 23, 1947.)

115. E. L. Cohen: *Sums of an even number of squares in* $GF[p^n, x]$.

The problem discussed in this paper is an extension of one considered by Carlitz (Trans. Amer. Math. Soc. vol. 35 (1933) pp. 397–410). Suppose $\alpha_1, \cdots, \alpha_{2s}$ are nonzero elements of $GF(p^n)$, $p\neq 2$, $\epsilon=\alpha_1+\cdots+\alpha_m$ $(2s\geqq m\geqq 1)$, and let F be a polynomial of $GF[p^n, x]$. The problem is to find the number of solutions of $\alpha F=\sum_1^{2s}\alpha_i X_i^2$ in primary polynomials $X_1, \cdots, X_m$ of degree k and arbitrary polynomials $X_{m+1}, \cdots, X_{2s}$ of degree less than k, for the cases in which (1) $\alpha=\epsilon\neq 0$, F primary of degree $2k$, and (2) $\alpha=1$, $\epsilon=0$, F arbitrary of degree less than $2k$. The number of solutions in either case is given by the function $\rho_{s-1}(F, \lambda)$, defined in the paper referred to above, where $\lambda=+1$ or -1 according as $(-1)^s\alpha_1\cdots\alpha_{2s}$ is or is not a square. (Received January 28, 1947.)

116. A. H. Diamond and J. C. C. McKinsey: *A relation between algebras and their subalgebras.*

Certain sets of postulates for Boolean algebra do not contain equations in more than three variables. It obviously follows that if an algebra has the property that every subalgebra generated by three elements is a Boolean algebra then the algebra is itself a Boolean algebra. The question arises whether the number of elements generating the subalgebra can be reduced to two. This question is answered in the negative by giving an example of an algebra having the property that every subalgebra generated by two elements is a Boolean algebra while the algebra itself is not. A corollary is that every set of equations defining Boolean algebra must include at least one equation in three variables. Analogous results are established for groups, rings, and other algebras. (Received December 5, 1946.)

117. R. J. Levit: *The nonexistence of a certain type of odd perfect number.*

Let $n=a_0a_1\cdots a_t$ be an integer with prime power factors $a_i=p_i^{\alpha_i}$, where the p_i are distinct primes, $i=0, 1, \cdots, t$. For n to be perfect it is necessary that exactly one of the prime powers, say a_0, have an even divisor sum $\sigma(a_0)$. Then, if $\sigma_0=\sigma(a_0)/2$, $\sigma_i=\sigma(a_i)$, $i=1, 2, \cdots, t$, the condition for n to be perfect may be written $n=a_0a_1\cdots a_t=\sigma_0\sigma_1\cdots\sigma_t$. All even perfect numbers are known to be of the form $n=2^{q-1}(2^q-1)$, where 2^q-1 is prime, so that the σ_i are the a_i in reverse order. The question presents itself whether there may exist odd perfect numbers for which analogously the σ_i are a mere rearrangement of the a_i. In this paper it is proved by elementary methods that no odd perfect numbers of this type can exist. (Received January 17, 1947.)

118. E. G. Straus: *On the existence of square-free numbers of the form* ax^2+b.

It is shown that there are infinitely many square-free integers among the numbers of the form ax^2+b, $x=1, 2, \cdots$, where a, b are integers whose g.c.d. (a, b) is square-free. An elementary counting process used in the proof yields an asymptotic formula for a number $S(n)$ of square-free numbers of the form ax^2+b, $x \leqq n$. The method of proof can be used for several generalizations. (Received January 7, 1947.)

119. R. D. Wagner: *The generalized Laplace equations in a function theory for commutative algebras.*

The author considers the generalization of the theory associated with the Laplace equation $\partial^2 u/\partial x^2+\partial^2 u/\partial y^2=0$ in the theory of functions of a complex variable to that of functions in a commutative linear associative algebra with a principal unit. The usual theory extends readily if the algebra is a Frobenius algebra. Using matric methods introduced by Ward (Duke Math. J. vol. 7 (1940)), the author obtains the generalized Cauchy-Riemann equations as necessary and sufficient conditions for analyticity of a function in the algebra A of order n. The generalized Laplace equations are then obtained as necessary and sufficient conditions that a function u of n variables shall be a component of an analytic function. One form of the generalized Laplace equations is equivalent to the statement that the Hessian matrix of u shall be the parastrophic matrix of some number of A. (Received December 10, 1946.)

Analysis

120. Richard Bellman: *On the boundedness of solutions of nonlinear differential and difference equations.*

The purpose of the author is to discuss the behavior of solutions of the nonlinear system of differential equations: $dx_i/dt=\sum_{j=1}^{N}a_{ij}x_j+f_i(x_1, x_2, \cdots, x_N, t)$, $i=1, 2, \cdots, n$, as $t \to \infty$, under various restrictions upon the matrix (a_{ij}), the functions $f_i(x, t)$, and the initial values. The more general case where the right-hand side contains the derivatives dx_i/dt is also considered. The three methods used are the method of successive approximations, the fixed-point method due to Birkhoff and Kellogg, and the method of approximating to a differential equation by a difference equation. Analogous results are derived for nonlinear difference equations. (Received January 22, 1947.)

121. Stefan Bergman: *Functions satisfying linear partial differential equations and their properties.*

The author investigates functions $\psi(\theta, \mathrm{H})$ of two real variables which satisfy the equation $(-\mathrm{H})^s\psi_{\theta\theta}+\psi_{\mathrm{HH}}=0$ for $\mathrm{H}\leqq 0$, and $(-\mathrm{H})^s\psi_{\theta\theta}+\psi_{\mathrm{HH}}=0$, for $\mathrm{H}\geqq 0$, $s>-2$, $\psi_{\theta\theta}\equiv(\partial^2\psi/\partial\theta^2)$, $\cdots$. In this paper the initial-value problem is considered. To this end $\psi(\theta, \mathrm{H})$ is expressed in terms of $T^{(0)}(\theta)$ and $T^{(1)}(\theta)$, the prescribed values of ψ and $\partial\psi/\partial\mathrm{H}$ respectively on the line $\mathrm{H}=0$. It is shown that the function ψ satisfying the conditions $\psi(\theta, 0)=0$, $\psi_{\mathrm{H}}(\theta, 0)\equiv[\partial(\theta, \mathrm{H})/\partial\mathrm{H}]_{\mathrm{H}=0}=T^{(1)}(\theta)$, $\theta^{(0)}\leqq\theta\leqq\theta^{(1)}$, where $T^{(1)}(\theta)$ is an analytic function of the real variable θ, can be written in the form $2\pi^2 i(2+s)\psi(\theta, \mathrm{H}) = \mathrm{H}\int_{\chi=0}^{1}\int_{t=0}^{\pi}\int_C(1-\chi)^{(s+2)^{-1}}T^{(1)}(\vartheta)dt\,d\chi d\vartheta/(\vartheta-\theta)[1+4(2+s)^{-2}(\vartheta-\theta)^{-2}(-\mathrm{H})^{s+2}\chi\sin^2 t]$, $\mathrm{H}<0$, and a similar expression holds for $\mathrm{H}>0$. C is a simple closed curve in the regularity domain of $T^{(1)}(\vartheta)$ (considered as a function of the complex variable $\vartheta=\theta+i\Theta$), which curve includes the interval $\theta^{(0)}\leqq\theta\leqq\theta^{(1)}$ of the real axis. An analogous formula holds for the solution ψ satisfying the condition $\psi(\theta, 0)=T^{(0)}(\theta)$, $\psi_{\mathrm{H}}(\theta, 0)=0$. Using these formulae, the author investigates the connections which exist between the location and the nature of the singularities of $T^{(0)}(\vartheta)$ and