ABSTRACTS OF PAPERS

The abstracts below are abstracts of papers presented by title at the Fifty-Third Summer Meeting of the American Mathematical Society. Abstracts of papers presented in person at that meeting will be included in the report of the meeting which will be published in the November issue of this BULLETIN.

Abstracts are numbered serially throughout this volume.

ALGEBRA AND THEORY OF NUMBERS

297. A. A. Albert: *Power-associative rings.* I.

A nonassociative ring is said to be *power-associative* if every quantity of A generates an associative subring. It is proved that if the characteristic of A is zero (and counterexamples are given otherwise) then A is power-associative if and only if $xx^2 = x^2x$, $(x^2x)x = x^2x^2$ for every x of A. If A is commutative, the characteristic need only be prime to 30. If A is any ring in which the equation $2x = a$ has a unique solution x, attach to A a commutative ring $A^{(+)}$ which is the same additive group as A, but whose product $x \cdot y$ is defined in terms of the product xy of A by $2(x \cdot y) = xy + yx$. Similarly attach $A^{(-)}$ defined by the product $(x, y) = xy - yx$. Use the decomposition relative to an idempotent in $A^{(+)}$ to obtain a decomposition relative to an idempotent for any power-associative ring, with more complete results in the case of *flexible* rings, that is, rings such that $x(yx) = (xy)x$. Commutative algebras of shrinkability index two are discussed and a structure theory given for the class of algebras in which $x^2y^2 + (xy)^2 = x(xy^2) + y(yx^2)$. The second part of this paper will be concerned with the structure of those flexible power-associative algebras A such that either $A^{(+)}$ is a semisimple Jordan algebra or $A^{(-)}$ is a semisimple Lie algebra. (Received July 21, 1947.)

298. Richard Bellman and H. N. Shapiro: *On the probability that k integers are relatively prime.*

The authors consider the problem of determining the number of lattice points in the n-dimensional region, $1 \leqq x_i \leqq m_i$, $i = 1, 2, \cdots, n$, with the property that the greatest common divisor of the n-uple $(x_1, x_2, \cdots, x_n)$ is 1. The m_i are allowed to tend to ∞ independently, and it is shown that, as might be expected on the basis of a simple heuristic argument, the density of these lattice points is $1/\zeta(k)$, where $\zeta(s)$ is the Riemann zeta-function. Various cases of this problem had already been considered by Lehmer and Chowla. The number of lattice points in the above region possessing the property that every 2-uple (x_i, x_j) has greatest common divisor equal to one, $x_i \neq x_j$, is also determined. The method can be extended to yield similar results for spherical, and general classes of symmetrical, regions, but does not seem to be applicable to the general convex region. (Received June 10, 1947.)

299. R. H. Bruck: *An application of loop theory to geometry.* Preliminary report.

If (P) is a property of loops, the loop G will be said to have property (P′) if and only if every loop isotopic to G has property (P). The following properties of multiplicative loop G are considered: (A): $xy=1$ when $yx=1$, where 1 is the unit; (B): $(xx)x=x(xx)$ for every x of G; (C): every element of G generates a group; and it is shown algebraically that the corresponding properties (A′), (B′), (C′) are equivalent. Thus, in geometrical language, a loop has property (C′) if and only if it defines a hexagonal net, or Sechseckgewebe (Blaschke and Bol, *Geometrie der Gewebe*, Berlin, 1938)—where, for present purposes, all continuity assumptions are inappropriate. The result will be applied to simplify and extend some geometrical work of Ruth Moufang (*Zur Struktur der projektiven Geometrie der Ebene*, Math. Ann. vol. 105 (1931) pp. 538–601). (Received July 21, 1947.)

300. Leonard Carlitz and Eckford Cohen: *Cauchy products of divisor functions in* $GF[p^n, x]$.

For given arithmetic functions $\phi(A)$, $\psi(A)$, where $A \in GF[p^n, x]$, the Cauchy product $\zeta=\phi\cdot\psi$ is defined by means of $\sum\phi(A)\psi(B)=\zeta(F)$, where $A+B=F$. Let $r>0$ be fixed. Three cases are considered according as $\deg A=\deg B=r$; $\deg A=r>\deg B$; deg A, $\deg B<r$; these are referred to as C_1, C_2, C_3, respectively. The functions considered in this paper are all of the form $\sum_0^r a_i\delta_i(A)$, where $\delta_i(A)$ denotes the number of divisors of A of degree i. In every case the Cauchy product is also a divisor function. Relative to C_3 the set of divisor functions forms a ring which is a direct sum of fields. Applications are made to the representation of a polynomial in quadratic forms with coefficients in $GF(p^n)$. (Received July 11, 1947.)

301. Eckford Cohen: *Sums of an even number of squares in* $GF[p^n, x]$. II.

In a previous paper (Duke Math. J. vol. 14 (1947)) the author considered the following problem: Let $\alpha_1, \cdots, \alpha_{2s}$ be nonzero elements of $GF(p^n)$, $p\neq 2$, such that $\alpha_1+\cdots+\alpha_l=0$ $(1\leqq l\leqq 2s)$, and let F be a polynomial of $GF[p^n, x]$ of degree less than $2k$. It was then required to find the number of solutions of $F=\sum_1^{2s}\alpha_i X_i^2$ in polynomials $X_1, \cdots, X_{2s}$ where $X_1, \cdots, X_l$ are primary of degree k and $X_{l+1}, \cdots, X_{2s}$ are arbitrary of degree less than k. The excluded case, namely $l=0$, is treated in the present paper. The result for this case is found to differ considerably from the result obtained in the previous paper. Thus, for $F\neq 0$, the number of solutions is found to be $R_{s-1}(F, \mu)=(p^{ns}-\mu)p^{n(s-1)(2k-1)}\sum_{z=0}^{k-1}\mu^z p^{-nz(s-1)}g_z(F)$, where $g_z(F)=\delta_z(F)-\delta_{2k-z-1}(F)$, and $\delta_i(F)$ is the number of primary divisors of F of degree i. A similar result is obtained for $F=0$. (Received June 1, 1947.)

302. R. P. Dilworth: *An imbedding theorem for semimodular lattices.*

The following theorem is proved: *Let L be a lattice with bounded chains over which an integer-valued semimodular functional ρ is defined. Then L is a sublattice of a semimodular point lattice M with rank function r such that $r(a)=\rho(a)$ for all a in L.* This theorem has as a corollary: *Every finite lattice is a sublattice of a finite semimodular point lattice.* (Received June 27, 1947.)

303. Samuel Eilenberg and Saunders MacLane: *Cohomology and Galois theory.* I. *Symmetry of algebras and Teichmüller's cocycle.*

Teichmüller (Deutsche Mathematik vol. 5 (1940) pp. 138–149) has considered central simple algebras A over a field N which have the property that every automorphism of a finite group Q of automorphisms of N can be extended to an automorphism of A. Each such algebra (termed a "Q-symmetric algebra") determines a 3-dimensional cohomology class of Q with coefficients in the multiplicative group of N. The cohomology classes are characterized as those composed of cocycles which become coboundaries when regarded as cocycles of a larger field $K \supset N$ normal over the field P of elements of N invariant under Q. If G and S are respectively the Galois groups of K over P and N respectively, then any central simple A over N can be represented as a crossed product $A = (f, S, K)$, where f is a 2-cocycle (factor set) of S in K. Such a crossed product is Q-symmetric if and only if there is a function h of two variables in G with values in K which agrees with f on the subgroup S and which has its coboundary δh a function only of arguments in $G/S = Q$. Under these circumstances, δh is the Teichmüller cocycle of A. (Received June 5, 1947.)

304. Saunders MacLane: *Symmetry of algebras over a number field.*

Let K be a finite normal extension of the algebraic number field k, with Galois group Q. A central simple algebra A over K is Q-symmetric if every automorphism of Q can be extended to an automorphism of A. If B is a central simple algebra over the ground field k, its scalar extension B_K is always Q-symmetric in this sense; hence any algebra A similar to a B_K may be termed trivially Q-symmetric. It is shown in this paper that there exist Q-symmetric algebras not trivially Q-symmetric, for suitable fields K. Further, it is proved that the group of Q-symmetric algebra classes, modulo the subgroup of trivially Q-symmetric classes, is cyclic. The order s of this cyclic group is determined as the greatest common divisor of all $S(p)$, where $S(p)$ is the number of factors in K of the prime divisor p of k. The proofs depend on the invariants of an algebra over a number field. (Received June 23, 1947.)

305. B. H. Neumann: *A group connected with ordered semigroups.*

In a fully ordered semigroup in which a product is greater than each factor every well-ordered set of elements (in the given semigroup order) generates a well-ordered semigroup. In this latter every element has only a finite number of factors out of the former. A multiplication is defined between well-ordered sets of elements. With this multiplication they form a group, the null-set being the unit element. The group is commutative if and only if the semigroup is communitative. Its order is at least the cardinal of the continuum. (Received April 23, 1947.)

306. Sam Perlis: *A note on the radical of an ideal.*

If A is an associative algebra or ring and B is an ideal of A, the radical of B is proved to be the intersection of B with the radical of A. The Jacobson definition of the radical is used. This result is applicable several places in the literature where it is proved under special hypotheses. (Received July 25, 1947.)

307. J. O. Reynolds: *On the irreducibility of certain polynomials.*

G. Pólya (Jber. Deutschen Math. Verein. vol. 28 (1919) pp. 31–40) proved the following theorem: If for n integral values of x, the integral polynomial $f(x)$ of degree n has values which are different from zero and, without regard to sign, less than a certain bound which depends only on n, then $f(x)$ is irreducible in the rational domain. This result was improved for positive definite polynomials by H. Ille (Jber. Deutschen Math. Verein. vol. 35 (1926) pp. 204–208), for arbitrary polynomials by Tatuzawa

(Proc. Imp. Acad. Tokyo vol. 15 (1939) pp. 253–254) and finally by A. T. Brauer and G. Ehrlich (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 844–856). In this paper a still lower bound is obtained, if the n considered values of $f(x)$ are positive. (Received July 28, 1947.)

308. H. E. Salzer: *An "empirical theorem" which is true for the first 618 cases, but fails in the 619th.*

The statement that integers of the form $10n+6$, $n\geqq 0$, are the sum of four non-negative tetrahedral numbers, that is, $m(m+1)(m+2)/6$, $m\geqq 0$, holds for $n=0, 1, 2, \cdots$ up to $n=617$, fails for $n=618$, and has been verified for $n=619$ up to 2000. Thus this property of the number 618, of being the sole exception to that statement among all numbers ending in 6 which are not greater than 2006, affords an example of an empirical theorem which holds as far as the first 618 cases, before failing in the 619th. (This may encourage those seeking to disprove Fermat's last theorem, since in the latest results to date mentioned by H. S. Vandiver, Amer. Math. Monthly vol. 53 (1946) p. 575, it has been verified for exponents $3+n$, $n=0, 1, 2, \cdots$, up to 615, which are the first 616 cases.) This investigation of numbers $10n+6$ was performed by suitable grouping into pairs of addends, each addend being the sum of two tetrahedrals, and by employing special stencil-like devices for the addition of a number to hundreds of other numbers simultaneously. Every case was checked in two or more ways, involving the use of independently designed stencils. (Received June 10, 1947.)

309. R. D. Schafer: *Note on derivation algebras.*

A derivation D of a non-associative algebra $\mathfrak{A}$ determines a derivation Δ of the (associative) transformation algebra $T(\mathfrak{A})$ as follows: $T\rightarrow T\Delta=[T, D]=TD-DT$ for T in $T(\mathfrak{A})$. Results resembling those for automorphism groups in Bull. Amer. Math. Soc. vol. 53 (1947) pp. 573–583 are obtained. Application is made to derivations of semi-simple (non-associative) algebras over nonmodular fields. (Received July 1, 1947.)

310. P. A. Smith: *The extension of local groups.*

Call a topological local group G extendable if it is locally isomorphic to a topological group. Let N be a closed normal local subgroup of G. It is shown that if G/N and N (together with certain automorphisms of N) are extendable, then so is G. By taking N as the center, one obtains a new proof of the existence of Lie groups in the large. (Received July 3, 1947.)

311. G. E. Wall: *Notes on binomic equations over an E_m-field.* I.

An E_m-field is a field of characteristic 0 over which the mth cyclotomic polynomial is irreducible. A. Capelli has established necessary and sufficient conditions for the irreducibility of a binomic equation over a field of characteristic 0 (cf. Math. Ann. vol. 54 (1901) pp. 602–603). By means of this criterion the following theorem is proved: The polynomial x^m-a, irreducible over the E_m-field R, is reducible over $R(\epsilon)$ (ϵ=primitive mth root of unity) if, and only if, (i) m is even and (ii) $a=b^2P$, where $b\in R$ and P is a product of different prime numbers dividing m (condition (ii) is to be modified if 8 does not divide m). G. Darbi (Annali di Matematica pura ed applicata (4) vol. 4 (1926)) considered x^m-a over the rational field and settled the cases, m odd, $m=2^n$, or m quadratfrei. H. Hopf and H. Schwerdtfeger (not yet published) proved (i) to be necessary over any E_m-field by consideration of the structure of the full linear permutation group. Extending this method, all Galois groups of x^m-a over R, possible under the conditions of the theorem, are established. (Received May 1, 1947.)

312. G. E. Wall: *Notes on binomic equations over an E_m-field.* II.

Let x^m-a be irreducible over the E_m-field R, and reducible over $R(\epsilon)$ (ϵ=primitive mth root of unity), and let $\alpha^m=a$. The form of the coefficient a is determined so that $R(\alpha, \epsilon)$ has a given Galois group $\mathfrak{G}$ over R (out of a class of admissible groups $\mathfrak{G}$ established in part I of this paper). Firstly, using the criteria of G. Darbi for normal binomic equations over an E_m-field (Annali di Matematica pura ed applicata (4) vol. 4 (1926)), the form of a is determined so that $\mathfrak{G}$ is of given order. These results are then applied to certain binomic subfields of $R(\alpha, \epsilon)$ in order to distinguish between groups $\mathfrak{G}$ of equal order. (Received May 1, 1947.)

ANALYSIS

313. R. P. Boas: *Some complete sets of analytic functions.*

In the following theorems are generalized results of Ibragimov (Bull. Acad. Sci. URSS. Sér. Math. vol. 11 (1947) pp. 75–100). Let $f(z)$ be analytic in $|y|<\pi$ and of period 2π. Let η and ζ be positive numbers with $\eta<\pi$, $\zeta\leqq\pi-\eta$. Let a set of lower density ζ/π of the Fourier coefficients of positive index of $f(z)$ not vanish. Let $\alpha_n=\beta_n+i\gamma_n$ be a sequence of complex numbers, $0\leqq\beta_n<2\pi$. Then $\{f(z+\alpha_n)\}$ is complete in $|z|<\zeta$ in the following cases. (1) The set $\{\gamma_n\}$ has a limit point in $|y|<\eta$. (2) The function $f(z)$ is entire, of order ρ and type σ_1, and $2\sigma_1<\rho \liminf n|\gamma_n|^{1-\rho}$. (Received July 22, 1947.)

314. R. P. Boas and K. Chandrasekharan: *Derivatives of infinite order.*

Let $f(x)$ have derivatives of all orders in (a, b). The following theorems are proved. (1) If $f^{(n)}(x)\to g(x)$ for each x in (a, b), where $g(x)$ is finite, then $g(x)=Ae^x$. (2) If $f(x)$ belongs to a Denjoy-Carleman quasi-analytic class in the open interval (a, b), and $\lim_{n\to\infty} f^{(n)}(x_0)=L$ exists for a single x_0 in (a, b), then $f^{(n)}(x)\to Le^{x-x_0}$ in (a, b). These theorems answer in the affirmative questions raised by V. Ganapathy Iyer (J. Indian Math. Soc. N.S. vol. 8 (1944) pp. 94–108) and remain true if $f^{(n)}(x)\to g(x)$ in a more general sense. If $\{\lambda_n\}$ is a given sequence of constants, the following theorems are proved. (3) Let (*) $\lim_{n\to\infty} f^{(n)}(x)/\lambda_n=g(x)$, $a\leqq x\leqq b$. If $\liminf |\lambda_{n-1}/\lambda_n|=0$ and (*) holds uniformly, $g(x)\equiv 0$ in (a, b). If $\liminf |\lambda_{n-1}/\lambda_n|>0$ and (*) holds dominatedly, $g(x)=Ae^{Bx}$. (4) If $\limsup n^{-1}|\lambda_n|^{1/n}<\infty$ and (*) is true for each x in $a<x<b$, then $g(x)=Ae^{Bx}$. (Received June 26, 1947.)

315. V. F. Cowling: *Some results for factorial series.*

Let $f(z)=\sum_{n=0}^{\infty}a_n n!/z(z+1)\cdots(z+n+1)$ with abscissa of convergence $\tau<\infty$. Let $l-1<h<l$ where l is integral and positive but otherwise arbitrary. Denote by D the domain in the w-plane $\psi_2\leqq\operatorname{Arg}(w-h)\leqq\psi_1$ where $0<\psi_1\leqq\pi/2$ and $-\pi/2\leqq\psi_2<0$. Let $a(w)$ be a function regular in D, with the possible exception of the point at infinity, for which $a(n)=a_n$, $n=0, 1, \cdots$. Suppose for $w=h+Re^{i\psi}$ in D and $R\geqq R_1$ that $|a(h+Re^{i\psi})|\leqq R^k \exp(-LR\sin\psi)$ where k is arbitrary and $0<L<2\pi$. Then