THE OCTOBER MEETING IN NEW YORK

The four hundred twenty-eighth meeting of the American Mathematical Society was held at Hunter College, New York City, on Saturday, October 25. The attendance was over two hundred, including the following one hundred eighty-five members of the Society:

Milton Abramowitz, Leonidas Alaoglu, E. B. Allen, C. B. Allendoerfer, R. L. Anderson, R. G. Archibald, L. A. Aroian, W. A. Asprey, P. T. Bateman, F. P. Beer, Stefan Bergman, Lipman Bers, Gertrude Blanch, R. P. Boas, Salomon Bochner, G. L. Bolton, A. D. Bradley, H. W. Brinkmann, Paul Brock, A. B. Brown, Hobart Bushey, J. H. Bushey, S. S. Cairns, J. D. Campbell, K. Chandrasekharan, Herman Chernoff, J. A. Clarkson, I. S. Cohen, L. W. Cohen, T. F. Cope, Richard Courant, A. P. Cowgill, W. H. H. Cowles, A. R. Craw, J. E. Crawford, H. B. Curry, E. H. Cutler, M. D. Darkow, F. H. Davidson, A. S. Day, Bernard Dimsdale, M. P. Dolciani, C. H. Dowker, Y. N. Dowker, Arnold Dresden, R. J. Duffin, James Dugundji, W. F. Eberlein, Samuel Eilenberg, Paul Erdös, J. M. Feld, E. J. Finan, N. J. Fine, Emanuel Fischer, R. M. Foster, K. O. Friedrichs, Gerald Freilich, Abe Gelbart, B. P. Gill, O. E. Glenn, H. E. Goheen, V. D. Gokhale, A. W. Goodman, Saul Gorn, M. J. Gottlieb, W. H. Gottschalk, M. C. Gray, Harriet Griffin, E. J. Gumbel, F. C. Hall, P. R. Halmos, Carl Hammer, Gerald Harrison, K. E. Hazard, Alex Heller, Erik Hemmingsen, L. S. Hill, Einar Hille, Joseph Hilsenrath, W. M. Hirsch, A. J. Hoffman, T. R. Hollcroft, E. M. Hull, Witold Hurewicz, S. A. Joffe, Fritz John, R. A. Johnson, A. W. Jones, Aida Kalish, Hyman Kamel, E. L. Kaplan, Samuel Kaplan, M. E. Kellar, D. E. Kibbey, J. R. Kline, E. G. Kogbetliantz, E. R. Kolchin, B. O. Koopman, H. K. Kutman, A. W. Landers, M. K. Landers, F. X. Larkin, J. R. Lee, Solomon Lefschetz, Joseph Lehner, Howard Levi, M. A. Lipschutz, E. R. Lorch, Lee Lorch, A. W. McMillan, Brockway McMillan, L. A. MacColl, Saunders MacLane, Henry Malin, A. J. Maria, M. H. Maria, Imanuel Marx, W. S. Massey, F. I. Mautner, K. A. Morgan, F. J. Murray, D. S. Nathan, C. A. Nelson, Morris Newman, P. B. Norman, E. R. Ott, O. G. Owens, J. C. Oxtoby, J. S. Oxtoby, L. G. Peck, A. M. Peiser, Anna Pell-Wheeler, C. R. Phelps, Everett Pitcher, E. L. Post, Hans Rademacher, R. C. Rand, Irving Reiner, Daniel Resch, R. R. Reynolds, Moses Richardson, J. F. Ritt, S. L. Robinson, Saul Rosen, Herman Rubin, H. E. Salzer, Arthur Sard, Samuel Schechter, M. M. Schiffer, Lowell Schoenfeld, Pincus Schub, Abraham Schwartz, G. E. Schweigert, I. E. Segal, Max Shiffman, James Singer, P. A. Smith, J. J. Sopka, E. H. Spanier, George Springer, E. P. Starke, N. E. Steenrod, J. J. Stoker, Walter Strodt, M. M. Sullivan, Olga Taussky-Todd, C. B. Tompkins, Hing Tong, L. V. Toralballa, Annita Tuller, H. E. Vansant, W. G. Warnock, W. R. Wasow, G. C. Webber, Louis Weisner, A. M. Whelan, M. E. White, A. L. Whiteman, Norbert Wiener, John Williamson, G. M. Wing, J. W. Young, J. J. Zeig, H. J. Zimmerberg, Leo Zippin.

On Saturday morning there were two sections, one for papers in Analysis in which Professor Norbert Wiener presided, and one for papers in Algebra, Topology, and Logic in which Professor H. B. Curry presided.

On Saturday afternoon Professor C. B. Allendoerfer of Haverford

College gave an address on *Global theorems in Riemannian geometry.* President Einar Hille presided. At the beginning of this session a motion by Professor C. A. Nelson was passed unanimously authorizing the Secretary to express to President George N. Shuster of Hunter College the thanks and appreciation of the Society for the use of the facilities of Hunter College for this meeting.

Abstracts of the papers read follow below. Papers whose abstract numbers are followed by the letter "*t*" were read by title. Professor Godement was introduced by Professor Claude Chevalley, Dr. Lorentz by Professor R. P. Agnew, and Professor Wright by Professor Saunders MacLane.

Algebra and Theory of Numbers

1t. Reinhold Baer: *The role of the center in the theory of direct decompositions.*

It has been noticed for some time that the center plays a fundamental part in the theory of direct decompositions of operator groups and loops. In particular it has been found that the existence of isomorphic refinements of direct decompositions can be assured by imposing conditions which refer solely to the center. It is the object of the author to give an explanation for these phenomena by proving that quite generally the validity of the refinement theory, properly stated, in an operator loop is a consequence of the validity of this theory in the center. (Received August 12, 1947.)

2t. Leonard Carlitz: *The singular series for sums of squares of polynomials.*

In several papers the writer had considered the problem of the number of representations of a polynomial in $GF[p^n, x]$ as the sum of first an even number of squares, and later as the sum of an odd number of squares. In a previous (unpublished) communication, the writer constructed an analog of the Hardy singular series for squares of rational integers. By this method it was possible to treat the even and odd cases simultaneously. The present paper is an improvement of the previous one in several respects. For one thing it is now shown that the singular series gives the correct result for one square. Secondly, in the even case it is shown to yield a theorem of Cohen (Duke Math. J. vol. 14 (1947) pp. 251–267). Finally, the evaluation of the series in the odd case is considerably simplified. This leads in particular to an elegant result on the number of *primitive* solutions. (Received August 22, 1947.)

3t. Leonard Carlitz: *Cauchy products of arithmetic functions in* $GF[p^n, x]$.

In a previous paper (L. Carlitz and Eckford Cohen, *Cauchy products of divisor functions in* $GF[p^n, x]$, Duke Math. J. vol. 14 (1947)) three kinds of Cauchy multiplication were defined; in particular the C_3-product of two arithmetic functions is defined by $\sum f(A)\, g(B)$, the summation extending over all polynomials A, B of degree less than r such that $A+B=M$. It follows that the set of functions $\{f\}$ forms a commutative ring $\boldsymbol{R}$; the divisor functions of the previous paper form a proper sub-ring of $\boldsymbol{R}$. It is proved that $\boldsymbol{R}$ is a direct sum of fields. Among the other results of the present paper may be mentioned the construction of a set of "orthogonal functions" ϵ_{GH} which seem well-suited for applications. Application is made to certain problems on sums of squares. It is now possible to handle certain cases excluded by the "singular series" method. (See preceding abstract.) (Received August 22, 1947.)

4. O. E. Glenn: *Phases of the inverse theory of rational invariants.*

If an n-ary quantic $f=a_x^m=a_1x_1^m+\cdots$ has arbitrary coefficients, and a set $g_1(y)$, $\cdots$, $g_s(y)$ of rational, integral polynomials in variables $y_1, y_2, \cdots$ is cogredient with the set $(a_1, \cdots, a_s)$ under the induced or unspecified transformations on the x_i, then $\Delta=(g_1\partial/\partial a_1+\cdots+g_s\partial/\partial a_s)$ is an invariant operator. If c_x^r is a covariant of degree g in the a_i, then $F=\Delta^{g-1}c_x^r$ is a ground-quantic, that is, is of degree unity in the a_i. But F in general has three types of factors, universal covariants of linear transformations on functions of y_i (and of x_i), universal covariants of cognate transformations on x_i, and an irreducible invariant function whose coefficients are the a_i. The transformations can then be determined from these universal covariants and ultimately from c_x^r. Applied to a case in which $g_1=(y_3-y_1)^m$, $g_2=(y_1-y_2)^m$, $g_3=(y_2-y_3)^m$, and f is ternary, a factor of F is $d=(y_3-y_1)x_1+(y_1-y_2)x_2+(y_2-y_3)x_3$. Therefore the group is a certain formal group on x_i, in four parameters, under which any ternary quantic will have a fundamental system. The group can be generalized. (Received July 14, 1947.)

5t. V. L. Klee: *On a conjecture of Carmichael.*

Carmichael (Bull. Amer. Math. Soc. vol. 28 (1922) pp. 109–110) conjectured that for no integer N can the equation $\phi(x)=N$ have exactly one solution, ϕ denoting Euler's totient. In support of his conjecture, this note shows by elementary methods that each N for which there is a unique solution must satisfy various restrictions, one of which implies $N>10^{400}$. (Received August 28, 1947.)

6. Saunders MacLane: *The group of abelian group extensions.*

The structure of the group Ext (T, G) of all abelian group extensions of the abelian group G by a torsion group T is studied. Assume T denumerable and p-primary, let T_n be the subgroup of all elements of order at most p^n, ωT the subgroup of all elements of infinite height. Then Ext (T, G) may be mapped onto the group of all homomorphisms of the system of groups $T_n \leftrightarrow T_{n+1}$ into the system of groups $G/p^nG \leftrightarrow G/p^{n+1}G$. The kernel of the mapping is the set $\mathrm{Ext}_f(T, G)$ of extensions trivial on finite subgroups of T; this kernel may be mapped onto the group of all homomorphisms of ωT into $G_\infty/\alpha G$, where G_∞ is the inverse limit of the sequence G/p^nG, and αG the natural image of G in G_∞. The kernel of this second mapping is isomorphic to $\mathrm{Ext}(\omega T, \omega G)$. To the latter group the same reduction may be applied again. (Received September 22, 1947.)

7t. F. I. Mautner: *The Peter-Weyl theorem for discrete groups.*

It is shown that any countable discrete group G has a system S of irreducible unitary representations (in Hilbert space) which is complete in $L_2(G)$ in the sense of the Parseval equation. In general S does not contain all irreducible unitary representations and the theory for L_2 can differ strongly from the theory of almost periodic functions on G. Use is made in the proofs of a decomposition theory for Hilbert spaces and rings of operators by von Neumann and recent results by Segal. The Murray-von Neumann theory of relative trace leads to a theory of group-characters, in particular their completeness in the space of class-functions in L_2. If the number of finite classes of conjugate elements of G is finite then S is finite (though G may have infinitely many inequivalent unitary irreducible representations) and orthogonality relations for group-characters hold. An improvement for discrete groups of the Gelfand-Raikov-Segal existence theorem for irreducible representations follows. (Received September 17, 1947.)

8*t*. C. E. Rickart: *One-to-one mappings of rings and lattices.*

Let $\mathfrak{R}$ and $\mathfrak{S}$ denote arbitrary rings and let $\phi(r)$ denote a one-to-one, multiplicative mapping of $\mathfrak{R}$ onto $\mathfrak{S}$. If $\mathfrak{R}$ is a Boolean ring, then $\phi(r)$ is additive as well as multiplicative. Also $\phi(r)$ is additive if $\mathfrak{R}$ contains a family $\mathfrak{J}$ of minimal right ideals with the following properties: (1) $Rr=(0)$, for each $R\in\mathfrak{J}$, implies $r=0$; (2) Each R is of dimension greater than one over the division ring of all endomorphisms of R which commute with each endomorphism induced in R through right multiplication by elements of $\mathfrak{R}$. It is also shown that one-to-one, meet preserving mappings of certain distributive lattices onto a distributive lattice are also join preserving. (Received September 15, 1947.)

9*t*. R. M. Robinson: *The critical numbers for unsymmetrical approximation.*

If ξ is an irrational number, the modulus of approximability from the right, $M^+(\xi)$, is defined as the least upper bound of the values of μ for which the inequality $0<A/B-\xi<1/\mu B^2$ has infinitely many solutions. In a similar way, $M^-(\xi)$ is defined measuring the approximability of ξ from the left. The number ξ is called *critical* if there is no other irrational number ξ' for which $M^+(\xi')<M^+(\xi)$ and $M^-(\xi')<M^-(\xi)$. A sequence of integers $r_1, r_2, r_3, r_4, \cdots$ will be called *derivable* if ultimately just two different numbers occur in the sequence, and these are consecutive integers k and $k+1$. The sequence $s_1, s_2, s_3, \cdots$ giving the number of $k+1$'s between consecutive k's is called the derivative. It is proved that *the number ξ is critical if and only if its expansion as a continued fraction has the form $\xi=[q_0, q_1, \cdots, q_{u-1}, 1, r_1, 1, r_2, 1, r_3, 1, \cdots]$, where the sequence $r_1, r_2, r_3, \cdots$ either has infinitely many derivatives, or its highest derivative is a sequence $t_1, t_2, t_3, \cdots$ such that $t_n\to k$, where k is a positive integer, or $t_n\to\infty$*. This paper is a continuation of *Unsymmetrical approximation of irrational numbers*, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 351–361. (Received August 22, 1947.)

10. Lowell Schoenfeld: *On the function which enumerates the number of partitions of an integer into distinct kth powers.* Preliminary report.

Let $F_k(x)=\prod_{m=1}^{\infty}(1+x^{m^k})=1+\sum_{m=1}^{\infty}P_k(m)x^m$ and $f_k(x)=\prod_{m=1}^{\infty}(1-x^{m^k})^{-1}=1+\sum_{m=1}^{\infty}p_k(m)x^m$. Then $P_k(m)$ is the number of partitions of m into kth powers with distinct summands and $p_k(m)$ is the number of unrestricted partitions of m into kth powers. Wright (Acta Math. vol. 63 (1934) pp. 143–191) obtained an exact transformation equation for $f_k(x)$ which exhibits the behavior of $f_k(x)$ in the neighborhood of its singularities at the rational points of the unit circle. In this paper, the author obtains a similar equation for $F_k(x)$. This can be done in two ways: either derive this equation directly in a manner similar to that used by the author for $f_k(x)$ (Duke Math. J. vol. 11 (1944) pp. 873–887), or observe that $F_k(x)=f_k(x)/f_k(x^2)$ and use the transformation equation for $f_k(x)$. The second method, though formal in character, is not trivial since as simple an equation as possible is required. While most of Wright's work is applicable to the determination of an aysmptotic series for $P_k(m)$, the oscillatory character of certain coefficients makes it difficult at the present time to ascertain the dominant term of this series. (Received October 8, 1947.)

11*t*. A. R. Schweitzer: *Remarks on groups of ordered dyads.*

The author constructs sets of postulates for groups of ordered dyads of which postulates 1, 2, 3 are the same as postulates 1, 2, 4 of a previous paper (Bull. Amer. Math. Soc. Abstract 52-11-352). Definition: $\alpha\beta\times\gamma\delta=\lambda\mu$ means: There exist ξ, η, ζ in S such that $\alpha\beta=\xi\eta$, $\gamma\delta=\eta\zeta$, $\lambda\mu=\xi\zeta$. This definition is effective relative to postulates 1, 2, 3, 4 and 1, 2, 3, 4′, 5′; 4. $\alpha\beta$, $\gamma\delta$ in T imply the existence of ξ, η in S such that $\alpha\beta=\eta\gamma$, $\gamma\delta=\beta\xi$, $\alpha\xi=\eta\delta$. 4′. $\alpha\beta$, $\gamma\delta$ in T imply the existence of ξ, η, ζ in S such that $\alpha\beta=\xi\eta$, $\gamma\delta=\eta\zeta$; $\alpha\beta$, $\alpha'\beta'$, $\alpha''\beta''$ in T imply the existence of ξ, η, ζ, τ in S such that $\alpha\beta=\xi\eta$, $\alpha'\beta'=\eta\zeta$, $\alpha''\beta''=\zeta\tau$. 5′. $\xi\eta=\xi'\eta'$, $\eta\zeta=\eta'\zeta'$ imply $\xi\zeta=\xi'\zeta'$. Basic for the proof of sufficiency of each set of postulates is the theorem: If $\alpha\beta\times\gamma\delta=\lambda\mu$ then $\alpha\beta=\alpha'\beta'$ implies $\alpha'\beta'\times\gamma\delta=\lambda\mu$; $\gamma\delta=\gamma'\delta'$ implies $\alpha\beta\times\gamma'\delta'=\lambda\mu$; $\lambda\mu=\lambda'\mu'$ implies $\alpha\beta\times\gamma\delta=\lambda'\mu'$. The independence of the first part of 4′ is discussed. It is assumed that the relation symbolized by "=" is reflexive, symmetric and transitive. Postulates 1, 2, 3, 4′, 5′ permit interpretation of dyads as rational numbers, excluding zero. (Received September 20, 1947.)

12. L. V. Toralballa: *On Newton's interpolation series in n variables.*

The finite difference analog of Taylor's expansion theorem in several variables is the theorem: If $f(x_1, x_2, \cdots, x_s)$ possesses partial derivatives of all kinds and orders at the origin, and moreover the limit as i approaches infinity of the square root of the absolute value of $\partial^i f/\partial x_1^{i_1}\partial x_2^{i_2}\cdots\partial x_s^{i_s}$ is zero for all sets $i_1, i_2, \cdots, i_s$ such that $i_1+i_2+\cdots+i_s=i$, then $f(x_1, x_2, \cdots, x_s)=f(0, 0, \cdots, 0)+(x_1\Delta_{x_1}+x_2\Delta_{x_2}+\cdots+x_s\Delta_{x_s})f(0, 0, \cdots, 0)+(1/2!)(x_1\Delta_{x_1}+x_2\Delta_{x_2}+\cdots+x_s\Delta_{x_s})^{(2)}f(0, 0, \cdots, 0)+\cdots$, where $(x_1\Delta_{x_1}+x_2\Delta_{x_2}+\cdots+x_s\Delta_{x_s})^{(i)}f(0, 0, \cdots, 0)=\sum(i!/\alpha_1!\,\alpha_2!\cdots\alpha_s!)\cdot x_1^{(\alpha_1)}x_2^{\alpha_2}\cdots x_s^{\alpha_s}\Delta_{x_1}^{\alpha_1}\Delta_{x_2}^{\alpha_1}\cdots\Delta x_s^{\alpha_s}f(0, 0, \cdots, 0)$, where the summation extends over all the compositions of i, zeros being allowed. This is proved by induction. As a special case one has $x_1^{m_1}x_2^{m_2}\cdots x_s^{m_s}=\sum_{i_1=1}^{m1}\sum_{i_2=1}^{m2}\cdots\sum_{i_s=1}^{m_s}(x_1^{(i_1)}/i_1!)(x_2^{(i_2)}/i_2!)\cdots(x_s^{(i_s)}/i_s!)\Delta_{x_1}^{i1}\,\Delta_{x_2}^{i2}\cdots\Delta_{xs}^{i_s}0^{m_1}0^{m_2}\cdots 0^{m_s}$. (Received September 22, 1947.)

13. G. C. Webber: *Non-existence of odd perfect numbers of the form* $p^\alpha 3^{2\beta}p_1^{2\beta_1}p_2^{2\beta_2}p_3^{2\beta_3}$.

Sylvester (C. R. Acad. Sci. Paris vol. 106) proved that an odd integer n, divisible by 3, cannot be perfect unless it contains at least five prime divisors. In this paper it is proved that the number of prime divisors of such n must be at least six. The principal tools used are, first, results concerning the form of the cyclotomic polynomial when that polynomial is divisible by an odd prime; second, certain inequalities obtained by consideration of $\sigma(n)<2n$ and $\sigma(n)>2n$. (Received September 11, 1947.)

14*t*. E. M. Wright: *Equal sums of like powers.*

Let $s\geqq 2$ and let $P(k, s)$ be the least value of j such that the equations $\sum_{i=1}^{j}a_{i1}^{h}=\sum_{i=1}^{j}a_{i2}^{h}=\cdots=\sum_{i=1}^{j}a_{is}^{h}$ $(1\leqq h\leqq k)$ have a nontrivial solution in integers. Prouhet (C. R. Acad. Sci. Paris vol. 33 (1851) p. 225) showed that $P(k, s)\leqq s^k$. The author proves that (i) $P(k, s)\leqq(k^2+k+2)/2$, (ii) if k is odd, $P(k, s)\leqq(k^2+3)/2$, (iii) $P(2, s)=3$, (iv) $P(3, s)=4$. (Received September 25, 1947.)

Analysis

15. R. J. Duffin: *On a question of Hadamard concerning super-biharmonic functions.*

A function w satisfies the equation $\Delta^2 w(x, y) = p(x, y)$ in a region of the plane and vanishes together with its first derivatives on the boundary of the region. Consider then the assertion "$p \geqq 0$ implies $w \geqq 0$." In mechanical language the assertion can be translated to say that if a positive distribution of pressure p is applied to the surface of a thin plate which is clamped on its boundaries, then the displacement w of the plate is nowhere negative. It is shown that the assertion is not true for some regions. Of special interest is the region bounded by two infinite parallel lines. If a positive pressure is applied over a sufficiently limited portion of such a strip, then the resultant displacement at large distances is essentially a damped sine wave whose wavelength and decrement depend only on the breadth of the strip and not on the particular form of the distribution of pressure. (Received September 22, 1947.)

16. Paul Erdös: *On bounded polynomials.*

Let x_i^n $(1 \leqq i \leqq n)$ be a point group in $(-1, 1)$. It has *property* A if for every nth degree polynomial f satisfying $|f(x_i^m)| < 1$ for $m > (1+c_1)n$ we have $|f(x)| < c_2$ in $(-1, 1)$, with c_2 independent of f and n but dependent on c_1. Bernstein proved the roots of the Tchebycheff polynomials, Zygmund the roots of the Legendre polynomials, have property A. The following property is proved necessary and sufficient for property A: if the y_i^n are the projections of the x_i^n upon the upper half of the unit circle then for any sequence of positive numbers a_n approaching infinity and sequence of arcs of length a_n/n the number of y_i^n on the nth arc is greater than $a_n/\pi - o(a_n)$. (Received September 11, 1947.)

17t. Roger Godement: *Les fonctions de type positif et la théorie des groupes.*

Let C be a locally compact topological group. A function ϕ on G is said to be of positive type if $\sum_{ij}\alpha_i\bar{\alpha}_j\phi(s_i^{-1}s_j) \geqq 0$ for all systems of points $(S_1, \cdots, S_n)$ of the group and of complex numbers $\alpha_1, \cdots, \alpha_n$. Continuous functions of positive type are shown to be closely related to representations of G by unitary operators in Hilbert space; by this method, the existence of irreducible unitary representations of G is established. These results duplicate to a large extent those of Gelfond and Raĭkow (*Irreducible unitary representations of arbitrary locally bicompact groups*, Rec. Math. (Mat. Sbornik) N.S. vol. 13 (1943)). The spectral theory of functions of positive type is generalized from the case of the group of real numbers to that of arbitrary locally compact groups, yielding a generalization to the latter case of Beurling's theorem. It is shown that a continuous ϕ of positive type is (in a unique way) the sum of an almost periodic function of positive type and a function ψ of positive type such that the mean value of $|\psi|^2$ is 0. Special properties of functions of positive type belonging to L^2 are studied. (Received October 23, 1947.)

18t. J. W. Lawson: *A sufficiency theorem for the Plateau problem.*

A hypersurface V_{n-1} is considered, embedded in the Euclidean space S_n, and the analogue of the Plateau problem set up in parametric form. For the multiple integral problem, a field is constructed using the sufficiency condition which is the main result of the paper. Hypersurfaces satisfying the Euler differential condition being termed minimal, the theorem is: If there exists one point in the space, through which no tangent hyperplane passes, then the minimal hypersurface is minimizing. (Received August 22, 1947.)

19t. Norman Levinson: *The asymptotic nature of solutions of linear differential equations.*

Let x denote a column vector. Let $dx/dt=(A+\Phi+R)x$ where (1) A is a constant matrix with simple characteristic roots μ_i; (2) $\Phi(t)$ is a complex matrix such that $\Phi\rightarrow 0$ as $t\rightarrow\infty$ and the elements of Φ are all of bounded variation over $(0,\infty)$; (3) the elements of the complex matrix, R, are all absolutely integrable $(0,\ \infty)$. Let the characteristic roots of the matrix $A+\Phi$ be denoted by $\lambda_i(t)$. If the real parts of μ_i are all distinct (or if certain less restrictive conditions are met) then there exist n independent vectors $x^{(k)}(t)$ each a solution of the system such that $x^{(k)}(t)\sim C^{(k)}$ exp $(\int_0^t\lambda_k(t)dt)$ as $t\rightarrow\infty$. Each $C^{(k)}$ is a characteristic vector of A associated with the root μ_k. (Received August 29, 1947.)

20t. G. G. Lorentz: *Tauberian theorems and Tauberian conditions.*

Let $\sigma_m=a_{m1}s_1+a_{m2}s_2+\cdots$ be a regular Silverman-Toeplitz sequence-to-sequence transformation A by which a sequence s_n is evaluable A to σ if lim $\sigma_n=\sigma$. A sequence $n_1<n_2<\cdots$ of positive integers determines a gap condition: $u_n=0$ when $n\neq n_1, n_2, \cdots$. A sequence $c_1, c_2, \cdots$ of positive constants determines an order condition: $u_n=o(c_n)$. Relations among the sequences n_k and c_k are determined such that if $\sum u_n$ converges whenever $\sum u_n$ is evaluable A and satisfies the gap (or order) condition, then $\sum u_n$ converges whenever $\sum u_n$ is evaluable A and satisfies the order (or gap) condition. For the Cesàro methods and the Euler-Abel power series method, there is given a characterization of the sequences c_k for which the order condition $u_n=O(c_n)$ is a Tauberian condition which, with summability, implies convergence. (Received September 30, 1947.)

21t. M. E. Munroe: *Homomorphisms on Banach spaces.*

If G is a closed linear subspace of the Banach space E, then G^* (the conjugate space to G) is algebraically isomorphic to the factor space E^*/Γ where $\Gamma=\{f\mid f(x)=0$ for $x\in G\}$. The natural homomorphism $T(E^*)=G^*$ is continuous and open when E^* has its norm topology and G^* has its norm topology. The above results are already known. It is shown that T is continuous and open with respect to the weak and weak* topologies in E^* and G^*. T is not closed for any of the usual topologies. In fact, it does not even map all closed convex sets into closed sets. However, T preserves weak* closure of linear subspaces. Remarks on weak and weak* convergence are appended, including an example to show that in any infinite-dimensonal E^* there is a directed set weak* convergent to zero but containing no bounded subset having zero as a point of accumulation. The same thing is true of weak convergence. (Received September 11, 1947.)

22. J. C. Oxtoby: *On the ergodic theorem of Hurewicz.*

Let μ be a measure in a set S and assume that S is σ-finite. Let T be a 1:1 measurable transformation of S onto itself, and let $F(X)$ be a countably additive finite function of a measurable set. Write $F_n(X)=\sum_0^n F(T^iX)$, $\mu_n(X)=\sum_0^n\mu(T^iX)$, and let $\phi_n(x)$ be a point function such that $\int_X\phi_n d\mu_n=F_n'(X)$, where F_n' is the absolutely continuous part of F_n with respect to μ_n. Such a function exists by the Radon-Nikodym theorem.

It is shown (1) that in any case the limit $\phi(x)=\lim_{n\to\infty}\phi_n(x)$ exists except on a set N such that $\mu(T^iN)=0$ for $i\geqq 0$, and (2) that if $\sum_0^\infty \mu(T^iX)=0$ or ∞ for each measurable set X, then $\mu(T^iN)=0$ for every integer i, $\phi(x)$ is integrable, and $\phi(x)=\phi(Tx)$ except on a set whose images all have measure zero. If in addition $F(X)=0$ for every invariant set of measure zero, then $\int_X \phi d\mu = F(X)$ for every invariant set of finite measure. Result (2) generalizes Hurewicz' theorem (Ann. of Math. (2) vol. 45 (1944) pp. 192–206) and directly implies the ergodic theorem of Halmos (Proc. Nat. Acad. Sci. U.S.A. vol. 32 (1946) pp. 156–161, Theorem 1). It is derivable from either of these theorems. The proof of (1) requires supplementary considerations. (Received September 10, 1947).

23. Everett Pitcher: *A proof of lower semicontinuity.*

A short direct proof is made of a general theorem of McShane (Duke Math. J. vol. 2 (1936) pp. 597–616, Theorem 3.1) on lower semicontinuity of simple integrals of the calculus of variations. (Received September 19, 1947.)

24. Arthur Sard: *The remainder in approximations by moving averages.*

Let $g(s, t)$ be a function which, for each real number t in a set T, is of bounded variation in s on each finite s interval. Given any function $x(s)$, put $y(t)=\int_{-\infty}^{\infty} x(s)d_s g(s, t)$ and $R[x]=x(t)-y(t)$, $t\in T$. Integrals on infinite ranges are to be understood either as Lebesgue-Stieltjes integrals or as improper Lebesgue-Stieltjes integrals. $R[x]$ exists if $y(t)$ and $x(t)$ exist and are finite for each $t\in T$. Assume that $R[x]$ exists and vanishes whenever $x(s)$ is a polynomial of degree $n-1\geqq 0$. Put $k(s', t)=R[\psi_{s'}]$, $\psi_{s'}(s)=0$ if $s\leqq s'$, $\psi_{s'}(s)=p(s, s')$ if $s>s'$, $p(s, s')=(s-s')^{n-1}/(n-1)!$. Let $x(s)$ be a function with absolutely continuous $(n-1)$th derivative on each finite s interval. Put $R^*[x]=\int_{-\infty}^{\infty} x^{(n)}(s')k(s', t)ds'$ and $I=\int_{-\infty}^{\infty} d_s g(s, t)\int_0^s p(s, s')x^{(n)}(s')ds'$. A necessary and sufficient condition that $R[x]$ and $R^*[x]$ exist and be equal is that I exist and that the order of integration in I be invertible, $t\in T$. A sufficient condition is that $\int_{-\infty}^{\infty} |x^{(n)}(s')|M(s', t)ds'$ be finite, $t\in T$, where $M(s', t)=\int_{-\infty}^{s'} |p(s, s')||d_s g(s, t)|$ if $s'\leqq 0$ $M(s'\ t)=\int_{s'}^{\infty} |p(s, s')||d_s g(s, t)|$ if $s'>0$. The sufficient condition also implies that $R^*[x]$ exists as a Lebesgue-Stieltjes integral. (Received September 19, 1947.)

25. I. E. Segal: *Operator algebras associated with locally compact groups.* Preliminary report.

Certain uniformly closed self-adjoint algebras of bounded operators on Hilbert spaces, determined either by a locally compact group or by a locally compact group of transformations of a locally compact space, with an invariant measure, are studied, from a viewpoint suggested by quantum kinematics. Some results for arbitrary groups are obtained, and it is also shown that the self-adjoint elements of the algebra determined by the covering group of the inhomogeneous de Sitter group constitute a model for the bounded observable associated with an elementary particle, with the following properties: (1) the usual relativistic and non-relativistic models are limiting cases, (2) space-time is relativistically invariant and space is discrete, (3) the rest-mass and spin of the particle are automatically determined. (Received September 2, 1947.)

26*t*. J. E. Wilkins: *The isoperimetric problem of Bolza with finite side conditions.*

In view of recent improvements in the theory of the problem of Bolza, due primarily to McShane and Hestenes, it is now possible to give a much simpler treatment of the problem of Bolza with auxiliary finite conditions than that originally given by Bower (*The problem of Lagrange with finite side condition*, Contributions to the calculus of variations 1933–1937, University of Chicago Press). (Received September 15, 1947.)

27*t*. G. M. Wing: *Summability with a governor of integral order.*

The series $\sum_{j=0}^{\infty} a_j$ is said to be summable by means of a governor of order k (k a positive integer), or (G, k) summable, if $\lim_{n\to\infty} \sum_{j=0}^{n} s_j p_{n-j}(k)/p_n(k+1) = \sigma$ exists and $\lim_{n\to\infty} p_n(k)/p_n(k+1) = 0$, where $s_n = \sum_{j=0}^{n} a_j$, $p_n(0) = |a_n|$, and $p_n(k) = \sum_{j=0}^{n} p_j(k-1)$ $(k = 1, 2, \cdots)$. The method (G, k) reduces to the method (G) of G. Piranian (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 882–889) in the case $k=1$, and is shown to preserve all of the characteristic properties of that method. It is proved that (G, k) includes the Cesàro method of order k, and several conditions are given under which the new method is stronger than the corresponding Cesàro mean. One of these yields a new proof of a theorem of Piranian on (G) summability of certain series derived from polynomials. An example is given to show that there exists no integer k' such that all Cesàro summable series are (G, k') summable. (Received August 29, 1947.)

Applied Mathematics

28*t*. Stefan Bergman: *An inversion formula for the integral operator of the second kind in the theory of a compressible fluid.*

The stream function $\psi(\lambda, \theta)$ of a compressible fluid flow (see NACA, TN972) can be represented in the region $[|\lambda| < 3^{1/2}|\theta|]$ in the form $\psi = \operatorname{Im} \int_{C_2} \mathbf{E}(Z, \bar{Z}, t) \cdot f(2^{-1}Z(1-t^2))dt/(1-t^2)^{1/2}$. Here $Z = \lambda + i\theta$, $\bar{Z} = \lambda - i\theta$, $\mathbf{E} = A_1\mathbf{E}^{(1)} + [2^{-1}Z(1-t^2)]^{2/3} \cdot A_2\mathbf{E}^{(2)}$, where A_κ are suitably chosen (complex) constants satisfying the condition $\operatorname{Im}\{i^{-4/3}A_1\bar{A}_2\} \neq 0$, and $\mathbf{E}^{(\kappa)} = \sum_{n=0}^{\infty} q^{(n,\kappa)}/(-t^2Z)^{n-1/2+2\kappa/3}$, $q^{(n,\kappa)}$ satisfying the equations $(n+2/3)q_{\bar{Z}}^{(n,k)} + q_{Z\bar{Z}}^{(n+1,\kappa)} + Fq^{(n+1,\kappa)} = 0$, $n = 0, 1, 2, \cdots$; C_2 is a curve in the complex $t-1$ plane connecting $t=-1$ and $t=1$. Let $\lim_{\lambda\to 0-}\psi = \chi_1(\theta)$, $\lim_{\lambda\to 0-}(\partial\psi/\lambda M) = \chi_2(\theta)$. The line $\lambda = 0$ (that is, $M=1$) is the transonic line. Then f can be expressed in terms of the χ_κ, namely, $f(\zeta) = 3^{1/2}[2\pi \operatorname{Im} (D_0\bar{D}_2)]^{-1}\{(-2i\zeta)^{7/6}\bar{D}_0\int_0^1 \chi'(\sigma)\tau^{-1/3}d\tau + (-2i\zeta)^{1/6} \cdot \sum_{\kappa=1}^{2} \bar{D}_\kappa \int_0^1 [\chi_\kappa(\sigma) - \alpha_0^{(\kappa)}](1-\tau)^{-1}\tau^{-2/3}d\tau - 3^{-1}(-2i\zeta)^{-1/6}\sum_{\kappa=1}^{2} \bar{D}_\kappa \int_0^1 [\chi^*(\sigma) - 3\alpha_0^{(\kappa)}] \cdot (1-\tau)^{-4/3}\tau^{-2/3}d\tau + (-2i\zeta)^{1/6}[\bar{D}_0\alpha_0^{(\kappa)} - \sum_{\kappa=1}^{2}\bar{D}_\kappa\alpha_0^{(\kappa)}]\}$, where $\chi_\kappa^*(\sigma) = \int_0^\sigma \omega^{-2/3}\chi_\kappa(\omega)d\omega$, $\sigma = -2i\zeta(1-\tau)$, and the D_κ and $\alpha_0^{(\kappa)}$ are suitably chosen constants. (Received October 25, 1947.)

29. Lipman Bers: *Subsonic gas flows past a straight cascade.* Preliminary report.

The author deals with a subsonic gas flow past a straight cascade of profiles. The gas is assumed to satisfy Chaplygin's simplified density-speed relation. The problem of finding the flow is reduced to a mapping problem involving the domain exterior to the profiles of the cascade and a certain standard Riemann surface. The mapping problem is shown to be equivalent to a nonlinear integral equation, similar to the one occurring in the theory of a single airfoil (cf. Bers, NACA TN No. 1006, 1946). Under the assumption that the profiles are convex, it is shown that the integral equation possesses a solution. (Received October 23, 1947.)

30*t*. Eric Reissner: *Note on the membrane theory of shells of revolution.*

The solution of the equilibrium equations of the membrane theory of shells of revolution is expressed in terms of a stress function which is slightly more general than Neményi's stress function (Bygningstatiske Meddelelser (1936)). The essential difference between Neményi's stress function and Pucher's stress function (Beton und Eisen vol. 33 (1934)) is set in evidence. It appears that Neményi's function is somewhat more convenient for the applications than is Pucher's function. Finally, a formula is derived which expresses explicitly Pucher's function in terms of Neményi's function. (Received September 9, 1947.)

GEOMETRY

31t. Reinhold Baer: *Projectivities of finite projective planes.*

Consider a projectivity p of the finite projective plane P in which the theorem of Desargues may or may not be valid. Denote by m the order of the projectivity p, by $n+1$ the number of points on a line in P and by F the set of fixed elements of the projectivity p. The author obtains relations between the arithmetic properties of m and n and the geometric properties of the configuration F. A typical example of such a relation is the following proposition: If m is a power of a prime q, and if F is not a complete subplane of P, then q is a divisor of $(1+n+n^2)n(n^2-1)$. (Received August 12, 1947.)

32t. Salomon Bochner: *Curvature and Betti numbers.*

If on a compact space S_n with a positive definite Riemannian metric the Ricci (mean) curvature is positive, and if the space is conformally flat, or more generally if the deviation from conformal flatness is a small fraction of the Ricci curvature, then all Betti numbers vanish, $B_p=0$, $1\leqq p\leqq n-1$. On the other hand, for n even, it is easy to give a criterion under which $B_{2l}\geqq 1$, $l=1, 2, 3, \cdots$. (Received September 17, 1947.)

33t. Edward Kasner: *Trajectories and catenaries.*

The author discusses analogies and differences between the theories of general trajectories and general catenaries in an arbitrary field of force. In each case, the focal locus is a circle. This circle becomes a straight line only when the field is constant or elastic. However, while the angular ratio is 1:1 for trajectories, it becomes 3:2 for catenaries. Again the fundamental ratio 1:3 for special (rest) trajectories becomes 1:2 for special catenaries in any field. Analogous results exist for brachistochrones, for velocity curves, and for systems S_k. The locus of the centers of all the focal circles for a given lineal element is determined. (Received September 26, 1947.)

LOGIC AND FOUNDATIONS

34t. Ira Rosenbaum: *Enumeration of first order classes, ordered q-ads, and q-adic relations in a universe of m objects* $(1, 2, \cdots, m)$.

Let $a^b \vdots (c-1)_m$ be the coefficient of a^b in the m-adic number-system expression of $c-1$. Define the nth ordered q-ad of an m-membered universe as follows: $q_n^m=(\{n, 1\}, \{n, 2\}, \cdots, \{n, q\})$ where $\{n, k\}=1+(m^{k-1} \vdots (n-1)_m)$ and $1\leqq n\leqq m^q$. Defining q-adic relations (predicates, propositional functions) extensionally as classes of ordered q-ads, $R_n^{q(m)}=(\{n, 1\}, \{n, 2\}, \cdots, \{n, m^q\})$, where now $\{n, k\}=1+(2^{k-1} \vdots (n-1)_2)$, and k denotes the ordered q-ad, q_k^m, and this q-ad is to be a member of $R_n^{q(m)}$ if and only if $\{n, k\}=1$. Here $1\leqq n\leqq 2^{m^q}$. A 1–1 relation is thus established between the numbers

1, 2, $\cdots$, m^q and the m^q ordered q-ads and between the numbers 1, 2, $\cdots$, 2^{m^q} and the 2^{m^q} q-adic relations definable in an m-membered universe. By methods like those of Bull. Amer. Math. Soc. Abstract 53-5-265, it is possible to determine the number n of any order q-ad whose elements and order are known, and the number n of any q-adic relation whose member q-ads are known. For $q=1$, the formula for $R_n^{q(m)}$ defines the nth class of individuals in an m-membered universe. $R_n^{q(m)}(q_k^m)$ is analytic or contradictory as $\{n, k\}$ is 1 or 2, that is $R_n^{q(m)}(q_k^m) \equiv \{n, k\}$. (Received August 18, 1947.)

35t. Ira Rosenbaum: *On a method of determining the nth rational p-valued q-adic relation in a universe with $\aleph_0$ elements.*

Both ordered q-ads of a universe with $\aleph_0$ elements and the rational numbers are effectively enumerable, the enumeration of the latter proceeding in familiar fashion, the fraction k/t occurring in the $[(k+t-1)(k+t-2)/2+k]$th place, fractions equal to a preceding fraction being considered subsequently struck from the array. A denumerably infinite array of columns, each containing a denumerable infinity of entries, may be envisaged, the value $\{n, r\}$ in the rth row and nth column being the rth digit in the p-adic decimal representation of the nth rational number. The denumerably infinite set of values $\{n, r\}$ obtained for fixed n and variable r defines the nth rational p-valued q-adic relation, $\{n, r\}$ indicating the degree in which the rth ordered q-ad is a member of the nth rational relation. Since every rational number when converted into a decimal, in general, becomes a terminated, or a pure repeating, or a mixed repeating decimal, one can represent the table of the nth rational p-valued q-adic relation in abbreviated form. Thus the table of the 18th rational 10-valued q-adic relation is $\dot{1}4285\dot{7}$. (Received September 8, 1947.)

36t. Ira Rosenbaum: *On a method of determining the number n of any arbitrary rational p-valued q-adic relation defined by a table.*

Any denumerably infinite set of integers coinciding with the digits of a terminated, pure or mixed repeating p-adic decimal may be taken as defining a rational p-valued q-adic relation of a universe with a denumerable infinity of individuals. To determine the number n of the rational p-valued q-adic relation in question requires two major steps, (1) transformation of the decimal into a fraction, this being reduced to its lowest terms, and (2) determination of the order number n of the resulting rational number in the enumeration of rational numbers. The number n of the resulting rational, say k/t, will be not greater than $[(k+t-1)(k+t-2)/2+k]$ and its determination hence a finite process. The transformation of the decimal into a fraction utilizes only known mathematical methods, for example, when $p=10$, the transformation of a mixed repeating decimal into a fraction is accomplished by multiplying the integer formed by the non-repeating digits by that formed by the repeating digits and then subtracting the former integer, subsequently dividing the result by $(10^r-1)10^s$ where s is the number of non-repeating digits, r the number of repeating digits. Related methods suffice in other cases. Reduction to lowest terms is also, generally, readily accomplished. (Received September 8, 1947.)

37t. Ira Rosenbaum: *On a method of enumerating ordered q-ads in a universe with a denumerable infinity ($\aleph_0$) of elements.*

In a universe with a denumerable infinity ($\aleph_0$) of elements, the set of ordered q-ads, for every finite integer q, is denumerable, since the number of q-ads is then $\aleph_0^q$ and $\aleph_0^q = \aleph_0$. Each q-ad may be regarded as a point with integral coordinates in a space of q dimensions. In order to enumerate the ordered q-ads of a universe of the type in question one first writes down the integers in the single q-partition of q, thus obtaining the first ordered q-ad. One then determines the number of q-partitions of $q+1$, the actual q-partitions of $q+1$, an order among these q-partitions, the permutations of each partition, an order among these permutations, and then lists the 1st, 2nd, $\cdots$ permutations of the first q-partition of $q+1$, of the second q-partition of $q+1, \cdots$, of the last q-partition of $q+1$; and similarly for $q+n$, $n=2, 3, 4, \cdots$. In this way every ordered q-ad will be listed once and only once. One may assign an order number to each ordered q-ad as it is written down and hence speak of the nth ordered q-ad in a universe with $\aleph_0$ elements. The above process embraces that of Gödel (Monatshefte für Mathematik und Physik vol. 37 (1930)). (Received September 8, 1947.)

38t. Ira Rosenbaum: *On absolute junctions of p-valued q-adic relations in an m-membered universe.*

A set of m^q integers, W_i, $1 \leqq W_i \leqq p$, may be regarded as defining a q-adic p-valued relation of an m-membered universe; each of the m^q integers W_i indicates the degree in which the ith ordered q-ad q_i^m is a member of the relation; the number n of the relation is determinable from the condition $n-1=\sum_{j=0}^{c} W_j \cdot p^{c-1-j}$ where $c=m^q$. Conversely, given the number n of a q-adic p-valued relation, the integer in the kth row of its table is $\{n, k\}=1+(p^{k-1} \vdots (n-1)_p)$, $1 \leqq k \leqq m^q$. One has: $R_n=R_s \cdot \equiv \cdot (k)(\{n, k\}=\{s, k\})$, $R_n \subset R_s \cdot \equiv \cdot (k)(\{n, k\} \geqq \{s, k\}) R_n \cap R_s \cdot = \cdot R_i$, $i=2x(k)(\{x, k\}=\max(\{n, k\}, \{s, k\}))$, $R_n \cup R_s \cdot = \cdot R_j$, $j=2x(k)(\{x, k\}=\min(\{n, k\}, \{s, k\}))$, $-R_n=R_{p^c-n+1}$. For $q=1$, the definitions reduce to those of p-valued class-junctives; for $p=2$, to those of the traditional (two-valued) theory of classes and relations. For $p=2$, $q=1$, classes defined by a table with only a single "1" in them are *unit* classes; for $q \geqq 2$, similarly, we obtain unit-relations (that is, those with only a single q-ad as member); and similarly for $p \geqq 3$. (Received September 8, 1947.)

39t. Ira Rosenbaum: *On converses, relative products and sums, powers and multiples of p-valued binary relations in an m-membered universe.*

Given two arbitrary p-valued binary relations R_n and R_t, letting $\{u, v\}=1+(p^{v-1} \vdots (u-1)_p)$, $x=i+[r/m]$ (where $[r/m]$ denotes the integral part of r/m) unless r/m is an integer z in which case $x=i+m(z-1)$, $y=\rho(r, m)+m(i-1)$ (where $\rho(r, m)$ denotes the remainder obtained on dividing r by m, unless r is exactly divisible by m, when $\rho(r, m)=m$) and adapting and generalizing a device of Schroeder's, one obtains (for the m^2 values of r $(=1, 2, \cdots, m^2)$ and hence of x and y) the m^2 values defining the relative product R_n/R_t and relative sum $R_n \oplus R_t$, by use of the formulae $\sum_{i=1}^{m}(\{n, x\} \cap \{t, y\})$ and $\prod_{i=1}^{m}(\{n, x\} \cup \{t, y\})$ respectively, $\sum$ and $\prod$ denoting p-valued logical sums and products. Using the m^2 values defining R_n/R_t and $R_n \oplus R_t$, determine the numbers of these tables (and relations) by previous methods. Letting $t=n$, one obtains the tables of the *square* R_n^2 and double $2R_n$, of R_n, from the formulae for relative product and sum respectively. Since $R_n^{k+1}=R_n^k/R_n$ and $(k+1)R_n=kR_n \oplus R_n$, powers and multiples of relations are definable generally. Letting $\rho(r, m)$ and $[r/m]$ have their previous significance, the converse, $Cnv'R_n$, of R_n has the table whose values, for the m^2 values of r, are given by $\{n, [r/m]+1+m(\rho(r, m)-1)\}$ and with these values the number of $Cnv'R_n$ is obtainable as previously. (Received September 5, 1947.)

40t. Ira Rosenbaum: *On p-valued q-adic relations in a universe with a denumerable infinity ($\aleph_0$) of elements.*

Although the set of ordered q-ads in a universe with $\aleph_0$ elements is denumerably infinite and, indeed, effectively so, the set of p-valued classes and p-valued q-adic relations is nondenumerably infinite, possessing the power of the continuum. This is easily seen since the number of p-valued q-adic relations in the universe in question is $p^{\aleph_0^q} = p^{\aleph_0} = 2^{\aleph_0}$. It follows that tables defining the p-valued q-adic relations cannot be completely enumerated. One may, however, consider a 1–1 correlation to exist between the real numbers of the interval $0 \leqq x \leqq 1$ and the set of p-valued q-adic relations. One may consider the relation correlated to a given real number, of the range indicated, to be defined by the successive digits in the p-adic decimal representation of the real number in question, the rth digit in the decimal indicating the degree in which the rth ordered q-ad is a member of the relation correlated with the real number. If the real number correlated with a p-valued q-adic relation is irrational, one may term the relation itself *irrational*; otherwise *rational*. The values in the tables defining irrational p-valued q-adic relations can be but incompletely determined, those of rational relations completely determined. (Received September 8, 1947).

41t. Ira Rosenbaum: *On the nth q-adic p-valued relation in an m-membered universe.*

Defining the kth ordered q-ad of an m-membered universe by the relation $q_k^m = (\{k, 1\}, \cdots, \{k, q\})$ where $\{k, i\} = 1 + (m^{i-1} \vdots (k-1)_m)$, generalize the definitions of classes and relations given in a preceding abstract. There any given individual (or ordered q-ad) either was, or was not, a member of a given class (or relation); so that an essentially two-valued theory of classes and relations resulted. The admission of more than these two possibilities (complete membership, complete non-membership) leads to many-valued theories of classes and relations as in the corresponding case of propositional logic. Let 1 denote complete membership, p complete non-membership, and intermediate integers intermediate degrees of membership. Then for the nth q-adic p-valued relation of an m-membered universe $R_n^{p,q(m)} = (\{n, 1\}, \{n, 2\}, \cdots, \{n, m^q\})$ where $\{n, k\} = 1 + (p^{k-1} \vdots (n-1)_p)$, k denotes the ordered q-ad q_k^m and this q-ad is a member of R_n in the degree indicated by $\{n, k\}$. For $q = 1$, the nth p-valued class of individuals in an m-membered universe is obtained. In all cases $R_n^{p,q(m)}(q_k^m) = \{n, k\}$. Given n^q integers i, $1 \leqq i \leqq p$, the number n of the q-adic p-valued relation defined by them is determinable as in Bull. Amer. Math. Soc. Abstract 53-5-265. (Received August 19, 1947.)

42t. Ira Rosenbaum: *The nth genus of rth order k-adic predicate in the functional calculi of higher order.*

The genus of an rth order k-adic predicate is here defined so as to depend solely on the pattern of orders of the arguments of the function. It is less complex, more general, than the concept of logical type. The nth genus of rth order k-adic predicate is either (1) undefined, or (2) is defined as that in which the order of the ith argument, for each i such that $1 \leqq i \leqq k$, is specified by $\{n, i\} = 1 + (r^{i-1} \vdots (n-1)_r)$. It is undefined or defined according as (1) for no value of i is $\{n, i\} = r-1$ or (2) for some i, $\{n, i\} = r-1$ and for no i is $\{n, i\} > r-1$. Given a set of k arguments of suitable orders the genus-number n of a function of those arguments is determinable. The number $G_{r,k}$ of genera of rth order k-adic functions is given by $G_{r,k} = r^k - (r-1)^k$ and by two recursive relations: $G_{r,k} = rG_{r,k-1} + (r-1)^{k-1}$ and $G_{r,k} = G_{r-1,k} + r^k - 2(r-1)^k$

$+(r-2)^k$. The number of genera of rth order functions of degree not greater than k is given by $\sum_{i=1}^{r} G_{r,i}=((r^{k+1}-1)/r-1)-(r-1)^{k+1}/(r-2)$. The number of genera of k-adic functions of orders not greater than r is given by $\sum_{i=1}^{r} G_{i,k}$ which is algebraically calculable with the help of the Euler-Maclaurin sum formula for $\sum_{m=1}^{n} m^k$. (Received August 18, 1947.)

Statistics and Probability

43t. Mark Kac: *On the distribution of certain Wiener functionals.* Preliminary report.

Let $x(t)$, $x(0)=0$, $0\leq t<\infty$, be elements of the Wiener space and $V(x)$ a non-negative piecewise continuous function satisfying certain additional conditions. Let $\sigma(\alpha;t)$ be the probability (Wiener measure) that $\int_0^t V(x(\tau))d\tau<\alpha$. It is shown that the double real Laplace transform $\int_0^\infty\int_0^\infty \exp(-u\alpha-st)d\sigma_\alpha(\alpha;t)dt$ is equal to $\int_{-\infty}^{\infty}\psi(s,u;x)dx$, where $\psi(s,u;x)$ is the (unique) fundamental solution of the differential equation $\psi''/2-(s+uV(x))\psi=0$ subject to the conditions $\psi(x)\to 0$ as $x\to\pm\infty$, $|\psi'(x)|<M$, $\psi'(-0)-\psi'(+0)=2$. The special cases $V(x)=x^2$, $V(x)=|x|$, $V(x)=(1+\text{sign } x)/2$ treated elsewhere by a somewhat different method are used as illustrations of the present method. Generalizations to more-dimensional Wiener processes and to Markoffian processes other than that of Wiener are indicated. (Received August 29, 1947.)

Topology

44. E. H. Spanier: *Borsuk's cohomotopy groups.*

Let X be a compact topological space with dim $X<2n-1$. Borsuk showed how the homotopy classes of continuous maps of X into the n-sphere S^n form a group $\pi^n(X)$, called the nth cohomotopy group of X. In a similar manner the relative Borsuk group $\pi^n(X, A)$ of a pair consisting of a compact space X and closed subset A is defined. The elements of this group are the classes of maps of X into S^n which map A into a point. A coboundary operator, which is a homomorphism from $\pi^n(A)$ to $\pi^{n+1}(X, A)$, is defined, as well as the homomorphism from $\pi^n(Y, B)$ to $\pi^n(X, A)$ induced by a continuous map from (X, A) to (Y, B). The cohomotopy groups with these auxiliary concepts satisfy the Eilenberg-Steenrod axioms for cohomology groups except that the cohomotopy groups are defined only for those integers n such that $n>(\dim X+1)/2$. The Hopf theorems about the classes of maps of an n-dimensional space into S^n and the Steenrod theorems about the classes of maps of an $(n+1)$-dimensional space into S^n are proved in the framework of the cohomotopy groups. These results are summarized in a mixed exact sequence involving cohomotopy and cohomology groups (Received September 10, 1947).

45t. A. H. Stone: *On products of paracompact metric spaces.*

A Hausdorff space is *paracompact* (J. Dieudonné, *Une généralisation des espaces compacts*, J. Math. Pures Appl. vol. 23 (1944) pp. 65–76) if it has arbitrarily fine neighborhood-finite open coverings. It is proved that a metrisable space is paracompact if and only if it has a basis for its open sets consisting of a countable number of families of open sets, the sets in each family being pairwise disjoint. The following two theorems are among those deduced from this characterization. A necessary and sufficient condition for a product of nonempty paracompact metric spaces to be paracompact is that all but a countable number (at most) of the factors be compact. For the product of a given metric space X with *every* compact Hausdorff space to be normal, it is necessary that X be paracompact; sufficiency here was proved by Dieudonné. (Received September 26, 1947.)

46. Hing Tong (National Research Fellow): *Some characterizations of normal and perfectly normal spaces.* Preliminary report.

Let R be a topological space. The following theorems are proved: (1) A necessary and sufficient condition for R to be normal is that if ψ and ϕ are respectively upper and lower semicontinuous functions over R such that $\psi \leqq \phi$ (for every point in R) there is a function ρ continuous over R so that $\psi \leqq \rho \leqq \phi$ (for every point in R). (2) The following properties concerning R are equivalent: (α) R is perfectly normal. (β) for every closed set A in R there is a function ϕ continuous over R such that the set of zeros of ϕ is precisely A. (γ) Every upper semicontinuous function over R is the limit of a monotonically decreasing sequence of continuous functions over R. (Received September 20, 1947.)

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