then the hypothesis is that the two configurations lead to the same sensation. Since elements of length appear to be equal, then if the apparent size is determined by a quadratic differential form, it follows that

$$g_{11}d\gamma^2 + g_{22}d\phi^2 + g_{33}d\theta^2 + \cdots = g_{11}d\gamma^{*2} + g_{22}d\phi^{*2} + g_{33}d\theta^{*2} + \cdots ,$$

where the coefficients $g_{ik}(\gamma, \phi, \theta)$ on the left are the same functions as $g_{ik}(\gamma^*, \phi^*, \theta^*)$ on the right. This leads to a non-Euclidean metric in general. The observation that two line elements on the same line of view which give the same $d\gamma$, $d\phi$ are seen as parallel, together with considerations of symmetry, leads to the quadratic differential form

$$ds^2 = M^2(\gamma)(\sigma^2 d\gamma^2 + d\phi^2 + \cos^2 \phi d\theta^2),$$

in which σ is a parameter of the observer which measures the relative effectiveness of convergence as compared with angular displacement in the estimation of length. Further considerations suggest that $M(\gamma) = 1/\sin h\sigma(\gamma + \mu)$, in which case the metric is hyperbolic.

Some of the topics which are treated in considerable detail are the horopter problem (geodesic lines), the alley problem, and rigid transformations of the hyperbolic visual space. In the alley problem with walls sensed as parallel it is pointed out that the meaning of "parallel" is ambiguous. Two cases are treated which correspond to different instructions to the subject. These in general give different results, the difference depending on the geometry chosen. Thus, while it is possible to account for apparently conflicting experimental results, it is also possible to make use of the experimental data to obtain restrictions on the metric. Using the hyperbolic metric, the author calculates the shape of distorted rooms which are congruent to rectangular rooms, that is, rooms with distorted walls and windows which appear (under fixed conditions) to be identical to rectangular rooms with rectangular windows.

Even if further experiments show that the metric derived is not adequate to account for the data, the author's efforts will greatly facilitate the task of determining a better approximation. The point of view developed, together with the many suggestions given, should prove to be of great help in determining the direction to be taken for further theoretical studies and experimental observations.

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Leçons de gêometrie différentielle. Vol. 1. By G. Vranceanu. Bucarest, Rotativa, 1947. 422 pp.

This volume, which is the first of two proposed volumes, is divided into two parts. The first part (Chapters I, II, III) deals with the author's topics of predilection; namely congruences, finite continuous groups, and equivalence problems of congruences. The second part (Chapters IV, V, VI) may be thought of as a report on metric, affine and projective differential geometry of curved spaces. It is written in a style which reminds one of the charm, as well as of the disadvantages, of Felix Klein's lithographed lectures. It may be labelled as one of the most attractive books on geometry, written in the style of pre-epsilon-delta era.

In the first chapter an informal approach is given to the fundamental concepts necessary for the calculus of congruences: Tensor properties, Pfaffians and the corresponding bilinear covariants. Poisson's brackets, and so on.

The second chapter deals with Lie groups. The author starts with a one-parameter Lie group and develops the theory of an r-parameter Lie group with all its fundamental concepts. The classical Lie theorems are explained in an intuitive way using the notion of the neighborhood of identity. The structural tensors C^e_{ab}, C^b_{ab}, C^d_{ab} C^b_{de} are investigated and corresponding topics, such as derived groups, center, characteristic equation, are introduced. A complete study of a G_3 (canonical forms) is presented. The framework of this chapter is strictly a classical one. The modern idea due to Chevalley (Pontrjagin) of considering a Lie group as an entity is not adopted, and consequently the Cartan-Schouten geometry of group space is not studied.

The last section of this chapter is devoted to the concept of primitivity and transitivity of a group with subsequent developments and applications on G_3.

The third chapter is devoted to the problem of equivalence of congruences. The author uses throughout this chapter the calculus with linear Pfaffian forms and gives many examples which illustrate this theory. In particular, he ends this chapter by a thorough study of the invariants of an equation $y''=F(x, y, y')$.

In the fourth chapter the author introduces the elementary concepts dealing with an affine curved space L_n, covariant derivative, parallelism,[1] curvature, and torsion. Moreover, as is his predilection, he introduces a topic which may be described as a geometry of L_n+ennuples.

The investigation is sometimes rather superficial (pp. 224–225),

[1] G. Hessenberg may be thought of as a godfather to the idea of parallelism which was discovered in 1917 by Levi-Civita and independently of Levi-Civita by J. A. Schouten in 1918 (reviewer's remark).

sometimes is dealt with by an unusually thorough treatment (pp. 249–254). A relatively large part is devoted to the Cartan spaces with covariant constant curvature tensor. The statement on "Fermi's theorem" (p. 219) is erroneous—for there is always a coordinate system $\bar{x}^a$ along a given open curve C for which $\bar{\Gamma}^a_{bc}=0$ along C *even in the case if C is not a path.*

In the fifth chapter the author confines himself to just classical results about a Riemann space: geodesics, curvature, the embedding of a V_n in a euclidean space E_N and the Levi-Civita deformation of geodesics. The embedding problem does not go beyond the classical elementary standpoint, where all the osculating spaces, as well as all the normal spaces, are mixed up. Incidentally, the E. E. Levi invariants $I_{klr}\ldots$ are not invariant with respect to parameter transformations of the V_n. The more recent and deeper results of W. Mayer, Tucker, T. Y. Thomas, Schouten-Struik are not discussed. In the last section the author reproduces in his own way the Levi-Civita deformation theory of geodesics. The resulting necessary and sufficient conditions (say $L_2^\nu=0$) are differential equations of second order. The reviewer would like to remark that in the case of a congruence they may be replaced by a system of order 1, say $L_1^\nu=0$.[2]

It seems to the reviewer that the topics being too familiar, the author hastened too much in their treatment, for there are some points in this chapter which have to be corrected: The equation (3), p. 298 is misleading. The formulae (4′) p. 299 and (5) p. 302 lead in the case of a non-definite metric to $|\cos\theta|>1$ (!). Page 305: "En désignant . . . les differentielles des arcs sur deux congruences nulles. . . . " The nonzero arc is uniquely defined (p. 300) only for non-null curves. The last formula on p. 317 is misprinted and the first formula on p. 318 is wrong but this fact does not affect the right result.[3]

The last chapter deals with the projective connection. There are different treatments of this topic: that of the Princeton school including Veblen's and T. Y. Thomas' theories, Cartan's method of the "repère mobile," Schouten-v. Dantzig algorithm which uses homogeneous coordinates. The author confines himself to the introductory

[2] If dealing with a congruence, we have $L_2^\nu=DL_1^\nu+L_1^\alpha D_\alpha i^\nu$, where D stands for covariant derivative along the curve to be deformed and i^ν is its unit tangential vector. (See Hlavatý, *Les courbes de la variété générale à n dimensions*, Mémorial des Sciences Mathématiques, no. 43, Paris, 1934, pp. 47–63.) Hence $L_1^\nu=0$ is a sufficient but not necessary condition.

[3] Some minor mistakes or misprints may be easily checked and corrected by the reader himself.

elements of the Princeton school and Cartan's method and uses the ennuple calculus to investigate thoroughly the one- and two-dimensional spaces. The approach to the projective connection is very intuitive but not a satisfactory one: The author obtains the projective connection "En considérant . . . les espaces euclidiens E_P and E_Q de deux points voisins P et Q et en désignant avec X_0^i les coordonnées de Q dans E_P · · · " (p. 372). This kind of heuristic[4] assumption has to be avoided in a modern mathematical book.[5]

The book is clearly meant for specialists. Any specialist will enjoy it for its large number of topics.

V. HLAVATÝ

[4] The assumption mentioned in the text above is mathematically incomprehensible to the reviewer. It seems to be described fully by the following argument: "Let Q be so close to P that it is incident with E_P, in spite of the fact that $E_P \neq E_Q$." No exact mathematical proof may be based on such contradictory requirements.

[5] The same remark holds for the author's approach to the affine connection.