THE NOVEMBER MEETING IN PASADENA

The four hundred fifty-second meeting of the American Mathematical Society was held at the California Institute of Technology, Pasadena, California, on Saturday, November 26, 1949. Approximately 130 persons attended, including the following 98 members of the Society:

W. G. Bade, M. M. Beenken, E. F. Beckenbach, E. T. Bell, Richard Bellman, H. F. Bohnenblust, S. G. Bourne, Herbert Busemann, W. D. Cairns, A. B. Coble, F. E. Cothran, F. G. Creese, J. H. Curtiss, Tobias Dantzig, P. H. Daus, J. E. Denby-Wilks, C. R. De Prima, R. J. Diamond, S. P. Diliberto, R. P. Dilworth, Avron Douglis, Roy Dubisch, C. L. Dunsmore, L. K. Durst, R. H. Edwards, Arthur Erdélyi, Harley Flanders, G. E. Forsythe, A. L. Foster, W. B. Fulks, W. H. Glenn, G. E. Gourrich, J. W. Green, Hugh Hamilton, A. R. Harvey, E. M. Henderson, M. R. Hestenes, P. G. Hoel, Alfred Horn, D. G. Humm, H. D. Huskey, D. H. Hyers, Rufus Isaacs, E. H. Jacobs, Glenn James, R. C. James, P. B. Johnson, William Karush, Samuel Karlin, A. J. Kempner, R. M. Lakness, Cornelius Lanczos, G. E. Latta, M. B. Lehman, D. H. Lehmer, Jack McLaughlin, A. B. Mewborn, C. B. Morrey, J. W. Odle, L. J. Paige, R. P. Peterson, R. S. Phillips, W. T. Puckett, H. R. Pyle, E. S. Quade, W. C. Randels, I. S. Reed, Julia Robinson, R. M. Robinson, E. B. Roessler, J. B. Rosser, Herman Rubin, Abraham Seidenberg, Max Shiffman, Harold Shniad, I. S. Sokolnikoff, R. D. Specht, D. V. Steed, Robert Steinberg, E. G. Straus, Irving Sussman, J. D. Swift, A.E. Taylor, F.G. Tricomi, A.W. Tucker, R. L. Vaught, S. S. Walters, Morgan Ward, W. R. Wasow, W. W. Weber, P. A. White, R. L. White, A. L. Whiteman, W. M. Whyburn, R. L. Wilder, G. M. Wing, František Wolf, J. R. Ziegler.

There was a general session in the morning for contributed papers and for the invited address *On the stability of solutions of differential equations*, by Professor Richard Bellman of Stanford University. Professor Arthur Erdélyi presided. In the afternoon there were two sectional sessions, at which Professor D. H. Hyers and A. L. Foster presided.

Following the meetings, those present were guests of the Department of Mathematics of the California Institute of Technology at a tea at the Athanaeum.

Abstracts of papers presented at the meeting follow. Those abstracts whose numbers are followed by the letter "*t*" were presented by title. Mr. Schweitzer, whose abstract was introduced by Professor Gabor Szegö, died on January 28, 1945.

ALGEBRA AND THEORY OF NUMBERS

70. Roy Dubisch: *A note on isotopy.*

The purpose of this note is to point out that isotopy of algebras may be handled from the point of view of a type of equivalence of matrices $X=(X_{ij})$ where X_{ij} is a linear form in $x_1, \cdots, x_n$ with coefficients in F. X and Y are called isotopic if X can be obtained from Y by a finite sequence of the usual three operations of ordinary equivalence theory plus the interchange of x_i with x_j, the replacement of x_i by $x_i+\xi x_j$, and the multiplication of x_i by any α in F. This technique is applied to show that every algebra which is not a zero algebra has an isotope with an idempotent. Also, all isotopically commutative and anti-commutative algebras (that is, algebras all of whose isotopes are commutative or anti-commutative respectively) are determined. (Received October 6, 1949.)

71. L. K. Durst: *Apparition of primes in elliptic sequences.* Preliminary report.

A study of the rank of apparition of primes in elliptic divisibility sequences (Morgan Ward, Amer. J. Math. vol. 70 (1948) pp. 31–74) was undertaken using lemniscate functions. These are the simplest elliptic functions admitting a complex multiplication. Computation indicates that for primes of the form $p=4k+3$ the rank is a divisor of $p+1$. This behavior is like that for Lucas' sequences. But for primes $p=4k+1$ no simple arithmetical restriction on the rank has yet been found. (Received October 14, 1949.)

72. Harley Flanders: *Unification of local class field theory.* Preliminary report.

The local class field theory is known to be valid for two types of fields: p-adic fields and formal power series fields over a finite constant field. In the first case, the theory has been developed by both commutative and non-commutative methods; however, in the second case, no commutative method has been given. The author shows how certain results of Hasse on higher ramification can be used to give a common treatment of the fundamental inequality for both cases. Having this, he applies the method of Chevalley's thesis to obtain the desired unified treatment. The methods are entirely commutative. (Received November 25, 1949.)

73. Alfred Horn: *Sentences which are true of direct unions of algebras*

For the terminology see J. C. C. McKinsey, *The decision problem for some classes of sentences without quantifiers*, Journal of Symbolic Logic vol. 8 (1943) pp. 61–76. Every sentence T corresponding to an algebra may be put in prenex form $Q^1_{x_1} \cdots Q^n_{x_n} S(x_1, \cdots, x_n)$ where the Q^i are quantifiers, and the matrix S results from a formula F in the propositional calculus by replacing in F the variables by equations. We say T is in prenex conjunctive form if F is in conjunctive normal form. Let A be a class of similar algebras and suppose Γ_i is an algebra in A for each $i \in I$. Denote by Γ the direct union of the Γ_i. Theorem: Let T be a sentence which has a prenex conjunctive form T' in which all equations are negated. Then if T is true of Γ_i for some $i \in I$, it is true of Γ. If the matrix of T' is a disjunction of inequalities, then T is true of Γ_i for some $i \in I$ if and only if it is true of Γ. Theorem: Let T have a prenex conjunctive form T' in whose matrix each disjunction contains at most one non-negated equation. Then if T is true of Γ_i for each $i \in I$, it is true of Γ. If the matrix of T' is a conjunction of equations, then T is true of Γ_i for each $i \in I$ if and only if it is true of Γ. Theorem: If for all $i \in I$, the algebras Γ_i are the same algebra Γ' and if T has a prenex form in which no universal quantifiers occur, then if T is true of Γ' it is true of Γ. These are the only results of their kind. (Received October 14, 1949.)

74. Robert Steinberg: *A geometric approach to the representations of the full linear group over a Galois field.* II.

In part I of this paper, a basic set of $p(n)$ irreducible representations of $LH(n, q)$, the group of all n-ary linear homogeneous nonsingular substitutions in the Galois field $GF(q)$, was determined through some properties of the associated finite geometry. In this part, through an application of the generating function technique used by Frobenius (G. Frobenius, Berliner Berichte, 1900, pp. 516–534) in his work on the symmetric group, the characters of one of these representations, that of degree $q^{n(n-1)/2}$ are determined explicitly. (Received October 12, 1949.)

75*t*. Robert Steinberg: *The representations of* $LH(3, q)$, $LH(4, q)$, $F(3, q)$, *and* $F(4, q)$.

In this paper, the representations and characters of $LH(3, q)$ and $LH(4, q)$, the groups of all third and fourth degree linear homogeneous nonsingular substitutions in the Galois field $GF(q)$, are determined, and, from them, those of $F(3, q)$ and $F(4, q)$, the corresponding fractional or collineation groups, are deduced. The method consists of the application of Frobenius' method of induced representations to linear representations of suitably chosen subgroups of $LH(3, q)$ and $LH(4, q)$. (Received October 12, 1947.)

76. A. W. Tucker: *Extensions of theorems of Farkas and Stiemke.*

Let S denote a linear subspace of an n-dimensional Euclidean vector space, S^* the orthogonal complement of S, and P the "positive orthant" consisting of all vectors with non-negative components. Then the following theorem and its converse hold: If S intersects P only in vectors whose first k components are zero, then S^* contains some vector of P whose first k components are all positive. For $k=1$ this reduces to a fundamental theorem of J. Farkas concerning the dependence of one homogeneous linear inequality on others [J. Reine Angew. Math. vol. 124 (1902) pp. 1–27] and for $k=n$ to a theorem of Erich Stiemke concerning positive solutions of a system of homogeneous linear equations [Math. Ann. vol. 76 (1915) pp. 340–342]. Let C denote the polyhedral convex "cone" consisting of all vectors that can be expressed as linear combinations of certain given vectors (finite in number) using non-negative coefficients only, and $-C^*$ the negative "polar" cone consisting of all vectors that make a non-acute angle with every vector of C. Then the theorem stated above and its converse hold also when C and $-C^*$ are substituted for S and S^*, respectively. This theorem on cones is closely related to papers on solutions of two-person games by H. F. Bohnenblust, S. Karlin, and L. S. Shapley, and by David Gale and Seymour Sherman [in a forthcoming Annals of Mathematics Study]. It also has application to econometric problems treated by G. B. Dantzig and T. C. Koopmans [in forthcoming *Proceedings of Linear Programming Conference,* Cowles Commission]. (Received October 12, 1949.)

77. A. L. Whiteman: *Theorems on quadratic partitions.*

Let $f(k)$ and $g(k)$ denote the number of distinct solutions of the congruences $x^2+x\equiv k \pmod{p}$ and $x^3+x^2\equiv k \pmod{p}$, respectively. In the present paper theorems of the following type are proved. (1) If p is a prime of the form $3n+1$, then the value of a in the quadratic partition $4p=a^2+3b^2$ is given by $a=-p+\sum_{s=0}^{p-1} f(s^3)$. (2) If p is a prime of the form $7n+1$, then the value of u in the quadratic partition $4p=u^2+7v^2$

is given by $3u = -p + \sum_{s=0}^{r-1} g(s^7)$. The formula $f(k) = 1 + ((4k+1)/p)$, where (m/p) denotes the Legendre symbol, leads at once to Chowla's formula $a = 1 + \phi_3(4)$, where $\phi_q(h) = \sum_{m=1}^{p-1} (m/p)((m^q+h)/p)$ is the Jacobstahl sum. The formula for $g(k)$ is more complicated and does not lead to a simple expression for u in terms of the Jacobstahl sum. The methods of this paper are based on the theory of cyclotomy. (Received September 20, 1949.)

ANALYSIS

78. E. F. Beckenbach: *On characteristic properties of harmonic functions.*

A lemma of L. V. Ahlfors is used in proving the following result: If $u(x, y)$ is of class C' in a domain D, and for each (x_0, y_0) in D we have $\int_0^{2\pi}\int_0^r[(\partial u/\partial x)^2 + (\partial u/\partial y)^2]\rho d\rho d\theta - \int_0^{2\pi} u(\partial u/\partial n)rd\theta = o(r^2)$, where the integrals are taken over the area and circumference, respectively, of the circle with center at (x_0, y_0) and radius r, then $u(x, y)$ is harmonic in D. (Received October 15, 1949.)

79. S. P. Diliberto: *Systems of ordinary differential equations.* II. Preliminary report.

The following theorem is established: *Let* (1) $dx_i/dt = P_i(x_1, x_2, x_3)$ $(i = 1, 2, 3)$ *where the* P_i *are of class* C^2 *in a neighborhood of* T, *a solid torus whose surface,* S, *is of class* C^3. *Assume* (H_1) *the vector* $V = (P_1, P_2, P_3)$ *has no singular points inside* T (*or on* S); (H_2) *at each point of* S, V *points inside* T; (H_3) *there exist surfaces* $F(x_1, x_2, x_3) =$ constant, *whose Gaussian curvature is strictly positive in a neighborhood of* T, *such that* grad $F = V$. *Then there exists a unique periodic solution* (*closed trajectory*) *of* (1) *inside* T; *further if* T *is the topological product* $S^1 \times E^2$ *of the* 1-*sphere* S^1 *with the disc* E^2 *of center* p, *then the closed trajectory is unknotted and homotopic* (*even isotopic*) (*inside* T) *to the "center-line"* $S^1 \times P$ *of the torus.* This is a partial generalization of the Poincaré-Bendixson Theorem. Proof—sketch: (1) The curvature condition implies that the distance of nearby trajectories decreases (but not that the flow decreases distance of representative points on the trajectories). (2) This implies all trajectories entering T have the same limit set, which in turn consists of a single trajectory. (3) The trajectories themselves are used to construct the homotopy. (Received October 26, 1949.)

80. R. C. James: *Bases and reflexivity of Banach spaces.*

It can be easily shown that a Banach space having a basis $\{x_n\}$ is reflexive if and only if: (1) $\sum_1^\infty a_i x_i$ converges if $\|\sum_1^n a_i x_i\|$ is bounded, and (2) $\lim_{n\to\infty} \|f\|_n = 0$, where $\|f\|_n$ is the norm of f on $x_n \oplus x_{n+1} \oplus \cdots$. By use of these conditions, it is shown that a Banach space with an unconditionally convergent basis is reflexive if no subspace is isomorphic with either (c) or $l^{(1)}$ and that the linear functionals $\{f_n\}$ which with the basis elements $\{x_n\}$ form a biorthogonal set is a basis for B^* if no subspace of B is isomorphic with $l^{(1)}$. A Banach space with a basis is reflexive if relative to any norm equivalent to the norm of B each linear functional attains its maximum on the unit sphere. (Received October 19, 1949.)

81. R. M. Lakness: *Unnecessary singularities of superharmonic functions.*

Let T be a domain in E^3 and let F be a closed bounded set of zero capacity contained in T. Let $u(M)$ be a function superharmonic and bounded below in $T'=T-F$. Then there exists a unique function $V(M)$ superharmonic in T and agreeing with $u(M)$ in T'. The values of $V(M)$ on F are given by $V(M)=\lim\inf_{M'\to M} u(M')$, for M' on T', M on F. The above theorem is an extension of the classical theorem on removable singularities of bounded harmonic functions. It states a corresponding property for superharmonic functions which are bounded below. The main tool used in the proof is the extension of Kellogg's uniqueness theorem, which has been proved by Professor G. C. Evans (Proc. Nat. Acad. Sci. U.S.A. vol. 33 (1947) p. 272). (Received October 13, 1949.)

82*t*. M. Schweitzer: *On the partial sums of second order of the geometric series.*

Szegö proved (Duke Math. J. vol. 8 (1941) p. 559) that the Cesàro sums of second order of the geometric series $\sum z^n$ have a monotonically decreasing real part for $z=e^{i\phi}$, $0\leqq\phi\leqq\gamma$, where $\sin^2(\gamma/2)=0.7$, $\pi/2<\gamma<\pi$. That is, the sine polynomial $\sum_0^n C_{n+2-\nu,2}\,\nu\sin\nu\phi$ is positive in the interval mentioned. In the present paper it is shown that the number γ can be replaced by $2\pi/3$; this is the largest number of this kind. (Received October 6, 1949.)

83*t*. Otto Szasz: *On a summation method of O. Perron.*

The following transform of a series $\sum_0^\infty a_\nu$ has been studied by Perron: $\phi_\lambda(x) =a_0+\sum_0^\infty a_\nu(x/(x+\lambda))\cdot((x+1)/(x+\lambda+1))\cdots((x+\nu-1)/(x+\lambda+\nu-1))$. If $\phi_\lambda(x)\to s$ as $x\to\infty$, then it is said that the series $\sum a_\nu$ has the sum s. The main result of this paper is that this summability method is more powerful than Cesàro summability of order λ. The essential tool is an integral representation of the function $\phi_\lambda(x)$. (Received October 12, 1949.)

84*t*. Gabor Szegö: *On certain special sets of orthogonal polynomials.*

F. Pollaczek has introduced in three Comptes Rendus notes (vol. 228 (1949) pp. 1363, 1553, 1998) various remarkable sets of orthogonal polynomials. In the simplest case they can be defined by the generating function $(1-ze^{i\theta})^{-1/2+i\phi}(1-ze^{i\theta})^{-1/2-i\phi}$ where $\phi=\phi(\theta)=(2\sin\theta)^{-1}(a\cos\theta+b)$, $a\geqq|b|$. They are orthogonal in the interval $-1\leqq x\leqq 1$, $x=\cos\theta$, with the weight function $(\cosh\pi\phi)^{-1}\exp\{(2\theta-\pi)\phi\}$. This weight becomes zero like $\exp(-\theta^{-1})$ as $\theta\to 0$ and the asymptotic behavior of the corresponding orthogonal polynomials is "irregular" in a certain sense. A new proof for the orthogonality relation is given and asymptotic properties are studied. (Received September 28, 1949.)

85. A. E. Taylor: *Banach spaces of analytic functions.* I.

The following notations are used. Δ: the set $|z|<1$ in the plane; $\mathfrak{A}$: the class of functions f analytic in Δ; r: a variable, $0\leqq r<1$; $u_n(z)=z^n$; $U_xf(z)=f(ze^{ix})$ (x real); $T_rf(z)=f(rz)$; $\gamma_n(f)=(1/n!)f^{(n)}(0)$; $B(f, g; z)=\sum_0^\infty\gamma_n(f)\gamma_n(g)z^n$; B: a normed linear space whose elements are members of $\mathfrak{A}$. B may satisfy one or more of the axioms P_1: $\gamma_n\in B^*$ (the conjugate space) and $\|\gamma_n\|$ is bounded; P_2: $u_n\in B$ and $\|u_n\|$ is bounded; P_3: U_x maps B isometrically into B; P_4: T_r maps B continuously into B, and $\|T_r\|$ is bounded in r. If B satisfies P_i, $1\leqq i\leqq k$, B is said to be of type $\mathfrak{A}_k$. For B of type $\mathfrak{A}_3$ define $N(g; r)=\sup_f|B(f, g; r)|$, $\|f\|=1$, $f\in B$, $g\in\mathfrak{A}$. Define B' as the class of $F\in\mathfrak{A}$ with $N(F)=\sup_r N(F; r)<\infty$. B' is a Banach space of type $\mathfrak{A}_4$ with norm $N(F)$. B' and B''' coincide. Define B^0 as the class of $F\in\mathfrak{A}$ for which $\lim_{r\to 1}B(f, F; r)$ exists for each $f \in B$. If B is complete of type $\mathfrak{A}_3$, B^0 is complete of type $\mathfrak{A}_4$ and a subspace of B'. Let B be complete of type $\mathfrak{A}_4$ such that $\lim_{r\to 1} \|T_r f - f\| = 0$ for each f. Then $B' = B^0$ and B' is equivalent to B^*. (Received October 12, 1949.)

86. S. S. Walters: *The space H^p with $0<p<1$.*

The space H^p is defined to be the class of all functions $f \equiv f(z)$ which are regular on the interior of the unit circle and such that $\int_0^{2\pi} |f(re^{i\theta})|^p d\theta$ is bounded in r on $0 \leqq r < 1$. For arbitrary $f \in H^p$ the "norm of f," $\|f\|$, is defined to be $\sup_{0\leqq r<1} ((1/2\pi)\int_0^{2\pi} |f(re^{i\theta})|^p d\theta)^{1/p}$. For the case $1 \leqq p$, it is known that said "norm" is a norm in the true sense, and that H^p is a Banach space. When $0<p<1$ it is shown that H^p is a complete, perfectly separable, linear topological space under the topology: $U \subseteq H^p$ is open in case for arbitrary $f_0 \in H^p$ it is true $\exists r>0 \ni E_f[\|f-f_0\|<r]$ lies in U. In fact H^p is linearly homeomorphic to a closed subspace of $L^p[0, 2\pi]$. Thus, since $(L^p[0, 2\pi])^*$, the space of linear functionals defined on $L^p[0, 2\pi]$, has no elements save the zero element, one might suspect the same to be said of the conjugate space of H^p, namely $(H^p)^*$. This, however, is not the case, and in fact there exists a countable set in $(H^p)^*$ which serves to distinguish elements of H^p, whence weak convergence in H^p makes sense when $0<p<1$. It is shown that if a sequence of elements in H^p converges weakly, then the sequence converges to its weak limit uniformly on all compact subsets of the unit circle. (Received October 7, 1949.)

87. Morgan Ward: *Note on real continuous iteration.*

The investigations of Fuller and Ward on the continuous iteration of all functions steadily increasing in $0 \leqq x < \alpha$ (Bull. Amer. Math. Soc. vol. 40 (1934) pp. 688–690 and vol. 42 (1936) pp. 393–396) and in C^0 are extended to functions in C^k and C^α. All continuous iterations for functions in these classes are specified, and it is proved that no analytic continuous iterations exist for the sub-class of functions regular in $z = x+iy$ along the positive axis $0 \leqq x < \alpha$ which are also regular along the positive axis. (Received October 14, 1949.)

88. František Wolf: *Analytic perturbation of operators in Banach spaces.* Preliminary report.

Let $A(\theta)$ be a bounded linear operator in a Banach space $\mathfrak{B}$ which depends analytically on θ in a neighborhood N of $\theta=0$. Suppose that $A(0)=A_0$ has μ_0 for an eigenvalue with an m-dimensional eigenspace $\mathfrak{S}(0)$. Then there exists an analytic idempotent $E(\theta)$ such that the range of $E(0)$ is $\mathfrak{S}(0)$. For small enough θ, $A(\theta)$ has eigenvalues $\mu_k(\theta)$ in an arbitrary small neighborhood of $\theta=0$. Their corresponding eigenspaces generate a space $\mathfrak{S}(\theta)$ which is also m-dimensional and is the range of $E(\theta)$. It is possible to find $f_1(\theta), \cdots, f_m(\theta) \in \mathfrak{B}$ analytic in θ which generate $\mathfrak{S}(\theta)$. In general, for $m>1$, it is not possible to define them so that all $f_i(\theta)$ are all eigenfunctions of $A(\theta)$ corresponding to $\mu_k(\theta)$. The functions $\mu_k(\theta)$ have in general a branch-point at $\theta=0$. In some special cases the $\mu_k(\theta)$ are all analytic in $\theta=0$ and their coincidence at $\theta=0$ is accidental. This is true in particular wherever it is known from the special character of $A(\theta)$ that $\mu_k(\theta)$ are real for real θ. For self-adjoint operators in Hilbert space this has been proved by F. Rellich (Math. Ann. vol. 113 (1936) p. 600). The author has found some conditions for this to occur in Banach spaces. (Received October 12, 1949.)

APPLIED MATHEMATICS

89*t*. H. W. Becker: *Planar and nonplanar cadences.*

Where $e^{tK}=1/(2-e^t)$ and $e^{tL}=e^{t/(1-t)}$, $K_n=L_n$ if $n<4$, otherwise $K_n>L_n$. The music theoretic interpretation is: that K_n is the number of cadences of n notes (melodies according to distinction of relative and not absolute pitch); while L_n is the number of planar cadences (having no crossovers in their Puttenham diagrams). Nonplanarities cannot arise, $n<4$; and $K_n \supset L_n$. The K_n are sums of differences of zero; the L_n are Laguerre numbers, and also sums of Stirling numbers of the third kind, Bull. Amer. Math. Soc. vol. 42 (1946) p. 826. Various other combinatory interpretations of K_n and L_n are tabulated. For example, they are the numbers of distributions of n men into externally and internally permutable crews, respectively. That is, in K_n, c crews may be ranked $c!$ ways; while in L_n, m members of a crew may be ranked $m!$ ways. Under this interpretation, the only overlap is $K_n \cap L_n = @_n$, where $e^{t@}=\exp(e^t-1)$. (Received October 12, 1949.)

90*t*. H. W. Becker: *The theory of the world series.*

"As the first game goes, so goes the series." The truth of this saying (S) in a four-win series is attacked combinatorially, and historically as of the 46 series to date. The winner of the first game can go on to win the series in 20 ways, or lose it in 15. So with the assumption (A) that the series is contested between teams of about equal strength and is won by "the breaks," (S) is 57% true in theory and 63% true on the record. However, the series durations have averaged almost a game shorter than they should be. This is attributable to departures from (A), and to "human equation" (B): any initial losers have a tendency to fall into brooding, and self-defeatism, though their remaining chances are better than they think. On the other hand, most of the series which go the limit have been *won* by the initial loser. That is ascribable to the "comeback equation" (C): an initial loser who has grimly fought back to eventually even terms, has now the greater momentum of victory. These considerations appertain to a mathematics, enriched to include all the problems of the mind: those of morale and gusto, as well as magnitude and order. (Received October 12 1949.)

91. G. E. Forsythe: *Round-off errors in numerical integration on automatic machinery.* Preliminary report.

Adapting numerical integration processes to automatic machinery ordinarily involves replacing a desired sum $\sum_{k=1}^{n} u_k$ by the computable sum $\sum_{k=1}^{n} N(u_k)$, where $N(u_k)$ is the nearest integer to u_k. Large round-off errors $R_n=\sum_{k=1}^{n}\{N(u_k)-u_k\}$ frequently accumulate near points $\{k'\}$ of "stationary phase" of $N(u_k)-u_k$. (H. D. Huskey, National Bureau of Standards Journal of Research vol. 42 (1949) pp. 57–62). The present author gives an approximate formula for R_n in terms of $\{N(u_{k'})-u_{k'}\}$ and $\{|\Delta u_{k'}|\}$, valid whenever Δu_k is a slowly changing function of k. The formula explains a posteriori most of the peculiarities of the round-off errors found in Huskey's integration of the system $\dot{x}=y$, $\dot{y}=-x$ on the ENIAC. Let $[u]$ be the greatest integer not exceeding u. The present author proposes a *random round-off*, whereby any real number $u=[u]+v$ is "rounded up" to $[u]+1$ with probability v, and "rounded down" to $[u]$ with probability $1-v$. The error r of random round-off is truly a random variable. Since $E(r)=0$ and $E(r^2)=v(1-v)\leqq 1/2$, simple probabilistic bounds on the accumulated error can be given without assuming a distribution for v. Tests with I. B. M. equipment indicate that random round-off probably eliminates a priori the peculiarities of round-off found by Huskey on the ENIAC. (Received October 12, 1949.)

92. Rufus Isaacs: *Transtability flutter of supersonic aircraft panels.*

For certain aero-elastic configurations it is possible to ascertain critical flutter conditions from static considerations alone. When the air speed exceeds a certain value, statically stable equilibrium—and sometimes equilibrium itself—is no longer possible. Such is the case for an aircraft structure panel, buckled by thermal expansion; here "one-dimensionalized" to a buckled beam with clamped ends. With no air velocity, the beam is the classical Euler column with its discrete set of possible deflections of which the first alone is stable. As the velocity increases the first two deflections become more alike until at a certain critical speed they coincide. At higher speeds they do not exist. As the only possibilities for the beam to be stationary lie in the unstable higher modes, it is reasonable to assert that flutter now occurs. A rigorous proof is impossible without entanglement in the complicated dynamic analysis, but strong plausibility arguments exist. For example, if the continuous beam is replaced by a discrete model made of hinged bars with only two degrees of freedom, there are only two eigenvalues. When the critical speed is exceeded both—and hence all possibility of equilibrium—disappear. Flutter is certain and this model should behave like the continuous one. (Received August 22, 1949.)

93. Cornelius Lanczos: *An iterative solution of Fredholm's integral equation.*

Consider Fredholm's integral equation $y-\lambda Ky=\phi$. Instead of mere iterations which would result in the Liouville-Neumann series, the method of "minimized iterations" is employed. This process assigns to any given $\phi(x)$ and $K(x, \xi)$ a sequence of biorthogonal functions ϕ_i and ϕ_i^* ($\phi_i\phi_k^*=0$ for $i\neq k$), together with a sequence of polynomials $p_i(x)$, generated by a recursion relation between three consecutive p_i. The roots of these polynomials converge to the reciprocal characteristic values $1/\lambda_j$ if these values exist, otherwise they converge to zero. A certain linear combination of the $\phi_i(x)$ converges to the solution of the given integral equation. The same combination, taken for $\lambda=1/\mu_j$ where μ_j is the jth root of the last polynomial $p_m(x)$ still evaluated (j fixed, m steadily increasing), converges to the characteristic solutions (defined by $\phi=0$) if these solutions exist. Numerical examples demonstrate the excellent speed of convergence for not too large values of λ. The convergence remains valid for arbitrarily large λ, although the number of iterations practically demanded becomes eventually excessive. (Received October 11, 1949.)

GEOMETRY

94. Abraham Seidenberg: *The hyperplane sections of normal varieties.*

Let V/k be a normal variety in n-space of dimension $r\geqq 2$ defined over an infinite ground-field k, and suppose further that the field of rational functions on V/k is separably generated. Then almost all hyperplane sections of V/k will also be normal, that is, the section by the hyperplane $a_0x_0+\cdots+a_nx_n=0$, $a_i\in k$, will be (irreducible and) normal provided the a_i satisfy a certain proper algebraic inequality over k. The above separability assumption may be removed upon restricting the a_i to k^p;

also, with appropriate formulation, the finiteness condition on k proves unnecessary. The more general case of an arbitrary linear system of V_{r-1}'s on V/k, which yields an analogue to the theorem of Bertini on the variable singular points of a linear system of varieties (obtained by replacing the word *singular* with the work *non-normal*), is left to a second paper. (Received October 10, 1949.)

Logic and Foundations

95. J. D. Swift: *Analogues of the Sheffer stroke in the three-valued case.*

There are precisely ninety commutative analogues of the Sheffer stroke (binary independent generating functions) in the three-valued case. These lie in fifteen classes each consisting of six conjugate functions. (Received October 12, 1947.)

Statistics and Probability

96t. C. J. Everett and S. M. Ulam: *On an application of a correspondence between matrices over real algebras and matrices of positive real numbers.*

Since the representation of real numbers by pairs (a, b) of positive reals has an essentially matric character under addition and multiplication, it is possible to define a (many-one) correspondence from the set of all $2n\times 2n$ matrices consisting of n^2 blocks, each of form $a_{11}=a_{22}>0$, $a_{12}=a_{21}>0$, to the set of all real $n\times n$ matrices, which preserves $(+)$ and $(\cdot)$. Thus, ring operations on matrices whose elements belong to a real linear associative algebra admitting a faithful real matrix representation (C. C. MacDuffee, *On the independence of the first and second matrices of an algebra*, Bull. Amer. Math. Soc. (1929)) may be replaced by operations on corresponding real positive matrices of higher order. Similar remarks apply to matrix $\times$ vector operations. The advantage for characteristic vector problems lies in the fact that a positive matrix may be realized as the first moment matrix of a multiplicative system (Bull. Amer. Math. Soc. Abstracts 55-1-41, 42, 43), and as such, its "first" characteristic vector represents the limit population of the system, which may be investigated by "Monte Carlo" methods (cf., for example, N. Metropolis, S. Ulam, *The Monte Carlo method*, Journal of the American Statistical Association vol. 44 (1949) pp. 335–341). (Received October 13, 1949.)

97t. C. J. Everett and S. M. Ulam: *Random walk and the Hamilton-Jacobi equation.*

Let $v(x)$ be a continuous function on $(-\infty, +\infty)$ with $0<a\leqq v(x)\leqq 1/2$. For every subdivision s of norm d, a random walk is defined, in which a particle moves, every d^2 seconds (left or right with equal probability), a distance $d\cdot v(x_{i-1})$ when on the subdivision interval (x_{i-1}, x_i). The probability $W_s(z, x, t)\Delta x$ of the particle being in $(x, x+\Delta x)$, t seconds after starting from z, has limit $W(z, x, t)\Delta x=[\Delta x/v(x)(2\pi t)^{1/2}]$ $\exp(-S^2(z, x)/2t)$ as $d\rightarrow 0$, where $S(z, x)=\left|\int_z^x dy/v(y)\right|$ is the S function of optics satisfying the Hamilton-Jacobi equation $S_x^2=1/v^2(x)$. Setting $W_t=0$, we obtain $S^2(z, x)=t$. Thus, the positions x which are at maximum probability density at time t, in the limiting case of random walk with variable length of step, coincide with the positions at which light would arrive from the same origin z in time $t^{1/2}$. Extension to higher dimensions, now under way, may open problems of mechanics, via the Hamilton-

Jacobi equation, as well as optics itself, to statistical sampling by high speed machine techniques. (Received October 13, 1949.)

Topology

98. R. L. Wilder: *Local orientability.*

Properties called *local orientability* were defined on p. 281 of the author's Colloquium book *Topology of manifolds* (hereafter referred to as "T. M." The word "connected" should be inserted before "neighborhood" in Definition 6.1 and "nonempty" before the second "open" in D''). An extensive study of these properties is made in this paper, they being found to be equivalent in an n-gm to each of six other properties (one of these being an axiom of Čech; cf. T. M., 289 Bibl. Comm. §6). Applications are made to orientability of n-gms. For example, in a locally orientable n-gm concurrent orientations of the local n-gms may be introduced so as to define orientability of an n-gm by means of an indicatrix and chains of n-gms analogous to the chains used by Poincaré in studying torsion (Proc. London Math. Soc. vol. 32 (1900) pp. 277–308) and in fashion similar to the classical orientability definition. From this definition a new proof of Begle's theorem (p. 251, T. M.) may be obtained. For connected n-gms, it should be noted that the assumption of local orientability implies conditions D and D' of T. M. (pp. 250, 254). (Received October 12, 1949.)

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