THE ANNUAL MEETING OF THE SOCIETY

The fifty-sixth Annual Meeting of the American Mathematical Society was held at Columbia University, New York City, Tuesday to Thursday, December 27–29, 1949, in conjunction with the annual meetings of the Mathematical Association of America and the American Association for the Advancement of Science. Registration headquarters for the Society and the Association were in the Lobby of Earl Hall, Columbia University. Residence headquarters were at the Hotel Governor Clinton. Over seven hundred persons registered, including the following six hundred sixty nine members of the Society (a record attendance for an Annual meeting):

Milton Abramowitz, C. R. Adams, R. B. Adams, R. P. Agnew, L. V. Ahlfors, M. I. Aissen, E. J. Akutowicz, A. A. Albert, R. G. Albert, E. S. Allen, C. B. Allendoerfer, Bernard Altshuler, Warren Ambrose, D. B. Ames, R. D. Anderson, R. L. Anderson, T. W. Anderson, T. M. Apostol, R. G. Archibald, K. J. Arnold, L. A. Aroian, Nachman Aronszajn, M. G. Arsove, Natascha Artin, Max Astrachan, F. E. Atkins, Helmut Aulbach, Silvio Aurora, Frank Ayres, W. L. Ayres, Frederick Bagemihl, F. E. Baker, S. F. Barber, Joshua Barlaz, Iacopo Barsotti, A. F. Bartholomay, F. D. Bateman, P. T. Bateman, M. R. Bates, H. P. Beard, Ralph Beatley, F. S. Beckman, F. P. Beer, E. G. Begle, Isaiah Benjamin, A. A. Bennett, Stefan Bergman, R. R. Bernard, Lipman Bers, R. H. Bing, Garrett Birkhoff, Archie Blake, D. W. Blackett, W. E. Bleick, I. E. Block, R. P. Boas, Salomon Bochner, H. W. Bode, H. F. Bohnenblust, G. L. Bolton, W. M. Boothby, Frank Bothwell, T. A. Botts, J. G. Bowker, C. B. Boyer, A. D. Bradley, Richard Brauer, N. A. Brigham, H. W. Brinkmann, Paul Brock, A. B. Brown, R. H. Bruck, Charlotte Brudno, H. D. Brunk, W. J. Bruns, J. R. Buchi, H. J. Buck, C. T. Bumer, L. H. Bunyan, R. S. Burington, W. K. Burroughs, L. J. Burton, L. E. Bush, Jewell H. Bushey, J. H. Bushey, S. S. Cairns, J. W. Calkin, E. B. Callahan, R. C. Campbell, M. E. Carlen, P. G. Carlson, L. V. Carleton, P. W. Carruth, H. C. Carter, K. V. Casey, J. O. Chellevold, K. T. Chen, Y. W. Chen, Herman Chernoff, Peter Chiarulli, W. L. Chow, Sarvadaman Chowla, K. L. Chung, Alonzo Church, Randolph Church, Edmund Churchill, F. E. Clark, G. R. Clements, E. A. Coddington, A. C. Cohen, H. J. Cohen, I. S. Cohen, L. W. Cohen, Harvey Cohn, R. M. Cohn, Nancy Cole, J. B. Coleman, Esther Comegys, H. R. Cooley, Philip Cooperman, T. F. Cope, A. H. Copeland, Natalie Coplan, Robert Cortell, Byron Cosby, Richard Courant, L. M. Court, R. R. Coveyou, W. H. H. Cowles, V. F. Cowling, E. H. Crisler, C. W. Curtis, E. H. Cutler, F. T. Daly, J. M. Danskin, D. A. Darling, D. R. Davis, H. C. Davis, Philip Davis, B. V. Dean, A. H. Diamond, J. B. Diaz, M. P. Dolciani, C. L. Dolph, J. L. Doob, H. L. Dorwart, C. H. Dowker, Y. N. Dowker, R. H. Downing, F. W. Dresch, Arnold Dresden, B. M. Drucker, J. S. Dwork, John Dyer-Bennet, Nelson Dunford, Aryeh Dvoretzky, P. S. Dwyer, W. F. Eberlein, Samuel Eilenberg, Carolyn Eisele, B. J. Eisenstadt, L. N. Enequist, M. P. Epstein, Paul Erdös, R. W. Erickson, W. H. Fagerstrom, A. L. Fass, F. D. Faulkner, Herbert Federer, J. M. Feld, William Feller, F. G. Fender, W. E. Ferguson, F. A. Ficken, N. J. Fine, D. T. Finkbeiner, C. H. Fischer, Emanuel Fischer, W. B. Fite, Edward Fleischer, W. W. Flexner, E. E. Floyd, Tomlinson Fort, M. C. Foster, R. M. Foster, J. S. Frame, J. C. Freeman, Bernard Friedman, J. B. Freier, Gerald Freilich, K. O. Friedrichs, Orrin Frink, R. E. Fullerton, A. S. Galbraith, David Gale, L. M. Garfunkel, G. N. Garrison, L. L. Gavurin, H. M. Gehman, Hilda Geiringer, Abe Gelbart, B. H. Gere, H. H. Germond, F. J. Gerst, J. C. Gibson, H. A. Giddings, E. N. Gilbert, B. P. Gill, Leonard Gillman, R. E. Gilman, Wallace Givens, A. M. Gleason, G. H. Gleissner, Casper Goffman, H. E. Goheen, Michael Goldberg, Samuel Goldberg, A. W. Goodman, D. B. Goodner, R. D. Gordon, W. O. Gordon, Daniel Gorenstein, Arthur Grad, H. S. Grant, J. B. Greeley, H. J. Greenberg, Harriet Griffin, George Grossman, Emil Grosswald, C. C. Grove, M. M. Gutterman, Carl Hammer, R. W. Hamming, G. H. Handelman, Frank Harary, T. E. Harris, O. G. Harrold, F. S. Hawthorne, G. E. Hay, R. M. Hayes, W. K. Hayman, K. E. Hazard, G. A. Hedlund, A. E. Heins, M. H. Heins, Alex Heller, G. C. Helme, Olaf Helmer, R. G. Helsel, Henry Helson, Arthur Hendler, Leon Henkin, A. H. Henry, L. H. Herbach, Aaron Herschfeld, I. R. Herschner, I. N. Herstein, A. D. Hestenes, M. R. Hestenes, T. H. Hildebrandt, L. S. Hill, Einar Hille, A. J. Hoffman, W. C. Hoffman, Banesh Hoffmann, T. R. Hollcroft, M. W. Hopkins, E. M. Hove, C. C. Hsiung, H. T. Hsü, R. C. Huffer, E. M. Hull, M. E. Hull, Ralph Hull, M. G. Humphreys, T. R. Humphreys, Mildred Hunt, Witold Hurewicz, Solomon Hurwitz, W. A. Hurwitz, L. C. Hutchinson, M. A. Hyman, W. H. Ingram, Eugene Isaacson, S. L. Isaacson, Nathan Jacobson, Wenceslas Jardetzky, Walter Jennings, E. D. Jenkins, Børge Jessen, Fritz John, M. M. Johnsen, R. A. Jóhnson, R. B. Johnson, F. E. Johnston, A. W. Jones, B. W. Jones, F. B. Jones, Bjarni Jonsson, M. L. Juncosa, Mark Kac, Robert Kahal, Shizuo Kakutani, Walbert Kalinowski, Aida Kalish, L. H. Kanter, Irving Kaplansky, S. N. Karp, Gilbert Kaskey, Edward Kasner, W. H. Keen, M. E. Kellar, J. B. Kelly, E. S. Kennedy, D. E. Kibbey, J. F. Kiefer, B. F. Kimball, E. K. Kinney, A. R. Kirby, M. D. Kirby, S. C. Kleene, J. R. Kline, Morris Kline, E. G. Kogbetliantz, E. R. Kolchin, Horace Komm, H. S. Konijn, B. O. Koopman, Jacob Korevaar, John Kronsbein, R. R. Kuebler, Wouter van der Kulk, A. W. Landers, M. K. Landers, R. E. Langer, C. W. Langley, E. H. Larguier, H. D. Larsen, V. V. Latshaw, A. L. Lax, Solomon Lefschetz, C. H. Lehmann, Marguerite Lehr, H. R. Leifer, R. B. Leipnik, Benjamin Lepson, Max LeLeiko, C. A. Lester, M. E. Levenson, W. V. LeVeque, Howard Levi, D. C. Lewis, M. A. Lifschutz, C. C. Lin, B. W. Lindgren, Samuel Linial, S. R. Lipsey, H. D. Lipsich, Charles Loewner, A. J. Lohwater, E. R. Lorch, Lee Lorch, A. N. Lowan, R. B. Lowe, C. I. Lubin, Eugene Lukacs, E. D. McCarthy, N. H. McCoy, S. W. McCuskey, Brockway McMillan, E. J. McShane, L. A. MacColl, C. C. MacDuffee, H. M. MacNeille, Nathaniel Macon, Irwin Mann, Murray Mannos, A. J. Maria, M. H. Maria, R. H. Marquis, W. T. Martin, F. I. Mautner, Kenneth May, D. G. Mead, P. E. Meadows, A. E. Meder, Jr., L. E. Mehlenbacher, C. C. Miesse, E. J. Miles, E. P. Miles, A. N. Milgram, Joseph Milkman, Frederic H. Miller, K. S. Miller, W. H. Mills, P. D. Minton, H. J. Miser, L. I. Mishoe, J. M. Mitchell, E. E. Moise, H. F. Montague, M. D. Montgomery, C. N. Moore, T. W. Moore, K. A. Morgan, R. H. Morris, Nathan Morrison, Marston Morse, G. D. Mostow, Wolfe Mostow, T. S. Motzkin, H. T. Muhly, J. H. Mulligan, F. J. Murray, W. R. Murray, D. J. Myatt, D. S. Nathan, Zeev Nehari, C. A. Nelson, Paolo Nesbeda, J. D. Newburgh, J. J. Newman, Morris Newman, C. V. Newsom, Jerzy Neyman, H. K. Nickerson, O. M. Nikodým, H. M. Nodelman, P. B. Norman, I. L. Novak, C. O. Oakley, G. G. O'Brien, L. F. Ollmann, Paul Olum, A. F. O'Neill, E. R. Ott, G. K. Overholtzer, J. C. Oxtoby, O. O. Pardee, N. G. Parke, Anna Pell-Wheeler, Jacqueline Penez, A. J. Penico, F. W. Perkins, C. L. Perry, G. W. Petrie, B. J. Pettis, H. R. Phalen, C. R. Phelps, C. G. Phipps, Everett Pitcher, Harry Polachek, H. O. Pollak, J. C. Polley, Walter Prenowitz, C. M. Price, G. B. Price, R. C.

Prim, Hans Rademacher, Tibor Rado, H. V. Rådstrom, L. R. Raines, Gordon Raisbeck, K. G. Ramanathan, O. J. Ramler, J. F. Randolph, G. N. Raney, H. E. Rauch, L. T. Ratner, G. E. Raynor, M. J. Rees, M. S. Rees, C. F. Rehberg, Irving Reiner, I. M. Reiner, Eric Reissner, G. W. Reitwiesner, Russell Remage, R. W. Rempfer, Daniel Resch, Helene Reschovsky, C. N. Reynolds, D. P. Richardson, Moses Richardson, C. E. Rickart, P. R. Rider, R. F. Rinehart, J. F. Ritt, R. K. Ritt, E. K. Ritter, H. E. Robbins, R. E. Roberson, J. H. Roberts, R. A. Roberts, J. E. Robinson, S. L. Robinson, V. N. Robinson, C. A. Rogers, Saul Rosen, Ira Rosenbaum, Murray Rosenblatt, P. C. Rosenbloom, J. H. Rosenbloom, M. A. Rosenlicht, Arthur Rosenthal, J. B. Rosser, M. F. Rosskopf, H. L. Royden, Herman Rubin, Herbert Ruderfer, Walter Rudin, C. W. Saalfrank, Bernard Sachs, Charles Salkind, Charles Saltzer, J. E. Sammet, R. G. Sanger, Arthur Sard, S. S. Saslaw, F. E. Satterthwaite, A. C. Schaeffer, A. T. Schafer, R. D. Schafer, Robert Schatten, Samuel Schecter, Henry Scheffé, Eugene Schenkman, Albert Schild, E. R. Schneckenburger, J. P. Scholz, K. C. Schraut, Pincus Schub, A. L. Schurrer, Abraham Schwartz, B. L. Schwartz, G. E. Schweigert, C. H. W. Sedgewick, I. E. Segal, Wladimir Seidel, George Seifert, D. B. Shaffer, H. S. Shapiro, I. M. Sheffer, L. W. Sheridan, Seymour Sherman, L. P. Siceloff, K. M. Siegel, J. A. Silva, L. L. Silverman, Annette Sinclair, James Singer, M. H. Slud, L. L. Smail, P. A. Smith, R. E. Smith, W. M. Smith, Ernst Snapper, A. K. Snyder, W. S. Snyder, Andrew Sobczyk, E. S. Sokolnikoff, J. J. Sopka, D. E. South, E. H. Spanier, D. E. Spencer, V. E. Spencer, M. R. Spiegel, A. H. Sprague, E. R. Stabler, E. P. Starke, J. R. K. Stauffer, M. P. Steele, N. E. Steenrod, I. A. Stegun, S. K. B. Stein, Fritz Steinhardt, Wolfgang Sternberg, R. W. Stokes, R. R. Stoll, R. C. Strodt, Walter Strodt, D. J. Struik, D. M. Studley, E. A. Sturley, Gabor Szegö, T. T. Tanimoto, Olga Taussky-Todd, J. S. Taylor, M. E. Taylor, A. J. Terzuoli, Feodor Theilheimer, J. M. Thomas, D. L. Thomsen, W. J. Thron, Michael Tikson, G. L. Tiller, John Todd, L. F. Tolle, C. B. Tompkins, M. M. Torrey, J. I. Tracey, W. R. Transue, H. M. Trent, W. J. Trjitzinsky, C. A. Truesdell, A. W. Tucker, J. W. Tukey, Annita Tuller, J. L. Ullman, F. E. Ulrich, Eugene Usdin, E. P. Vance, J. L. Vanderslice, Henry Van Engen, H. E. Vansant, A. H. Van Tuyl, S. I. Vrooman, M. C. Waddell, D. H. Wagner, G. L. Walker, S. E. Walkley, J. L. Walsh, R. M. Walter, W. G. Warnock, D. R. Waterman, G. C. Webber, J. V. Wehausen, Alexander Weinstein, Louis Weisner, M. J. Weiss, F. P. Welch, David Wellinger, J. G. Wendel, E. H. Wetherell, F. J. Weyl, George Whaples, M. E. White, D. E. Whitford, P. M. Whitman, A. M. Whitney, D. R. Whitney, Hassler Whitney, G. T. Whyburn, L. S. Whyburn, W. M. Whyburn, D. V. Widder, V. A. Widder, N. A. Wiegmann, Norbert Wiener, Albert Wilansky, E. S. Wolk, Y. K. Wong, M. Y. Woodbridge, M. A. Woodbury, Arthur Wouk, F. M. Wright, M. A. Wurster, Bertram Yood, D. M. Young, J. W. Young, J. W. T. Youngs, N. J. Zabb, L. A. Zadeh, Oscar Zariski, Arthur Zeichner, Daniel Zelinsky, P. W. Zettler-Seidel, A. D. Ziebur, J. A. Zilber, H. J. Zimmerberg, Leo Zippin, Antoni Zygmund.

The twenty-third Josiah Willard Gibbs Lecture, entitled *The problem of sensory prosthesis*, was delivered by Professor Norbert Wiener of the Massachusetts Institute of Technology at 4:30 P.M., Wednesday, December 28. Professor J. L. Walsh, President of the American Mathematical Society, was the presiding officer.

At a joint session of Section A and the Society at 2:00 P.M., Tuesday, December 27, Professor G. T. Whyburn of the University of

Virginia gave an address, *The open mapping medium in topological analysis*, as a retiring Vice President of the American Association for the Advancement of Science. Professor E. J. McShane, Chairman of Section A, presided.

The Committee to Select Hour Speakers for Annual and Summer Meetings invited two speakers. On Wednesday, December 28, Professor Sarvadaman Chowla of the University of Kansas gave an address on *The Riemann zeta and allied functions* at 9:30 A.M. Vice President W. T. Martin presided. At 2:00 P.M. Professor L. V. Ahlfors of Harvard University gave an address on *The classification of open Riemann surfaces*. Vice President Hassler Whitney presided.

The annual Business Meeting and Election of Officers was held at 10:00 A.M., Thursday, December 29, with President J. L. Walsh in the chair. Details of proceedings are reported in the sequel. The Cole Prize in Algebra was awarded to Professor Richard Brauer of the University of Michigan for his work in group theory and in particular for the following paper: *On Artin's L-series with general group characters*, Ann. of Math. (2) vol. 48 (1947) pp. 502–514. Professor Brauer gave a brief address on his prize-winning paper.

A tea for members of the mathematical organizations and their guests was given by Columbia University at 4 P.M. Tuesday at the Faculty House.

On Wednesday evening at the Casa Italiana, there was a musicale by Mr. and Mrs. Everett Anderson and Mr. Harold Triggs of the Columbia University Department of Music.

There was a conducted tour of the Museum of Modern Art on Thursday afternoon.

The dinner Thursday evening in the John Jay Hall Dining Room was attended by 240 members and guests. The toastmaster, Professor P. A. Smith, introduced Dr. George B. Pegram, Vice President of Columbia University who gave a brief address of welcome. He was followed by Professor J. L. Walsh, President of the American Mathematical Society; Dr. H. M. MacNeille, Executive Director of the American Mathematical Society; Professor Jerzy Neyman, President of the Institute of Mathematical Statistics; and Professor R. E. Langer, President of the Mathematical Association of America. The toastmaster then introduced Professor M. R. Hestenes who offered resolutions of thanks and appreciation to Columbia University, the Committee on Arrangements, and all who had helped make the meetings successful and enjoyable. This resolution was adopted by a rising vote.

The Council met at 6:30 P.M. on December 28, 1949.

The Secretary announced the election of the following one hundred and thirty-one persons to ordinary membership in the Society:

Mr. John Pruyn van Alstyne, Hamilton College, Clinton, N.Y.;
Professor Henry Adalbert Antosiewicz, Montana State College;
Mr. Elihue Barden, Alcorn College, Alcorn, Miss.;
Mr. James Earl Barney, II, University of Kansas;
Dr. Howard Gordon Bergmann, City College, New York, N.Y.;
Miss Irma Berkowitz, University of Alabama;
Mr. Alvin Kilian Bettinger, Creighton University, Omaha, Neb.;
Miss Laura Blakeley, Armstrong College, Savannah, Ga.;
Dr. Felix Earl Browder, Massachusetts Institute of Technology;
Mr. Beverley Merl Brown, Tusculum College, Greeneville, Tenn.;
Mr. Howard Harry Brown, Franklin Institute, Philadelphia, Pa.;
Mr. Robert William Bryant, Miami University, Oxford, Ohio;
Dr. J. Richard Buchi, Ripon College, Ripon, Wis.;
Mr. Alberto Pedro Calderón, University of Chicago;
Mr. Jean Mitchener Calloway, University of Pennsylvania;
Mrs. Dorothy Wolking Campbell, University of Wisconsin;
Mr. Richard Crawford Campbell, University of Colorado;
Mr. George Casseb, Saint Mary's University of Texas, San Antonio, Tex.;
Mr. Bruce Bampton Clark, Grinnell College, Grinnell, Iowa;
Professor Willie E. Clark, Agricultural, Mechanical and Normal College, Pine Bluff, Ark.;
Mr. Daniel Edmund Coffey, Montana State University;
Mr. Warren Alan Couch, Washington University;
Mr. John Norris Crawford, Texas Southmost College, Brownsville, Tex.;
Professor Dorothy Ada Curry, Wilberforce University, Wilberforce, Ohio;
Professor Rene Joseph De Vogelaere, Laval University, Quebec, Que., Canada;
Mr. Robert Benjamin Davis, Massachusetts Institute of Technology;
Dean Louis Aspell Deesz, Youngstown College, Youngstown, Ohio;
Mr. Leroy John Derr, Tulane University of Louisiana;
Mr. Raymond Allen Dibrell, Jr., Southwest Texas State College, San Marcos, Tex.;
Mr. Robert John Dickson, Jr., California Institute of Technology;
Professor Israel Edward Drabkin, City College, New York, N.Y.;
Mr. Georges Roland Dubé, Yale University;
Mr. James Eells, Jr., Amherst College, Amherst, Mass.;
Professor Gustav Elfving, University of Helsingfors, Helsingfors, Finland;
Mr. John W. Forman, University of Kansas;
Mr. Harold Forstat, Purdue University;
Mr. Colonel Douglas Gardner, Okolona College, Okolona, Miss.;
Mr. Eugene Joseph Germino, Saint John's University, Brooklyn, N.Y.;
Mr. Walter Adelbert Glass, Drew University, Madison, N.J.;
Mr. William Goldstein, State Teachers College, Trenton, N.J.;
Mr. Jacob Festus Golightly, Jacksonville Junior College, Jacksonville, Fla.;
Mr. Edward Gordon Goman, College of Puget Sound, Tacoma, Wash.;
Miss Lillian Gough, University of Buffalo;
Mr. Richard Briggs Grekila, University of Maryland;
Mr. Vincent Harold Haag, Franklin and Marshall College, Lancaster, Pa.;
Mr. Herman David Hartstein, School of Business, Washington University;
Mr. Arthur Hendler, Rensselaer Polytechnic Institute, Troy, N.Y.;

Professor Boyd Herbert Henry, Parsons College, Fairfield, Iowa;
Mr. John Stephenson Hicks, Socony-Vacuum Laboratories, Paulsboro, N.J.;
Mr. Harley A. Hill, Portland, Ore.;
Miss Jean E. Hirsh, Queens College, Flushing, N.Y.;
Mr. Cullen Squaere Hodge, Fisk University, Nashville, Tenn.;
Mr. Robert James Howerton, Regis College, Denver, Colo.;
Dr. Sze Tsen Hu, Tulane University of Louisiana;
Professor Gilbert Agnew Hunt, Cornell University;
Mr. Edward Ormond Hynard, Manhattan College, New York, N.Y.;
Professor Wendell Gilbert Johnson, Phillips University, Enid, Okla.;
Mr. John Ernest Kelley, University of Miami, Coral Gables, Fla.;
Mrs. Rachel Geck Kirkendall, Highland Park School, Highland Park, Mich.;
Mr. John Sharpless Klein, Williams College, Williamstown, Mass.;
Mr. Karl John Klein, Loras College, Dubuque, Iowa;
Mr. Micha Klein, Mekoroth Water Co., Ltd., Tel-Aviv, Israel;
Dr. Paul Koditschek, Long Island University, Brooklyn, N.Y.;
Mr. Allan Theodore Kovitz, Polytechnic Institute of Brooklyn;
Mr. Andrew V. Kozak, Concord College, Athens, W. Va.;
Professor Anthony Edward Labarre, Jr., University of Idaho, Moscow, Idaho;
Mr. James Richard Larkin, University of Kansas, Lawrence, Kan.;
Mr. Sim Lasher, Chicago, Ill.;
Mr. Eric Liban, New York University;
Mr. Johnathan Wattam Lindsay, Texas Technological College, Lubbock, Tex.;
Professor Wilhelm Ljunggren, University of Bergen, Bergen, Norway;
Mr. Manice de Forest Lockwood, III, New York, N.Y.;
Dr. Henry Francis Joseph Löwig, University of Tasmania, Tasmania, Australia;
Dr. George Lorentz, University of Toronto;
Professor Fred Wilbur Lott, Jr., Iowa State Teachers College, Cedar Falls, Iowa;
Mr. Cyrus Ray McAllister, University of Idaho, Moscow, Idaho;
Mr. Charles Wilson McArthur, Tulane University of Louisiana;
Miss June Marie McArtney, University of Buffalo;
Dean (Henry) Stirling McCall, Norman College, Norman Park, Ga.;
Mr. (James) Edward McGaughy, Lawrence College, Appleton, Wis.;
Mr. Kenneth M. McMillin, University of Minnesota;
Mr. Nathaniel Macon, University of North Carolina;
Professor David Middleton, Physics Department, Harvard University;
Mr. Henri Monik, Detroit Institute of Technology;
Miss Mary Prudence Moseley, Southern Methodist University, Dallas, Tex.;
Miss Martha Rheuba Mouser, Wilson Teachers College, Washington, D.C.;
Mr. Howard Warren Moyer, Lost City, W. Va.;
Mr. Robert George Needels, Loyola University of Los Angeles;
Mr. Edward Peter Neuburg, University of Vermont and State Agricultural College;
Miss Virginia Anne Ohlson, Purdue University, Lafayette, Ind.;
Mr. Clellie Curtis Oursler, Gary Center, Indiana University;
Dr. Ting Kwan Pan, University of California, Berkeley, Calif.;
Mr. Kenneth Stephen Phelan, Saint John's University, Brooklyn, N.Y.;
Professor Willis Lloyd Pickard, State University of New York, New Paltz, N.Y.;
Mr. John F. Purdy, Goodyear Tire and Rubber Co., Akron, Ohio;
Mr. Edward Roy Rang, University of Wisconsin;
Mr. Roy Franklin Reeves, Iowa State College of Agriculture and Mechanic Arts;
Professor Thomas Robert Richards, Wilkes College, Wilkes-Barre, Pa.;

Professor Charles Lathan Riggs, Kent State University, Kent, Ohio;
Dr. Julien Ashton Ripley, Jr., Physics Dept., Montgomery Junior College, Bethesda Md.;
Miss Oral Maye Robbins, State Teachers College, Milwaukee, Wis.;
Mr. David Rosen, University of Pennsylvania;
Professor Jules Perry Russell, Polytechnic Institute of Brooklyn;
Miss Cecile Blanche Salwen, Hunter College;
Mr. Leonard Schieber, Jr., Lees-McRae College, Banner Elk, N.C.;
Mr. Robert Charles Seber, Rockford College, Rockford, Ill.;
Mr. Bernard David Seckler, Long Island University, Brooklyn, N.Y.;
Mr. Nathan Thomas Seeley, Jr., Agricultural, Mechanical and Normal College, Pine Bluff, Ark.;
Dr. Esther Seiden, University of California, Berkeley, Calif.;
Mr. Sharad Shankar Shrikhande, College of Science, Nagpur, India;
Mr. Luiz Oswaldo Teixeira de Silve, Polytechnic School, Catholic University, Rio de Janeiro, Brazil;
Professor Ralph Axel Staal, University of New Brunswick;
Mr. George Henry Stanley, Jr., Colby College, Waterville, Me.;
Mr. David Sternberg, Physics Department, Columbia University;
Professor Joseph Talacko, Marquette University, Milwaukee, Wis.;
Mr. Nien Yee Tang, University of Washington;
Professor Abdelnour Simon Thomas, Boston College, Chestnut Hill, Mass.;
Mr. Morris Edward Tittle, Agricultural and Mechanical College of Texas, College Station, Tex.;
Mr. John Edward Vollmer, University of Maryland, College Park, Md.;
Professor Enrico Giovanni Volterra, Illinois Institute of Technology;
Mr. Frank Joseph Wagner, Creighton University, Omaha, Neb.;
Mr. Charles Kenneth Wasserman, University of Idaho, Moscow, Idaho;
Miss Betty Rose Weber, University of South Carolina;
Mr. Edwin Weiss, Massachusetts Institute of Technology;
Mr. William Joseph Wells, State Teachers College, Mankato, Minn.;
Mr. George Nichols White, Jr., Brown University;
Professor Mary Elizabeth Williams, Skidmore College, Saratoga Springs, N.Y.;
Mr. Robert Joel Wisner, University of Washington;
Mr. George E. Witter, University of Idaho, Moscow, Idaho;
Professor Gerhard Norval Wollan, North Georgia College, Dahlonega, Ga.;
Sister Mary Rupert Zeiser, Mount Saint Clare Academy, Clinton, Iowa.

It was reported that the following one hundred forty-one persons had been elected to membership on nomination of institutional members as indicated:

University of Alabama: Professor James Clifton Eaves, Mr. Judson James Hart;
University of British Columbia: Mr. James Lewin McGregor;
Brooklyn College: Mr. Leonard Geller;
Brown University: Dr. Jacques Heyman, Mr. John Allen Lewis;
University of California, Berkeley: Messrs. Hewitt Kenyon and Maurice Sion;
University of California, Los Angeles: Messrs. James Richard Jackson and Elmer Edwin Osborne;
California Institute of Technology: Messrs. Benjamin Bernholtz, Robert Yost Dean, William Thomas Guy, Robert Hunter Owens, Carleton Bernard Solloway, and Richard Bennett Talmadge;

University of Chicago: Messrs. Leonard David Berkovitz, Murray Gerstenhaber, Oscar Irving Litoff, Ernest Arthur Michael, and I. Alex Rosenberg;
University of Cincinnati: Professor Emilio Baiada;
City College, New York: Messrs. Robert John Aumann, Leon Ehrenpreis, Carl Engelman, and Leopold Flatto;
Columbia University: Messrs. Maurice Auslander and Alfred Lorenzo Duquette, Miss Alice Krikorian, Mr. Burt Jules Morse;
Cornell University: Mr. Jerome Blackman, Miss Joanne Elliott, Messrs. Carl Samuel Herz and Ralph Gordon Selfridge;
Harvard University: Messrs. Henry Nelson Browne, Jr., Robert Lawrence Graves, Alan Francis Kay, Lawrence Friedman Markus, Robert Osserman, Eoin Laird Whitney, and Norman Zachary Wolfsohn;
Illinois Institute of Technology: Messrs. Robert Alonzo Eubanks and Nathaniel Roy Goodman, Miss Elizabeth Catherine Kleinhans, Messrs. Howard S. Levin, Jack Milton Miller, Seymour Victor Parter, Stewart Schlesinger, Leo Andrew Schmidt, and William Barry Snarr;
University of Illinois: Mr. Martin Nixon Chase, Miss Marjorie Baldwin Djorup, Messrs. Hans Carl Flesch, Donald Gordon Higman, Joseph Hope Hornback, Mitsuru Kadoyama, Jerome Henry Manheim, Merle Wallace Milligan, William Rolen Orton, Jr., Donald Lawrence Sullivan, and Robert John Weeks;
Institute for Advanced Study: Drs. Raoul Bott and Raouf Doss, Professors Olof Hanner and Kunihiko Kodairo;
Iowa State College of Agriculture and Mechanic Arts: Mr. Robert J. Lambert;
State University of Iowa: Mr. Robert Vincent Hogg;
Johns Hopkins University: Messrs. Edward Daniel Carey, Ralph Elwood Keirstead, Jr., William Edward Moore, Norman L. Rabinovitch, Alexander Cormac Smith;
University of Kansas: Mr. Robert Charles Fisher;
Kenyon College: Mr. Daniel Rightor Mason;
Lehigh University: Messrs. Benjamin Collins Kenny and Hubert Howard Snyder;
University of Maryland: Professors George Gerald O'Brien and John Aloysius Tierney;
Massachusetts Institute of Technology: Mr. John Weber Carr, III, Mrs. Violet Bushwick Haas, Messrs. Flavio Botelho Reis and William Lucas Root;
Michigan State College: Mr. Henry Arthur Hanson;
University of Michigan: Messrs. Fred Brafman and Joshua Chover, Miss Margaret Louise Comstock, Mr. Arthur Covington Downing, Jr., Rev. Lester Joseph Heider, Messrs. Robert Maxwell Lauer, John Francis Riordan, and Shih-Hua Tsao;
University of Missouri: Mr. H. C. Griffith;
New York University: Messrs. Ignace I. Kolodner and Chankey Nathaniel Touart;
Ohio State University: Mr. Edley Wainright Martin, Jr., Miss Vera Marie Tozzer;
Oklahoma Agricultural and Mechanical College: Mr. George B. Pedrick;
University of Oregon: Mr. Luther Peh-Hsiun Cheo;
Pennsylvania State College: Mr. Joseph D. E. Konhauser;
University of Pennsylvania: Mr. Athanasios Papoulis;
Purdue University: Mr. Obert Benjamin Moan;
College of St. Thomas: Mr. Richard Anthony Zemlin;
Smith College: Miss Elinor Constance Martin;
Stanford University: Messrs. Kenneth Lloyd Cooke, William James Firey, Howard Ashley Osborn, and Edgar C. Smith, Jr.;
Swarthmore College: Mr. Albert Ross Eckler;

Syracuse University: Mr. Frank Pao-ming Pu;

University of Tennessee: Mr. Eldon Eugene Posey;

Department of Pure Mathematics, University of Texas: Miss Lida Baker Kittrell, Messrs. Herbert Allen Morris, Ernest Tilden Parker, and Spurgeon Eugene Smith;

University of Toronto: Messrs. Geoffrey Eric Norman Fox, Harold Stanley Heaps, Steve Alexander Kushneriuk, and Adrian Eugen Scheidegger;

University of Virginia: Messrs. Frank Thomas Dresser, David Barrington Lowdenslager, Leonard Robert Schlauch, and Robert Fones Williams;

University of Washington: Messrs. Chi-Yuan Lee and Fred H. Ringey;

Wellesley College: Miss Joyce B. Friedman;

University of Wisconsin: Messrs. William Francis Ames, Paul Axt, Robert Joseph Buehler, William F. Donoghue, Jr., Wendell Helms Fleming, Melvin Henriksen, Erwin Kleinfeld, and Paul William Knaplund, Miss Joan K. Ross, Messrs. Garth Hollis Markle Thomas and Arnold Wendt;

Yale University: Messrs. John Daniel Baum, Thaddeus Bankson Curtz, Leon William Green, Henry George Jacob, Jr., and Sol Schwartzman.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: Deutsche Mathematiker Vereinigung: Dr. Wilhelm R. Fischer, University of Pennsylvania (nominee); Professor Alwin Walther, Technische Hochschule, Darmstadt, Germany; London Mathematical Society: Professor Patrick Du Val, University of Georgia; Dr. H. G. Eggleston, Swansea University College, Wales; Mr. David Emrys Evans, University College of North Wales; Mr. Marcus Campbell Goodall, Institute for Advanced Study (nominee); Mr. Walter Kurt Hayman, Brown University (nominee); Miss Marjorie Littleton, H. H. Wills Physical Laboratory, University of Bristol; Dr. Claude Ambrose Rogers, University College, London, and member, Institute for Advanced Study (nominee); Dr. Hsien-Chung Wang, Louisiana State University; Polish Mathematical Society: Professor Adam Feliks Bielecki, University of Lublin; Professor Mieczysław Kwiryn Biernacki, University of Marie Curie-Skłodowska; Mrs. Helena Alina Raś, University of Warsaw; Dr. Roman Sikorski, University of Warsaw; Société Mathématique de France: Dr. Shmuel Agmon, Rice Institute; Mr. Jean Arbault, University of Poitiers; Professor Claude Gilbert Chabauty, University of Strasbourg; Mr. Robert Louis Croisot, University of Poitiers; Professor Leonce Lesieur, University of Poitiers; Dr. Jacques Riquet, Institut Henri Poincaré; Professor Laurent Schwartz, University of Nancy; Wiskundig Genootschap te Amsterdam: Dr. Jacob Korevaar, Purdue University.

The following universities were elected as institutional members of the Society: University of Massachusetts; Princeton University; University of Wichita.

The Secretary is pleased to report at this time that the ordinary membership of the Society is now 4064, including 339 nominees of institutional members and 49 life members. There are also 107 institutional members. The total attendance at all meetings in 1949 was 2470; the number of papers read was 580; there were 13 hour addresses, 5 addresses and 16 papers at the Applied Mathematics Symposium, 1 Gibbs Lecture, and 4 Colloquium Lectures; the number of members attending at least one meeting was 1470.

It was reported that the Council had voted by mail to approve the list of Honorary Presidents and Honorary Chairmen for the 1950 International Congress, as recommended by the Organizing Committee. The Council had also voted by mail to accept for publication in Mathematical Surveys a manuscript by Claude Chevalley entitled *Introduction to the theory of algebraic functions of one variable.*

The following appointments by President J. L. Walsh of representatives of the Society were reported: Professor H. A. Rademacher on the Editorial Board of the American Year Book for a term of three years beginning January 1, 1950; Professor T. R. Hollcroft at the inauguration of the Very Reverend Juvenal Lalor as President at St. Bonaventure College on September 22, 1949; Professor R. H. Marquis at the inauguration of William Bay Irvine as President of Marietta College on October 15, 1949; Professor E. G. Olds at the inauguration of Will W. Orr as President of Westminster College on October 14, 1949; Professor D. S. Morse at the One Hundred Twenty-fifth Anniversary of the founding of Rensselaer Polytechnic Institute on October 13–15, 1949; Professor W. M. Whyburn at the inauguration of Arthur Hollis Edens as President of Duke University on October 22, 1949; Professor G. E. Raynor at the inauguration of David E. Weinland as President of Moravian Seminary and College for Women on October 22, 1949; Professor David Blackwell at the inauguration of Alonzo G. Moron as President of Hampton Institute on October 29, 1949; Professor B. W. Jones at the inauguration of Albert Charles Jacobs as Chancellor of the University of Denver on November 19, 1949; Professor Gweneth Humphreys at the inauguration of George Tyler Miller as President of Madison College on December 10, 1949.

The following additional appointments by the President were reported: Professors R. E. Langer (Chairman), E. F. Beckenbach, and S. S. Wilks as a committee to nominate Society representatives on Policy Committee; Dr. A. S. Householder (Chairman), Professors J.

A. Cooley, G. A. Garrett, W. S. Snyder, J. M. Thomas, and W. M. Whyburn as Committee on Arrangements for meeting at Oak Ridge, Tennessee, on April 21–22, 1949; Professors W. M. Whyburn (Chairman), Tomlinson Fort (term to expire December 31, 1951), and J. M. Thomas (term to expire December 31, 1950) as Committee to Select Hour Speakers for Southeastern Sectional Meetings; Professor B. P. Gill as Chairman of the committee to consider non-editorial problems of the Memoirs (to replace Professor A. W. Tucker); Professor Einar Hille as Chairman of the Committee on the Role of the Society in Mathematical Publication (to replace Professor R. L. Wilder); Miss Grace Bolton and Professor Arthur Sard as tellers for the 1949 annual election; Professors C. G. Phipps (Chairman), R. G. Blake, W. A. Gager, H. M. Gehman, W. R. Hutcherson, F. W. Kokomoor, H. A. Meyer, and W. M. Whyburn as a Committee on Arrangements for 1950 Annual Meeting; Professor A. A. Albert as a member of the Committee to Select Hour Speakers for Summer and Annual Meetings for the period 1950–1951 (committee now consists of Professors J. R. Kline, Chairman, A. A. Albert, and G. A. Hedlund); Professor E. G. Begle as member of the Committee to Select Hour Speakers for Eastern Sectional Meetings for the period 1950–1951 (committee now consists of Professors T. R. Hollcroft, Chairman, E. G. Begle, and D. V. Widder); Professor Walter Leighton as member of the Committee to Select Hour Speakers for Western Sectional Meetings for the period 1950–1951 (committee now consists of Professors J. W. T. Youngs, Chairman, Ralph Hull, and Walter Leighton); Professor D. C. Spencer as a member of the Committee to Select Hour Speakers for Far Western Sectional Meetings for the period 1950–1951 (committee now consists of Professors J. W. Green, Chairman, E. F. Beckenbach, and D. C. Spencer); Professor H. S. Wall as a member of Committee on Visiting Lectureships for the period 1950–1952 (committee now consists of Professors Hassler Whitney, Chairman, H. S. Wall, and Norbert Wiener); Professor J. B. Rosser as liaison officer for Quarterly of Applied Mathematics for the period 1950–1952; Professors K. O. Friedrichs and John von Neumann as members of Committee on Applied Mathematics for the period 1950–1952; Professors Garrett Birkhoff and G. C. Evans as members of Committee on Role of the Society in Mathematical Publication for the period 1950–1952; Professor R. A. Johnson as a member of Committee on Printing Contracts for the period 1950–1952.

It was reported to the Council that: Professor G. E. Uhlenbeck has accepted the invitation to deliver the Gibbs Lecture at the 1950

Annual Meeting; Professor Einar Hille has been appointed a member of the Organizing Committee of the International Congress to replace the late Dean R. G. D. Richardson; Professor Einar Hille was substituted for Professor M. H. Stone as a nominee for the Mathematical Reviews Editorial Committee on the ballot for the annual election; the usual biennial membership campaign was held in October, 1949, at which time invitations to join the Society were sent to 4,203 teachers of collegiate mathematics (including 485 members of the Mathematical Association of America); during 1950 Professor G. T. Whyburn will act as Managing Editor of the Transactions and Memoirs, Professor C. C. MacDuffee will act as Chairman of the Colloquium Editorial Committee, Professor A. W. Tucker will act as Chairman of the Mathematical Surveys Editorial Committee, Dean E. B. Stouffer will act as Chairman of the Bulletin Editorial Committee, and Professor Nathan Jacobson will act as Chairman of the Proceedings Editorial Committee.

At the annual election which closed on December 29, and at which 744 votes were cast, the following officers were elected:

President Elect, Professor John von Neumann.

Vice Presidents, Professors E. J. McShane and R. L. Wilder.

Associate Secretaries, Professors J. W. Green, W. M. Whyburn, and J. W. T. Youngs.

Members of Editorial Committee of the Bulletin, Professors G. B. Price and A. C. Schaeffer.

Members of Editorial Committee of the Transactions, Professors L. V. Ahlfors and J. L. Doob.

Member of the Editorial Committee of the Colloquium Publications, Professor R. L. Wilder.

Member of the Editorial Committee of Mathematical Reviews, Professor Einar Hille.

Member of the Editorial Committee of Mathematical Surveys, Professor W. T. Martin.

Representative on the Editorial Board of the American Journal of Mathematics, Professor Samuel Eilenberg.

Members at large of the Council, Professors Warren Ambrose, P. R. Halmos, Mark Kac, S. B. Myers, and D. C. Spencer.

The Council voted to approve April 28, 1950, as an additional date for the spring meeting in the far west. Dates of other meetings in 1950 were set as follows: October 28 in New York City; November 24–25 in Chicago; November 25 in the far west; December 27–29 at the University of Florida. The Council also accepted an invitation to hold the October, 1951, meeting at the National Bureau of Standards in

Washington, D. C.

In an appendix to this report are excerpts from the report of the Treasurer for the fiscal year 1949 as verified by the auditors. A copy of the complete report will be sent, on request, to any member of the Society.

The American Journal of Mathematics, which is a joint enterprise of the American Mathematical Society and The Johns Hopkins University, reported that it had published 975 pages in 1949.

The Librarian reported the following additions to the Library: 309 bound volumes of periodicals, 124 books, and 139 pamphlets (including 41 dissertations).

Certain invitations to give addresses in 1950 were announced; Professor C. L. Siegel for the February meeting in New York City; Professors J. S. Frame, Sam Perlis, and M. F. Smiley for the February meeting in East Lansing, Michigan; Professors Atle Selberg and R. P. Boas for the April meeting in Washington, D. C.

The Bulletin Editorial Committee reported that the backlog of the journal was such that a period of approximately eighteen months must now elapse between acceptance and publication of a particular paper. A recommendation that a total of 1438 pages be authorized for the 1950 Bulletin and Proceedings was accepted by the Council and referred to the Trustees. This represents an increase of 249 pages over the 1949 volume of the Bulletin. Professors C. B. Allendoerfer, G. Hochschild, and J. B. Diaz were reported as new Assistant Editors for the Proceedings.

The Transactions Editorial Committee reported that the transfer of several long papers to the Memoirs had greatly relieved the backlog of papers awaiting publication in the Transactions. Additional funds for the publication of the Memoirs were requested and approved, subject to a vote of the Trustees. The Council voted to authorize the Committee on Printing Contracts, in consultation with the editors, to study the possibility of economizing in the cost of the Transactions by reducing the margins.

The Mathematical Reviews Editorial Committee reported that 766 pages of reviews had been published, as against 638 pages for 1948; the increase was due partly to the policy of encouraging longer reviews for Russian papers but also to the increase in mathematical production in most countries. During 1949 the Société Mathématique de France and the Unione Matematica Italiana were added to the list of sponsors. The subscription list as of November, 1949, was 2,126.

The Council authorized the Executive Committee, with the

Trustees, to negotiate a contract with Columbia University in which the interests of the Society with respect to housing would be protected and in which the library of the Society would be used as a proper bargaining asset.

A recommendation in connection with the Executive Editorship of Mathematical Reviews was accepted and referred to the Trustees for approval.

The Council voted to act as a sponsor of a symposium on the *Eigenvalue problem* to be held about the middle of June, 1950, at Oklahoma Agricultural and Mechanical College.

The Council voted to accept invitations to hold the 1951 Annual Meeting at Brown University and the 1953 Summer Meeting at Laval University; the latter meeting will be in conjunction with the meeting of the Canadian Mathematical Congress and the 100th anniversary of the charter of Laval University.

A report on activities of the Policy Committee was presented, including the following items: (1) Plans for a Union Conference to be held at Columbia University for three days immediately preceding the International Congress, to be attended by delegates of National Committees which have been discussing statutes and by-laws for a new International Mathematical Union; (2) Discussion of the possible establishment of a Mathematics Foundation, to receive and administer funds contributed for the general support of mathematics, by a subcommittee of the Policy Committee; (3) Study by a subcommittee of the needs of mathematics which might be met by a National Science Foundation.

The Organizing Committee of the International Congress reported that satisfactory progress had been made toward a solution of the difficult problem of visas. All indications point to a successful Congress of a truly international character at Cambridge in the summer of 1950.

On recommendation of the Committee on the Role of the Society in Mathematical Publication, the Council voted to recommend to the Trustees that for a period of three years the Society contribute an annual subsidy not to exceed $1500 to the proposed Pacific Journal of Mathematics.

The Council voted to omit the 1950 Symposium in Applied Mathematics.

It was voted to advance by forty-eight hours the deadline for papers to appear in the programs of Society meetings.

The President was authorized to appoint a committee to study the relation of the Bulletin and Proceedings to individual membership dues and to reconsider the matter of individual dues.

Abstracts of the papers read follow. Presiding officers at the sessions for contributed papers were Professors W. M. Whyburn, Arnold Dresden, C. A. Truesdell, Wallace Givens, R. H. Bruck, B. O. Koopman, F. W. Perkins, G. L. Walker, Dr. R. S. Burington, Professors A. A. Bennett and M. R. Hestenes.

Papers whose abstract numbers are followed by the letter "*t*" were read by title. Paper number 99 was presented by Mr. Macon, 131 by Professor Goffman, 135 by Mr. Rogers, 142 by Professor Rosenbloom, 147 by Professor Brunk, 154 by Professor Rado, 170 by Professor Lin, 180 by Professor Sard, 183 by Mr. O'Brien, 195 by Professor Kasner, 210 by Professor Cohen.

Professor Samuelson was introduced by Professor W. T. Martin and Professor Baiada by Professor C. N. Moore.

ALGEBRA AND THEORY OF NUMBERS

99. A. T. Brauer and Nathaniel Macon: *On the approximation of irrational numbers by the convergents of their continued fractions*. II.

Let ξ be any positive irrational number and A_n/B_n the convergents of its expansion as a regular continued fraction. If λ_n is defined by $|\xi - A_n/B_n| = \lambda_n^{-1}B_n^{-2}$, then the best possible lower bounds for the sums of any k consecutive λ_n are obtained. Hurwitz [Math. Ann. vol. 39 (1891) pp. 279–284] proved that the limit of the arithmetic means of the first n numbers λ_i equals $5^{1/2}$ for $(1+5^{1/2})/2$ and its equivalent numbers. It is proved here that the lower limit of these arithmetic means is not less than $5^{1/2}$ for every irrational number. (Received November 14, 1949.)

100. R. H. Bruck: *A numerical invariant for finite nets.*

The author initiates a study of "imbedding problems" for nets. A (finite) net N of degree k, order n is a system of "points" and k classes of "lines" such that: (i) each point is on one line of each class; (ii) two lines of distinct classes have a common point; (iii) a line has n points. (Finite affine planes, sets of orthogonal latin squares, finite loops, can be interpreted as nets.) An integer i is "represented" by N provided there exists an integer-valued point-function f which sums to i over each line of N. The invariant $\phi(N)$ is the l.p.i. represented by N; $\pi(N)$, the l.p.i. represented with a non-negative f. A n.a.s.c. that there exist n points of N, no two collinear, is that $\pi(N)=1$; a necessary condition, that $\phi(N)=1$. The invariant ϕ is explicitly calculated for affine planes and loops. Relations are obtained between the ϕ's of a net and its homomorphic image; sharper relations, for direct products. As an application: if n is a product of coprime prime-powers, the smallest of which is $k-1$, there exists a set of $k-2$ mutually orthogonal $n\times n$ latin squares to which no square can be added. (Received December 12, 1949.)

101. K. T. Chen: *Integration in free groups and commutator calculus.*

To each word in a free group F with n generators a path in a Euclidean n-space is assigned. The integral of any polynomial along this path depends only on the element of F represented by the word. The totality of elements of F for which the integral of every polynomial of degree less than d vanishes forms a subgroup of F which is proved to be the product of the dth lower central commutator group ${}^dF$ with the second upper central commutator group $F^{(2)}$. Using this it is found that ${}^{d-1}F\cdot F^{(2)}/{}^dF\cdot F^{(2)}$ is a free abelian group with $(d-1)C_{d+n-2,d}$ generators $(n\geqq 2)$. Further the theory can be applied to the computation of the above factor group for any group G given by a finite number of generators and relations, thus yielding numerical invariants of G. (Received November 10, 1949.)

102*t*. Sarvadaman Chowla and Paul Erdös: *A theorem on the distribution of the values of L-functions.*

The authors prove the following result: Let (d/n) denote Kronecker's symbol defined for $d\equiv 0$, 1 (mod 4) and d not equal to a perfect square. Define for $s>0$, $L_d(s) = \sum_1^\infty (d/n)n^{-s}$. Denote by $g_s(a, x)$ the number of integers $d\leqq x$ such that (i) $d\equiv 0, 1\,(4)$, (ii) $d\neq t^2$, (iii) $L_d(s)<a$. Then if $s>3/4$, $\lim_{x\to\infty} g_s(a, x)/2^{-1}x=g_s(a)$ exists; furthermore $g(-\infty)=0$, $g(\infty)=+1$, and $g_s(a)$, the distribution function, is a continuous and steadily increasing function of a. (Received November 14, 1949.)

103*t*. Sarvadaman Chowla: *On the nonvanishing of a certain infinite series.*

The author proves the following result: For every integer n let $f(n)$ be an integer such that (i) $f(n)=f(n')$ whenever $n'\equiv n$ (mod p), (ii) $f(n)=-f(-n)$. Then $\sum_1^\infty f(n)n^{-1}\neq 0$, unless all $f(n)$ are zero. Here p is a prime such that p and $(p-1)/2$ are both primes. The paper also contains a proof of an extension of this theorem due to Siegel. (Received November 14, 1949.)

104*t*. Harvey Cohn: *Finiteness conditions for a convex body in hyperspace.*

If a convex body in d-dimensional space contains a hypersphere of radius R about the origin, then unless it contains an integral lattice point within the hypersphere of radius C_d/R^{d-1}, it will lie entirely within the latter hypersphere. There is a best constant C_d as $R\to 0$, but it is more difficult to determine. Tentative bounds for C_d are found by letting the Dirichlet boxing in principle oppose the Perron transferal principle. (Received November 11, 1949.)

105*t*. Harold Davenport: *Euclid's algorithm in certain quartic fields.*

It is proved in this paper that Euclid's algorithm can hold in only a finite number of quartic fields of the totally complex type. The method is a development of that already applied by the writer to real quadratic fields and to cubic fields of negative discriminant, in a series of papers in course of publication. An additional complication, as compared with those cases, arises from the fact that the inequalities which naturally present themselves are not quite precise enough to be of service. (Received December 27, 1949.)

106. H. C. Davis: *Closure lattices.* Preliminary report.

Corresponding to a Boolean algebra B there is defined (uniquely) a lattice $L(B)$ with the following properties: (1) It contains the partition lattice over B as a sublattice; (2) it is pseudo-complemented; (3) if B is finite, $L(B)$ is semi-modular. There is a natural dual homomorphism of $L(B)$ into the lattice of closure algebras over B—the latter lattice being defined in immediate generalization of the case where B is the field of all subsets of some set. The extension to the case where B is replaced by a more general lattice is discussed. (Received December 14, 1949.)

107. D. T. Finkbeiner: *A general dependence relation for lattices.*

Let M be an arbitrary set, and let D be a relation between the elements and subsets of M. D is called a dependence relation if D satisfies (1) $mD(S\vee m)$ for arbitrary $S\subseteq M$, and (2) mDS and SDT imply mDT. A subset S is called closed if mDS implies $m\in S$, and the closed subsets form a complete lattice L. MacLane (Duke Math. J. vol. 4 (1938) pp. 455–468) has characterized dependence relations for which L is a semi-modular point lattice. In this paper M is considered to be partially ordered, and a corresponding characterization is given for dependence relations which preserve the ordering of M, in that L is a complete lattice whose set of completely join irreducible elements is isomorphic to M. Furthermore L satisfies a point-free exchange axiom which defines semi-modularity for lattices in which coverings may not exist. If M is not ordered, this characterization reduces to the classical result for semi-modular point lattices. (Received November 14, 1949.)

108*t*. A. L. Foster: *On n-ality theories in rings and their logical algebras; tri-ality principle in three-valued logics.*

The K-ality theory, as launched in a series of previous publications, represents a simple abstract framework which numbers among its realizations—and thus serves to unify—disciplines as widely dissimilar as, for example, the classical Boolean theory together with many new extensions thereof on the one extreme, and the traditional invariant-transformation theories (for example, tensors, groups, forms, geometries, and so on) on the other. The broad program involves the study of arbitrary disciplines (for example, rings, and so on) "in" different "coordinate systems" belonging to a group K of admissible "coordinate transformations" in the discipline. Thus, specialized to rings R and with K chosen as C, the "simple complementation" group (of order 2) generated by $x^*=1-x$, one obtains the C-ality (or "simple" duality) theory of rings, which further reduces to the classical Boolean duality when R is Boolean. The present communication, while still confined to rings, is not restricted to the C-level. The *K-ality theorems*, successors of the simple duality notions, are given, and various problems elevated to the general K-level. Among these is the concept *ring-logic* (mod K), generalizing the classical Boolean ring-Boolean algebra (logic) equivalence. (Received November 25, 1949.)

109*t*. A. L. Foster: *p-rings and their Boolean-vector representation.*

In connection with the K-ality theory (see previous abstract), the general structure of p-rings is studied. Using certain results previously obtained, it is found that all p-rings are isomorphic with a certain (generalized) hypercomplex "vector p-ring" over a suitable Boolean algebra—namely the idempotent Boolean sub-algebra of the p-ring. (In the vector representation, the vectors are componentially determined, but are not in general additive—hence the expression "generalized.") This leads to a generalized hypercomplex representation of p-rings which is a generalization of the familiar Stone representation for the special Boolean ($p=2$) case. Earlier results on p-rings of McCoy and Montgomery are subsumed and generalized. (Received November 9, 1949.)

110t. A. L. Foster: *Ring-algebras (logics) and p-rings.*

The ring-logic (mod N) status of p-rings for $p>3$ was left unsolved in a previous paper (see first abstract above). Applying the results on the structure (decomposition and p-vector synthesis) of p-rings obtained later (see preceding abstract), the general theorem is proved: each p-ring is a ring-logic (mod N). (Received November 9, 1949.)

111. Emil Grosswald: *On some algebraic properties of the Bessel polynomials.*

For Bessel polynomials (see paper by H. L. Krall and Orrin Frink, Trans. Amer. Math. Soc. vol. 65 (1949) pp. 100–115), two asymptotic formulas are established and upper bounds for the errors terms are found. Using recursion formulas, it is shown that all the roots of Bessel polynomials are simple and non-real, except a single root, x_n, of the polynomials of odd degree n, which is real and satisfies $-1 \leqq x_n \leqq -1/n$. Let p be an odd prime, m a positive integer, q an integer less than p^m; then, using mainly Dumas' theorem (Journal de Mathématiques (6) vol. 2 (1906) pp. 191–258), it follows that Bessel polynomials are algebraically irreducible in the field of rational numbers, at least for $n \leqq 323$, and also for $n>323$, if n is of one of the forms: p^m, p^m+1, $q \cdot p$, $q \cdot p-1$, p^m-1, 2^m-1, $q \cdot p^m$ (in the last 3 cases, with some restrictions); if not irreducible, any Bessel polynomial contains an irreducible factor of degree not less than $A \cdot (n-1)$, where $A>16/17$. Using theorems of I. Schur (Preuss. Akad. Sci. Sitzungsber. (1929) pp. 443–449) and Dedekind, it is found that all irreducible Bessel polynomials (except, perhaps, for $n=9, 11, 12$) lead to equations without affect. (Received November 7, 1949.)

112t, Franklin Haimo: *A representation for Boolean algebras.*

For a direct sum G of additive Abelian G_α, elements $x=\{x_\alpha\}$ and $y=\{y_\alpha\}$ are said to be in the relation $x \perp y$ if $x_\alpha \neq 0$ implies $y_\alpha=0$, and likewise with the x's and y's interchanged. Abelian groups which have a partial ordering like that of a direct sum, where $x \geqq y$ means that $y \perp x-y$, are called vector ordered groups (quasi vector ordered if the generalization is based on weak rather than on strong summation). It is proved that every complete Boolean algebra can be represented faithfully by the suitably ordered set of all endomorphisms bounded above by the identity endomorphism on a vector ordered group. Each of these endomorphisms is splitting, and their image spaces contain elements maximal therein. Conversely, such endomorphisms on a vector ordered group form a complete Boolean algebra. The not necessarily complete Boolean algebras are linked, in a somewhat similar fashion, to the quasi vector ordered groups. (Received November 14, 1949.)

113t. Franklin Haimo: *Decomposition of complete Boolean algebras.*

Direct decompositions of vector ordered groups (see the preceding abstract) are connected with join decompositions of the corresponding complete Boolean algebra. To each infinite cardinal corresponds a pair of decompositions, one for the group, the other for the algebra. One of the components of the group decomposition turns out to be a strong direct sum of subgroups, each of which is the image space of a splitting endomorphism. Correspondingly, every complete field of sets can be generated by a set of disjoint sets in the field. Each of these generators is small in that each contains, set-theoretically, only a limited number of such generators. This subsumes a result due to Tarski, Fund. Math. vol. 24 (1935) p. 197, Theorems 6. (Received November 14, 1949.)

114. I. N. Herstein: *On a conjecture on simple groups.*

If G is a finite group, let Γ_p be the group ring G over a field of characteristic p, and N_p the radical of Γ_p. Consider the following two assertions: (A) A simple group of odd order is of prime order. (B) If the order, n, of G is odd, then for some prime p, $p|n$, there exists a $g\in G$, $g\neq 1$, with $g-1\in N_p$. The result found in the paper is that (A) and (B) are equivalent; that (B) implies (A) follows easily by showing that $\{g\in\sigma | g-1\in N_p\}$ is a normal p-subgroup of G. To show (A) implies (B), use is made of Clifford's theorem on the representations induced in normal subgroups. (Received November 14, 1949.)

115. B. W. Jones: *An extension of Meyer's theorem on indefinite ternary quadratic forms.*

Let f be a properly or improperly primitive indefinite ternary quadratic form of determinant d such that its matrix has integer elements, Ω is the g.c.d. of its two-rowed minor determinants and $d=\Omega^2\Delta$ determines Δ. The principal result of this paper is: there is one class in the genus of f if neither Ω nor Δ is divisible by 4, they are not both even and if, for every prime factor p common to Ω and Δ, the following conditions hold: 1. p^2 divides neither Ω nor Δ; 2. There exists a q, depending on p, which is either a prime or double a prime, does not divide $2d$, and such that every solution of $x^2-qy^2\equiv 1 \pmod{p}$ is congruent (mod p) to a solution of $x^2-qy^2=1$. If $(p\pm 1)/2$ is a prime for proper choice of the ambiguous sign, condition 2 holds. Hence it seems likely that there is an infinite number of primes p satisfying this condition. (Received November 14, 1949.)

116. W. J. LeVeque: *On the equation $a^x-b^y=1$.*

It is shown that the Diophantine equation $a^x-b^y=1$ has at most one solution unless $a=3$, $b=2$, when it has just the two (1, 1) and (2, 3). If a is even and b is odd, the only possible solution is with $x=\beta/\alpha$, where 2^α is the highest power of 2 dividing a and 2^β is the highest power of 2 dividing $b+1$. If a is odd and b is even and $(a, b) \neq (3, 2)$, then the only possible solution is with $y=t$, where t is half the exponent to which b belongs (mod a). As an application, it is shown that the identity $\sum_{k=1}^n k^3 =(\sum_{k=1}^n k)^2$ is the only one of its kind; that is, that the only solution (r, s, t) of $\sum_{k=1}^n k^s=(\sum_{k=1}^n k^t)^r$ is (2, 3, 1). (Received November 14, 1949.)

117*t*. R. C. Lyndon: *Groups with one defining relation.*

If $R=Q^q$ for Q a word in a free group F and q maximal, then an identity $\prod T_iR_i^{e_i}T_i^{-1}=1$ (where T_i in F, $e_i=\pm 1$) implies that the indices fall into pairs (i, j) such that $e_i=-e_j$ and that, for some p_i, $T_i\equiv T_jQ^{p_i}$ modulo R. A slightly more general theorem is proved, by a method derived from Magnus (J. Reine Angew. Math. (1930)). From incidence matrices (with operators) one calculates the Eilenberg-MacLane groups $H^n(G, K)$ for G defined by a single relation, and any coefficients K. For $n>2$ these groups are the exact analogues of the special case G cyclic of order q (infinite cyclic if $q=1$), as given by Eilenberg and MacLane (see Bull. Amer. Math. Soc. vol. 55 (1949) p. 22). (Received November 14, 1949.)

118*t*. F. I. Mautner: *Unitary representation of locally compact groups. II.*

This is a continuation of *Unitary representations*. I. Whereas part I dealt to a large extent with discrete groups, the problems of the present part II are mainly relevant in the case of non-discrete groups. Indeed the main problem treated is the structure of the operator algebra W generated by any given unitary representation of a semi-simple Lie group. And after various preliminary sections dealing with general properties of unitary representations and generalized direct sums of Hilbert spaces (cf. von Neumann, *Reduction theory*, Ann. of Math. (1949)) it is shown that if G is the 2×2-unimodular group (or the Lorentz group), then the ring W generated by any continuous unitary representation of G gives rise to factors of type I only in the sense of von Neumann (loc. cit.). An easy consequence of this is a proof of the (known) Fourier-inversion formula for G, which uses relatively few special properties of G. The analogous problem for arbitrary semi-simple Lie groups is reduced to a problem about the behavior of invariants of the Lie algebra (such as the fundamental bilinear form) under unitary group-representations. (Received November 10, 1949.)

119. A. J. Penico: *The Wedderburn principal theorem for Jordan algebras.*

The title of this paper is the same as that of a paper by Albert (Ann. of Math. vol. 48 (1947) pp. 1–7). He considered the "special" Jordan algebras obtained from associative algebras by the quasi-multiplication $ab=(a\cdot b+b\cdot a)/2$. The algebras considered here are the "general" Jordan algebras defined by the identities: $ab=ba$, $a(a^2b)=a^2(ab)$. The following theorem is proved: If A is a Jordan algebra over a field of characteristic zero, and if N is the radical of A, then there exists a subalgebra S of A such that A is the direct sum $A=S+N$, and $S\cong A-N$. For $N^2\neq(0)$, and A with unity element, it is proved that AN^2 is an ideal of A properly contained in N, and, by the usual inductive proof, the theorem is reduced to the case $N^2=(0)$. This latter case is then established for $A-N$ any one of the five types of split Jordan algebras. (Received November 14, 1949.)

120t. P. A. Samuelson: *Solving linear equations by continuous substitution.*

To solve the n linear equations $ax=b$, define rectangular matrices $[{}_0A_{ij}]=[a, b]$ and $[{}_kA_{ij}] = [{}_{k-1}A_{ij}(1-\delta_{ik})+({}_{k-1}A_{kk})^{-1}{}_{k-1}A_{kj}\{-{}_{k-1}A_{ik}(1-\delta_{ik})+\delta_{ik}\}]$ where $k=1, 2, \cdots, n$ and $[\delta_{ij}]=I$, the identity matrix. Then the writer's solution is $[{}_nA]=[I, a^{-1}b]$, whatever the number of columns of b. This invariant ritual, easily mechanized, avoids the usual "back-solution" but at the cost of about $n^3/6$ extra multiplications beyond the $n^3/3$ involved in the closely related conventional Gauss-Chio-Doolittle-Crout-Aitken solution of asymmetrical equations. The validity of the present variant of one of Aitken's methods follows from the elementary fact that the typical successive substitution process can be used on *all* n equations, not just on the dwindling number of interdependent equations involved in the reduction process. It may be useful to note that if ${}_0A$ is bordered below by $[c, d]$, then ${}_nA$ will be bordered by $[0, d-ca^{-1}b]$; this tells how to interpret all parts of ${}_kA$ for any k. (Received December 23, 1949.)

121. Ernst Snapper: *Completely primary rings.* III.

Let R and S be completely primary rings, that is, commutative rings with unit elements whose radicals $N(R)$ and $N(S)$ are maximal ideals. Let $R\subset S$ and let $N(S)$ be nilpotent. Assume that the field extension $\overline{R}=R/N(R)\subset S/N(S)=\overline{S}$ either has characteristic zero or preserves p-independence. For any ideal $\mathfrak{a}\subset R$, the smallest ideal of S which contains $\mathfrak{a}$ is denoted by $\mathfrak{a}^*$. If $(N(R))^* = N(S)$, S is a *principal extension* of R. If furthermore $\mathfrak{a} = \mathfrak{a}^* \cap R$ for each $\mathfrak{a} \subset R$, S is a *canonical extension* of R. The theory of algebraic and transcendental ring extensions, reported at the October meeting of the Society in New York, is used to prove that two canonical extensions S and S' are equivalent ring extensions of R if and only if $\bar{S}$ and $\bar{S}'$ are equivalent field extensions of $\bar{R}$. The proof uses the theorem that, whether or not S is a canonical extension, there exists a principal, completely primary ring extension T of R, where $R \subset T \subset S$ and $T/N(T) = \bar{S}$; this fact is closely connected with a theorem of I. S. Cohen on local rings. (Received November 10, 1949.)

122*t*. M. A. Woodbury: *On games whose matrices are the sums of the matrices of other games.*

Let $G^{(k)}$ $(k=1, 2, \cdots, K)$ be the finite two-person zero-sum game with the matrix $(G_{ij}^{(k)}$ $(i=1, 2, \cdots, m; j=1, 2, \cdots, n)$. Let $v(G)$ be the value of the game G and by $X(G)$ and $Y(G)$ respectively denote the (convex) sets of good strategies for the first and second players for G. Define the sum of two games by the addition of their matrices. If $\bigcap_{k=1}^{K} X(G^{(k)}) \neq \phi$ then $(*) v(\sum_{k=1}^{K} G^{(k)}) \geqq \sum_{k=1}^{K} v(G^{(k)})$, since there exists a strategy x such that for all $k=1, 2, \cdots, K$, $xG^{(k)}y \geqq v(G^{(k)})$. If also $\bigcap_{k=1}^{K} Y(G^{(k)}) \neq \phi$ then the equality sign holds. Clearly $X(\sum_{k=1}^{K} G^{(k)}) \supset \bigcap_{k=1}^{K} X(G^{(k)})$ but the inclusion may be proper as is shown by the case $G^{(1)} = -G^{(2)} =$ a matrix whose first row is $1, -1$ and whose second row is $-1, 1$. Another example, $G_1 =$ a matrix whose first row is $1, -1$ and whose second row is $-1, 1$, $G_2 =$ a matrix whose first row is $2, -1$ and whose second row is $-2, 1$, due to D. Gale shows that the equality sign may hold in (*) even if $\bigcap_{k=1}^{K} Y(G^{(k)}) = \phi$. A result of A. W. Tucker follows as an easy consequence of this result (*), namely: Let the matrix of G be divided into blocks by subdividing the strategies of the first and second players into M and N groups respectively, denote by G_{ij} the game defined by the corresponding block and by v_{ij} the value of this game. Further let (v_{ij}) be the matrix of a new game V. Then the relation $v(V) \leqq v(G)$ holds provided that $\bigcap_{j=1}^{N} X(G_{ij}) \neq 0$ for $i=1, 2, \cdots, M$. (Received November 14, 1949.)

123. Daniel Zelinsky: *Rings with ideal nuclei.*

Decomposition theorems like those of Kaplansky and of van Dantzig are proved for certain complete topological rings with a neighborhood system at zero consisting of ideals. Such a ring decomposes as a (complete, usually infinite) direct sum exactly when its discrete homomorphs decompose concordantly. Such a concordant decomposition is at hand if the ring has a restricted minimum condition on open ideals and is either semisimple with first countability axiom or is commutative. The summands are respectively discrete classical simple rings, or primary rings like $\mathfrak{p}$-adic completions of the original ring. A corollary on Dedekind rings implies that an algebraic number field topologized so as to have a bounded open additive subgroup has as completion a local direct sum of $\mathfrak{p}$-adic fields. (Received November 15, 1949.)

Analysis

124. T. M. Apostol: *Identities involving the coefficients of certain Dirichlet series.*

The Dirichlet series $\phi(s) = \sum a(n)n^{-s}$ under consideration are of signature $(\lambda, \kappa, \gamma)$ in Hecke's terminology, converge absolutely for $\sigma > \kappa > 0$, have at worst a first order pole at $s=\kappa$, and have functional equations of the form $(\lambda/2\pi)^s\Gamma(s)\phi(s) = (\lambda/2\pi)^{\kappa-s}\Gamma(\kappa-s)\phi(\kappa-s)$. The author derives the identity $\sum_{n=0}^{x} a(n)(x-n)^q/q! = \rho x^{\kappa+q}\Gamma(\kappa)/\Gamma(\kappa+q+1)+\gamma(\lambda/2\pi)^q x^{(\kappa+q)/2}\sum_{n=1}^{\infty} a(n)n^{-(\kappa+q)/2}J_{\kappa+q}(4\pi\lambda^{-1}(nx)^{1/2})$ where J_ν is the ordinary Bessel function of order ν and ρ is the residue of $\phi(s)$ at $s=\kappa$. The series on the right converges absolutely for positive $q>\kappa-1/2$. This includes as special cases many known identities in analytic number theory. In particular, when $a(n)$ is the number of representations of n as the sum of two squares this gives the integrated form of Hardy's identity in the famous problem concerning lattice points in a circle. The proof is based on a certain Mellin integral for $J_\nu(z)$. The method used also applies to identities of a different type involving the functions $F_p(z) = \cos(\pi p/2)J_p(z) - \sin(\pi p/2)(Y_p(z)-2K_p(z)/\pi)$ and $V_p(z) = (F_{1-p}(z)-F_{1+p}(z))/p$ where Y_p and K_p are the usual Bessel functions. Mellin integrals for these functions are also obtained in the course of the analysis. (Received October 26, 1949.)

125t. Helmut Aulbach: *Some geometrical inequalities for sets in Hilbert space.*

Certain constants $\rho(S)$ and $\sigma(S)$ are introduced for any bounded set S in Hilbert space. $\rho(S)$ was first implicitly introduced by K. Loewner in *Grundzuge einer Inhaltstheorie im Hilbertschen Raume*, Ann. of Math. vol. 40 (1939), and $\sigma(S) = \lim_{n\to\infty} s_n^*$, where s_n^* is the least upper bound of the sides of n-dimensional simplices whose vertices are in S. By the side of an n-dimensional simplex T is meant the length of the edge of that regular n-dimensional simplex whose volume equals the volume of T. A lemma is then proved which states that the side of an n-dimensional simplex T is not greater than the geometric mean of the sides of the $(n-1)$-dimensional simplices which constitute the faces of T. Using this lemma, it is shown that $\rho(S) \leqq \sigma(S)/2^{1/2}$ and a set is exhibited for which the equality holds. Furthermore, a necessary and sufficient condition for a closed set to be compact is that it be bounded and $\sigma=0$. (Received November 18, 1949.)

126. Emilio Baiada: *An isoperimetric problem for ship-bodies.*

Let $y=\phi(x)$, $z=0$, $0\leqq x\leqq 1$, $\phi(0)=\phi(1)=0$, be an A.C. positive function (profile), and $x=f(z)$, $y=0$ (bow); $x=g(z)$, $y=0$ (stern) two arbitrary A.C. functions such that $g(0)=1$, $f(0)=0$. A suitable law being given (similitude in the simplest case), out of the profile and for any $0<z_1\leqq m$, we obtain curves $y=\Phi(x, z_1)$, $z=z_1$, similar to the profile and with first and second end points on bow and stern lines respectively. The solid [ship or wing-body] enclosed by the surface built up and planes $z=0$, $y=0$, $z=m$, has an "external" area A and volume V. Two problems are solved: I. Minimum of A, V being given. The author obtains (1) Existence theorem, (2) Euler equations, (3) When A is minimum, bow and stern are C_2 and convex, and $\phi(x)$ being convex, they meet in a point with tangent parallel to the x-axis. II. Same problem, but profile being given in shape but not in size; (1), (2), (3) are obtained and (4) length of minimum body is found, (5) Euler equation reduces to the first order. (Received November 14, 1949.)

127t. Stefan Bergman: *On solutions with algebraic character of linear partial differential equations. I.*

The author associates with every harmonic function $H(X)$, $X=(x, y, z)$, a function $\chi(Z, Z^*) = \chi_1(Z, Z^*)+(ZZ^*)^{1/2}\chi_2(Z, Z^*)$, $x=2(ZZ^*)^{1/2}$, $y=-i(Z+Z^*)$, $z=(Z-Z^*)$, of two complex variables. χ_k are analytic at $Z=Z^*=0$. $P_1(\chi) = 2\int_{T=0}^{1} u^{1/2}\{d[u^{1/2}$

$\cdot (u\zeta^{-1}T^2, u\zeta(1-T^2))]/du\}dT$, $u=x+\zeta Z+\zeta^{-1}Z^*$ transforms χ (the C_3-associate of H) into a function $f(u, \zeta)$ (the B-associate of H); $p_2(f)=(2\pi i)^{-1}\int_{|\zeta|=1} f(u, \zeta)d\zeta/\zeta$ transforms f into $H(X)$. The $\{\chi\}$ and $\{f\}$ form algebras, while $\{H\}$ forms only a linear space. The author studies functions H and harmonic vectors $H=H_1i+H_2j+H_3k$, div $H=0$, curl $H=0$, generated by P_2 from rational $f(u, \zeta)=p(u, \zeta)/q(u, \zeta)$, $p(u, \zeta) = \sum_{\nu=0}^{n} u^\nu(\sum_{s=-(N-\nu)}^{N-\nu} b_{\nu s}\zeta^\nu)$, $q(u, \zeta)=\sum_{\nu=1}^{m} u^\nu(\sum_{s=-(M-\nu)}^{M-\nu} a_{rs}\zeta^s))$. He obtains algebraic harmonic functions $H(X)=R(\zeta, X)=\sum_{\mu=+N-M-1}^{N+M-1} G_{\mu-N+M+1}(\zeta^\mu/(\partial Q/\partial\zeta))_{\zeta=\zeta^{(\nu)}}$ if $M<N$, where $G_s=\sum_\kappa b_{j,-M+j+s-\kappa}\Gamma_{j\kappa}$, $\Gamma_{\gamma,\lambda}=\sum_{j=[\lambda/2]}^{\min(\lambda,\nu)} 2^{-\nu}C_{\nu,j}C_{j,\lambda-j}(iy-z)^{\nu-j}(2x)^{2j-\lambda}(iy+z)^{\lambda-j}$, $\lambda\leqq 2\nu$, and the $\zeta^{(\nu)}(X)$ satisfy the equation $Q(\zeta; X)=\sum_{\nu=0}^{2N} A_\nu(X)\zeta^\nu=0$. Fixing q and letting p vary, one obtains a family of algebraic functions which are single-valued on the R-manifold (an analogue of a Riemann surface) $Q(\zeta; X)=0$. Canonical representation for $R(\zeta, X)$ is derived. An analogue of the Riemann-Roch theorem is established. (Received December 19, 1949.)

128*t*. Lipman Bers: *Abelian minimal surfaces*. Preliminary report.

Let $\phi(x, y)$ be a solution of the equation of minimal surfaces. If $\phi(x, y)$ can be continued analytically along any path in the (x, y)-plane which avoids a finite number of points, and if this continuation leads to only a finite number of determinations of $\phi_x - i\phi_y$, the minimal surface $z=\phi(x, y)$ is called Abelian. Relations are established between the number of critical points (points where $\phi_x=\phi_y=0$) of the surface and the number and character of the singularities. A parametric representation of all such surfaces is obtained in terms of Abelian integrals. This representation establishes a one-to-one correspondence between Abelian minimal surfaces and a class of real algebraic plane curves, and associates infinitely many Abelian minimal surfaces to any closed orthosymmetric Riemann surface of positive genus. Two Abelian minimal surfaces belong to the same (abstract) Riemann surface if and only if the corresponding algebraic curves are connected by a birational transformation which takes real points into real points. (Received November 14, 1949.)

129. Lipman Bers: *Partial differential equations and generalized analytic functions*.

The author considers functions $f=u(x, y)+iv(x, y)$ satisfying $u_x=\sigma(x, y)v_y$, $u_y=-\sigma(x, y)v_x$, σ being a given positive function defined over the whole Riemann sphere and possessing Hölder-continuous partial derivatives of first order. He proves the existence of such "pseudo-analytic" functions which behave like $a(z-z_0)^r$ (r arbitrary rational number). He also establishes theorems generalizing the theorems of classical function theory on the characterization of analytic functions by a differentiability requirement, on the existence of derivatives of all orders, on Taylor and Laurent expansions, on the classification of singularities, on representation of rational functions by partial fractions, on zeros of polynomials, on the Cauchy integral, on the logarithmic function, on approximation by polynomials, and on the uniformization of algebraic functions. (The case $\sigma=a(x)b(y)$ was treated by Gelbart and the author, cf. Trans. Amer. Math. Soc. (1944), by different methods.) (Received December 23, 1949.)

130. P. W. Carruth: *Maximal functions of ordinals*.

Let $\{a_\alpha\}$, $0\leqq\alpha<\beta$, be an ordinal sequence of ordinals. Every one-to-one map $\alpha(\alpha')$ of the ordinals less than β onto themselves reindexes the system $\{a_\alpha\}$ as $\{a_{\alpha(\alpha')}\}$, and defines a sum $\sum_{0\leqq\alpha'<\beta} a_{\alpha(\alpha')}$ and a product $\prod_{0\leqq\alpha'<\beta} a_{\alpha(\alpha')}$. Let S

denote the class of sums and P the class of products thus obtained. B. Dushnik, Trans. Amer. Math. Soc. vol. 62 (1947) pp. 240–247, has given a sufficient condition that S contain a maximum. Let $\{c_\alpha\}$, $0 \leqq \alpha < \tau$, be the ordinal sequence such that $c_\alpha \leqq c_\delta$ whenever $0 \leqq \alpha < \delta < \tau$, and such that any ordinal in $\{c_\alpha\}$ is "repeated" either k times or ω_σ times according as it is "repeated" in $\{a_\alpha\}$ the finite cardinal k times or $\aleph_\sigma$ times. Conditions that S contains a maximum are determined, these conditions being both necessary and sufficient whenever τ is such that the power of the set of ordinals less than τ is equal to that of the set of ordinals less than the smallest transfinite remainder of τ. Results analogous to most of these, concerning the class P, also are obtained. (Received November 14, 1949.)

131. L. W. Cohen and Casper Goffman: *On completeness in the sense of Archimedes.*

It is shown that an ordered abelian group G is complete in the sense of Archimedes if and only if, for every proper isolated subgroup $I \subset G$, G/I is non-discrete and topologically complete. (Received August 15, 1949.)

132. V. F. Cowling: *On some representations of functions defined by Taylor and Newton series.*

Let $f(z) = \sum_{n=K}^{\infty} a_n z^n$ (K integral and not less than 0) with finite nonzero radius of convergence R. Suppose there exists a single-valued function $a(t)$ analytic in some right half-plane $\text{Re}(z) \geqq L$ and there satisfying conditions (1) $a(n) = a_n$, $n = [L]+1, \cdots$, (2) $a(t) = O(t^{-h})$, $h \geqq 1$. Then choosing L non-integral and greater than K (if necessary) one can write $f(z) = a_K z^K + \cdots + a_{[L]} z^{[L]} + \int_0^\infty (\phi(u)(ze^{-u})^{[L]+1}/(1-ze^{-u}))\,du$ where $\phi(u) = (1/2\pi i)\int_{L-i\infty}^{L+i\infty} e^{ut} a(t)\,dt$, for all z not on that portion of the positive real axis from 1 to ∞. Similar results are obtained for Newton series. Examples are given. (Received November 14, 1949.)

133. J. M. Danskin: *On the existence of minimizing surfaces in parametric problems in the calculus of variations.*

The problem solved is that of minimizing a certain functional $I(S)$ among all Fréchet surfaces S bounded by a given Jordan curve g in space which bounds some surface of finite area. $I(S)$ is defined for all surfaces of finite Lebesgue area by an integral due to Cesari which reduces for sufficiently regular representations to the form $I(S) = \iint f(x^1, x^2, x^3, X^1, X^2, X^3)\,dudv$, where f is even, positively homogeneous in the generalized Jacobians X^i, positively regular, and positive definite. The lower semicontinuity of this functional with respect to uniform convergence is known. It thus suffices to obtain a uniformly convergent minimizing sequence. From an initial, arbitrary minimizing sequence results a minimizing sequence of polyhedra represented conformally on the unit circle. A subsequence of the representing sequence of vector functions converges uniformly on the boundary to a representation of g, and also along each of a countable everywhere dense set of parallels to the axes, and weakly in a Morrey space $\mathfrak{P}_2$ to a continuous limit function. Each surface of a further subsequence is modified by a geometrical process, so that the modified sequence is also minimizing and converges uniformly to the limit function obtained above. (Received November 21, 1949.)

134t. H. C. Davis: *Symmetry in Banach spaces.*

The following theorem is proved. A Banach space is a Hilbert space if and only if there exists, for any vector g and in any 2-space containing g, a nonzero f such that $\|f+\alpha g\| = \|f-\alpha g\|$ for all scalars α. The proof is by straightforward construction of an inner product. Two other symmetry characterizations of Hilbert space (one of them known previously) are corollaries. [It has been pointed out that this theorem was proved by R. C. James, Duke Math. J. vol. 12 (1945) pp. 291–302, Theorem 4.7 and Corollary 4.7.] (Received November 15, 1949.)

135. Aryeh Dvoretzky and C. A. Rogers: *Absolute and unconditional convergence in normed linear spaces.*

The well known conjecture that the equivalence of absolute and unconditional convergence of series in a Banach space implies the finite dimensionality of the space is established. This is done by exhibiting in every infinitely dimensional Banach space a series which is unconditionally but not absolutely convergent. The problem of constructing such a series is reduced to proving the following result: There exist positive constants $c_1, c_2, \cdots$ with $\lim\inf_{n\to\infty} c_n/n=0$, having the property that, if K is any bounded n-dimensional convex body with the origin as centre, there are n points $X_1, \cdots, X_n$ on the boundary of K such that all 2^n points $(\pm X_1 \pm X_2 \pm \cdots \pm X_n)/c_n$ are in K. The points of contact of K with an inscribed ellipsoid of maximum volume are considered, and the existence of such constants with $c_n = O(n^{3/4})$ is established. (Received November 14, 1949.)

136. W. F. Eberlein: *A note on ergodic means.*

Connections between Abel and Cesaro means on one-parameter semi-groups of operators have been obtained by an extension to Banach spaces of classical Tauberian theory. It is shown that such relations follow in a direct and much more transparent fashion from the author's abstract ergodic theorem. (Received November 14, 1949.)

137. D. B. Goodner: *Normed linear spaces which have property* P_1.

A normed linear space X has property P_s, $s \geqq 1$, if and only if for every normed linear space Y containing X there is a projection T, $\|T\| \leqq s$, of Y onto X. Theorem 1. If the normed linear space X has property P_1 and the unit sphere C of X has an extreme point, then C is the closed convex hull of its extreme points. Theorem 2. A finite-dimensional normed linear space X has property P_1 if and only if the unit sphere C of X is a "parallelepiped." Theorem 3. A normed linear space X does not have property P_1 if there exists in X a hyperplane $H = \{x \mid f(x) = 1 = \|f\|\}$ such that $H \cap C$ (C the unit sphere of X) is an n-dimensional simplex, $3 \leqq n$. (Received November 14, 1949.)

138. R. G. Helsel: *Remarks on the isoperimetric inequality.*

Radó [Trans. Amer. Math. Soc. vol. 61 (1947) pp. 530–555] has established the spatial isoperimetric inequality for a general closed surface in terms of its Lebesgue area and an enclosed volume defined as the integral of the absolute value of the topological index with respect to the surface. In the present paper it is shown that a stronger inequality is obtained if the enclosed volume is defined as follows. Let P be a polyhedron and $V_R(P)$ its enclosed volume as defined by Radó. Then for a general surface S, let the enclosed volume be $V^*(S) = \inf \lim\inf V_R(Pn)$, where the limit inferior is with respect to a sequence of polyhedra which converges in area to S and the infimum is taken for all such sequences. It follows from a paper of the author [Duke Math. J. vol. 16 (1944) pp. 111–118] that $V^*(S) = V_R(S)$ if S is a surface which occupies a point set of zero three-dimensional Lebesgue measure; however, the present paper includes an example to show that $V^*(S)$ can exceed $V_R(S)$ for surfaces occupying a point set with positive measure. (Received November 7, 1949.)

139. M. R. Hestenes: *Gradients of integrals in the calculus of variations.*

The purpose of the author is to study the concept of the gradient of a function $F(x)$ in a Hilbert space and to apply the results obtained to problems in the calculus of variations. In particular it is shown, under certain conditions, that if y_n is the gradient of F at x_n and α is a suitably chosen positive constant, the sequence $x_{n+1}=x_n-\alpha y_n$ will converge to a minimum point x_0. It is shown further that if the norm is properly chosen this sequence obtained is identical with one obtained by an application of Newton's method of differential corrections. The conditions imposed are too stringent to be applicable to the general integral in the calculus of variations, but is applicable to cases when the Euler equations are linear in the derivatives. The method appears to be amenable to numerical computations. (Received November 14, 1949.)

140t. R. C. James: *A non-reflexive separable Banach space isomorphic with its second conjugate space.*

For any sequence $x=(x_1, x_2, \cdots)$ of real numbers, let $\|x\|=[\text{l.u.b.} \{\sum_{i=1}^{n}(x_{p_{2i-1}}-x_{p_{2i}})^2+(x_{p_{2n+1}})^2\}^{1/2}$ over all finite increasing sequences of positive integers $p_1, p_2, \cdots, p_{2n+1}]$. Let B be the Banach space of all such x for which $\lim_{n\to\infty} x_n=0$ and $\|x\|$ is finite. Then B can be shown to have a basis $\{z^n\}$, where z^n has all components zero except the nth, which is 1. The sequence of linear functionals $\{f^n\}$, where $f^n(z^m)=\delta_m^n$, is a basis for B^*. B^{**} is a sequence space with a norm equivalent to that of B, but without the restriction $\lim_{n\to\infty} x_n=0$. However $\lim_{n\to\infty} x_n$ exists, and the natural image of B in B^{**} is a closed maximal linear subspace of B^{**}. B^{**} is isomorphic with B, and all conjugate spaces of B are separable. B does not have an unconditionally convergent basis, since it is known that any Banach space B is reflexive if B has an unconditionally convergent basis and B^{**} is separable. (Received November 14, 1949.)

141t. Herbert Jehle: *On the Majorana representation of Dirac matrices.*

Abandoning the requirement of covariance with respect to reflections, the relativistic Schroedinger equation can be put into a pair of first order differential equations with two complex components (Jehle, Physical Review vol. 75 (1949) p. 1609; Zeitschrift für Physik vol. 100 (1936) p. 702; Bull. Amer. Math. Soc. Abstract 50-1-57) $\gamma^\kappa(\partial/\partial x^\kappa-i\phi_\kappa)\psi=\mu\psi^*$ where $(\gamma^{\kappa*}\gamma^\lambda+\gamma^{\lambda*}\gamma^\kappa)/2=-g^{\kappa\lambda}1$, $\gamma^{\kappa*}=\gamma_R^\kappa-i\gamma_I^\kappa$, $\psi^*=\psi_R-i\psi_I$. The two-by-two matrices γ^κ have been extensively studied by O. Veblen (unpublished manuscript). It has been shown by J. Serpe (Physical Review vol. 76 (1949)) that the above equations are equivalent to an equation with four real components $\Gamma^\kappa(\partial/\partial x^\kappa-\Gamma\phi_\kappa)\Psi=\mu\Psi$, where the first two rows four columns of Γ^κ are γ_R^κ, $-\gamma_I^\kappa$, the second ones $-\gamma_I^\kappa$, $-\gamma_R^\kappa$, and the matrix $\Gamma=\mp\Gamma^1\Gamma^2\Gamma^3\Gamma^4$; Ψ is a column matrix with the four components ψ_R and ψ_I. The Γ^κ satisfy $(\Gamma^\kappa\Gamma^\lambda+\Gamma^\lambda\Gamma^\kappa)/2=-g^{\kappa\lambda}1$, everything being real. The hermitean matrices Γ^1, Γ^2, Γ^3, $i\Gamma^0$ give a Majorana representation (Nuovo Cimento vol. 14 (1937) p. 171) of Dirac matrices (H. A. Kramers, Proc. Amsterdam vol. 40 (1937) p. 814; F. J. Belinfante, Physica vol. 6 (1939) p. 859; W. H. Furry, Physical Review vol. 54 (1938) p. 56). (Received November 25, 1949.)

142. D. E. Kibbey and P. C. Rosenbloom: *A theorem of Doob on the ranges of analytic functions.*

In 1935 Doob introduced the class $K(\rho)$, $0<\rho\leqq 1$, of functions f analytic in $|z|<1$ such that $f(0)=0$ and there is an arc A of length $2\pi\rho$ on $|z|=1$ with the property that if $\{z_n\}$ is an arbitrary sequence approaching a point of A, then $\lim\inf |f(z_n)|\geqq 1$. He proved that if $0<r_2<r_3<1$, then there is an $r_1=\eta(r_2, r_3, \rho)>0$ such that the range of f covers either $|w|\leqq r_1$ or $r_2\leqq |w|\leqq r_3$. Also there is a $k(\rho)>0$ such that if $f\in K(\rho)$, then the range of f covers a circle of radius $k(\rho)$. Doob's proofs give no method of estimating the constants $\eta(r_2, r_3, \rho)$ and $k(\rho)$. In the present paper, by the use of the elliptic modular function, it is found that $\eta(r_2, r_3, \rho)>287r_2 \exp(-16(3.2 -\log(1-r_3))^2 65^{1/\rho}/\rho^2)$ and $k(\rho)>95\exp(-285(65^{1/\rho}/\rho^2))$. (Received September 28, 1949.)

143. B. O. Koopman: *Exponential limiting products in Banach algebras.*

Let $\{a_n\}$ be a sequence of elements in a Banach algebra containing the unit element, and let $\{\omega_n\}$ be a monotonic subsequence of positive numbers with $\omega_n\rightarrow\infty$ and $\omega_{n+1}/\omega_n\rightarrow 1$ as $n\rightarrow\infty$. Then it is shown that if $(a_1+\cdots+a_n)/\omega_n\rightarrow m$ and $[N(a_1)+\cdots+N(a_n)]/\omega_n$ remains finite as $n\rightarrow\infty$, the product $(1+a_1/\omega_n) \cdots (1+a_n/\omega_n)$ approaches e^m. A similarly explicit exponential result is established for the limit of the product $(b_1+a_1/\omega_n)\cdots(b_n+a_n/\omega_n)$, where $\{b_n\}$ are also in the Banach algebra. These theorems have applications to asymptotic distributions involving dependent probabilities. (Received October 31, 1949.)

144. B. W. Lindgren: *An integral on a space of continuous functions.*

Let C' denote the space of real, continuous functions $x(t)$ defined for $0\leqq t\leqq 1$. Let a "quasi-interval" Q be the set of functions of C' corresponding to real constants $a_1, \cdots, a_n, b_1, \cdots, b_n$, and $0\leqq t_1<\cdots<t_n\leqq 1$, satisfying the n inequalities $a_i<x(t_i)\leqq b_i$ $i=1,\cdots,n$. Let its measure be defined to be the integral in n-dimensional space $m(Q)=[\pi^{n-1}(t_2-t_1)\cdots(t_n-t_{n-1})]^{-1/2}\int_a^b \exp[-\sum_2^n(u_k-u_{k-1})^2(t_k-t_{k-1})^{-1}]\,du$. This definition leads to a Lebesgue type measure and integral over C' analogous to those defined by Wiener over the subspace C of $x(t)$ such that $x(0)=0$. Although $m(C') =\infty$, there is a countable set of quasi-intervals of finite measure whose union is C'. The new integral is related to the Wiener integral by the formula $\int_{C'}F[x]dx =\int_{-\infty}^{\infty}d\xi\int_C^w F[y+\xi]d_wy$, where $F[x]$ is summable. Using this relation and certain known formulas for the Wiener integral, the author obtains corresponding integration formulas for the integral over C'. (Received November 10, 1949.)

145*t*. B. W. Lindgren: *Transformation formulas for an integral on a space of continuous functions.*

Let C' denote the space of real, continuous functions $x(t)$ on $0\leqq t\leqq 1$. The author has defined (see previous abstract) an integral on C' related to the Wiener integral on $C=E_x[x\in C', x(0)=0]$ by the formula $\int_{C'}F[x]dx=\int_{-\infty}^{\infty}d\xi\int_C^w F[y+\xi]d_wy$. Using this relation and transformation formulas for the Wiener integral given by R. H. Cameron and W. T. Martin (Trans. Amer. Math. Soc. vol. 58 (1945) pp. 184–219), formulas are obtained for the integral on C' identical in appearance with those for the Wiener integral on C. (Received November 10, 1949.)

146t. H. F. MacNeish: *The application of the logarithmic method of integration to the binomial differential.*

From the Chebyshev theorem, $-U=\int x^p(1-x)^q dx$, p and q rational, is integrable in a finite number of terms containing only elementary functions, if at least one of p, q, and $p+q$ be an integer. For the binomial differential this theorem becomes: $I=\int x^m(a+bx^n)^{r/s}dx$, for m, n, r, s rational, $s\neq 0$, is integrable in a finite number of terms containing only elementary functions, if at least one of r/s, $i=(m+1)/n$, $j=(m+1)/n+r/s$ be an integer. The following results are proved by elementary methods: Case 1, i an integer. (a). If i is an integer not less than 1, I is algebraic and can be found by the logarithmic method of integration. (b). If i is an integer less than 1, I is transcendental. Case 2, j an integer. (a). If j is an integer less than 0, I is algebraic and can be found by the logarithmic method of integration. (b). If j is an integer not less than 0, I is transcendental. (Received November 14, 1949.)

147. Szolem Mandelbrojt and H. D. Brunk: *A composition theorem for asymptotic series.*

In a recent theorem (S. Mandelbrojt, *Quelques théorèmes de composition*, C. R. Acad. Sci. Paris vol. 226 (1948) pp. 1155–1157), Mandelbrojt gives conditions sufficient for one of two infinitely differentiable functions of a real variable to vanish identically, provided that the product of their $2k$th derivatives vanishes for each nonnegative integer k. The following analogue, for holomorphic functions having asymptotic representations in a half-plane, is proved: if (i) $F(z)$ and $\Phi(z)$ $(z=x+iy)$ are holomorphic in the half-plane $x>0$ and continuous on $x=0$, if (ii) there exists a positive number δ such that $|F(z)|=O(|z|^\delta)$ and $|\Phi(z)|=O(|z|^\delta)$ as $z\to 0$, if (iii) there exist sequences $\{a_n\}$ and $\{b_n\}$ of complex numbers such that $a_{2k-1}=b_{2k-1}=0$, $a_{2k}b_{2k}=0$ $(k=1, 2, \cdots)$, and sequences $\{M_n\}$ and $\{M'_n\}$ of positive numbers such that $\log M_n$ and $\log M'_n$ are convex functions of n, and such that $|F(z)-\sum_1^n a_k z^{-k}|<M_n|z|^{-n}$ and $|\Phi(z)-\sum_1^n b_k z^{-k}|<M'_n|z|^{-n}$ for $x>0$, and if (iv) $\sum M_n M'_n/M_{n+1}M'_{n+1}=\infty$, then either $F(z)\equiv 0$, in which case $a_k=0$ $(k=1, 2, \cdots)$, or $\Phi(z)\equiv 0$, in which case $b_k=0$ $(k=1, 2, \cdots)$. (Received October 25, 1949.)

148t. C. N. Moore: *A convergence factor theorem in general analysis.*

The purpose of the author is to prove a theorem in general analysis which includes as special cases a theorem concerning series due to Hardy and Bohr and an analogous theorem concerning integrals obtained by Hardy. Using the notations introduced in a previous paper (Trans. Amer. Math. Soc. vol. 24 (1922) pp. 79–88), this theorem deals with two classes of functions $\eta(\sigma)$, $\phi(\sigma)$, an operation $(J\eta)(\sigma)$, and the conditions under which the existence of the generalized limit $\lim_\sigma (C_n\eta)(\sigma)$ implies the existence of the limit $\lim_\sigma (C_n[\eta\phi])(\sigma)$. (Received November 4, 1949.)

149. C. N. Moore: *A generalized Tauberian theorem for Dirichlet's series, with application to the prime pair problem.*

In the present paper a Tauberian theorem for Dirichlet's series obtained by Ikehara and later somewhat generalized by Wintner is extended to series of the type $\sum a_n(x)e^{-\lambda_n s}$, which for a particular value of x are Dirichlet series but for a variable x constitute a generalization of such series. It is shown that if the function $f(s, x)$ defined by the above series for $R(s)>1$, in which region the series converges absolutely, tends to a continuous limit as $R(s)\to 1$, for $|I(s)|>0$, and if further $f(s, x)$

$\sim\phi(x)/(s-1)$ as $s\rightarrow 1$, then $(\sum a_n(x))_{\lambda_n \leqq x} \sim e^x\phi(x)$. This theorem may be applied to the problem of the distribution of prime pairs. (Received December 19, 1949.)

150*t*. Zeev Nehari: *Conformal mapping of open Riemann surfaces.*

It is shown that open Riemann surfaces of given genus and bounded by a finite number of closed curves can be mapped conformally onto certain types of multiply-covered slit-domains which are analogous to the classical schlicht slit-domains. It is likewise shown that many of the relations between various canonical mapping functions and domain functions have their counterpart in the theory of open Riemann surfaces. (Received December 7, 1949.)

151*t*. Zeev Nehari: *Extremum problems in the theory of bounded analytic functions.*

Let B be the family of functions $f(z)$ which are regular and single-valued and satisfy $|f(z)| \leqq 1$ in a multiply connected domain D. Generalizing the classical case in which D is the unit circle, it is possible to pose a large number of extremum problems for continuous functionals defined on B. Some problems of this kind were solved by L. V. Ahlfors (Duke Math. J. vol. 14 (1947) pp. 1–11), P. R. Garabedian (Harvard Thesis, 1948) and the author (Duke Math. J. vol. 15 (1948) pp. 1033–1042). In the present paper, a method is developed which permits one to solve a large variety of such problems. The method works equally well when the class B is restricted by various additional conditions. In all cases, the extremal functions yield $(1, m)$ conformal maps of D onto the unit circle, where m depends on the particular problem and can be explicitly stated in each case. The method consists in associating with each particular extremum problem an allied extremum problem within a family of specified functions involving a finite number of parameters. The boundary values of the solution of the associated problem are used to set up a Dirichlet problem, which is then shown to yield the solution of the original problem. (Received November 14, 1949.)

152*t*. Athanasios Papoulis: *On the strong differentiation of the indefinite integral.*

Given a function $f(x, y)$ integrable on a plane set E of positive measure, form $W(I) = \int\int_{I\cdot E} f(x, y)dxdy$ and $F(I) = \int\int_{I\cdot E}|f(x, y)|dxdy$. It can be seen that from the existence of the strong derivative $F'_s(x, y)$ pp. on E, the existence pp. of $W'_s(x, y)$ follows. The purpose of the author is to show that the converse is not true. (Received December 9, 1949.)

153. F. W. Perkins: *Properties of Nicolesco's means for certain classes of functions.* Preliminary report.

Nicolesco has defined a denumerable set of related integral means of particular interest in connection with the theory of polyharmonic functions (Bull. Soc. Math. France vol. 60 (1932) pp. 129–151). In the present paper the author gives a number of examples illustrating properties of these means as applied to polyharmonic functions and some metaharmonic functions. (Received November 14, 1949.)

154. Tibor Rado and P. V. Reichelderfer: *On n-dimensional concepts of bounded variation, absolute continuity, and generalized jacobian.*

Let f be a bounded continuous mapping from a bounded connected open set in

n-dimensional space into n-dimensional space. The purpose of the authors is to develop from f concepts of essential bounded variation, essential absolute continuity, and essential generalized jacobian which generalize those presently in the literature for the 2-dimensional case [T. Radó, *Length and area*, Amer. Math. Soc. Colloquium Publications, vol. 30, 1948]. Transformation formulas for n-tuple integrals and closure theorems for essentially absolutely continuous transformations are developed, which represent complete generalizations of those presented in Part IV of the above reference. (Received November 14, 1949.)

155. Walter Rudin: *A theorem on subharmonic functions.*

Let P denote a point in the plane. Put $\Delta_r f(P) = m(f; P, r) - f(P)$, where $m(f; P, r)$ denotes the mean of the function f on the circle of radius r, with center at P. The following theorem is proved. Suppose (i) the function u is upper semi-continuous in the finite plane domain D; (ii) $\limsup_{r\to 0} \Delta_r u(P)/r^2 \geqq 0$ for P in $D-E$, where E is a countable subset of D; (iii) $\limsup_{r\to 0} \Delta_r u(P)/r \geqq 0$ for P in E. Then u is subharmonic in D. The theorem is used to extend some of the author's previous results (Bull. Amer. Math. Soc. Abstracts 55-7-303 and 55-11-541). (Received November 14, 1949.)

156. Robert Schatten: *Some Banach spaces of operators.* Preliminary report.

The conjugate space of the Banach space of all linear bounded operators on a Hilbert space is characterized. In it the "trace-class" of operators on a Hilbert space can be imbedded in a "natural" fashion. (Received November 14, 1949.)

157*t*. I. E. Segal: *Equivalences of measure spaces.*

Any one of the following conditions on a measure space M implies all the others, and defines a "localizable" space: (a) completeness of the lattice of all measurable sets modulo null sets, (b) being strongly equivalent to (that is, having a measure ring isomorphic to that of) a direct sum of finite measure spaces, (c) the algebra $A(M)$ of all multiplications by bounded measurable functions on $L_2(M)$ being maximal abelian in the algebra of all bounded linear operators, (d) validity of the conclusion of the Radon-Nikodym theorem for M. Any uniformly locally compact space is localizable, and so also is any "perfect space," this being a regular locally compact measure space in which for each measurable set of finite measure there is a unique continuous function equal nearly everywhere to the characteristic function. Any measure space is "metrically" equivalent (equivalent as far as integration is concerned) to a unique perfect space (for finite spaces the existence part of this is due to Kakutani). Two measure spaces M_1 and M_2 are strongly equivalent if and only if the corresponding multiplication algebras $A(M_1)$ and $A(M_2)$ are algebraically isomorphic, which in turn is the case if and only if they are unitarily equivalent. (Received November 14, 1949.)

158*t*. Seymour Sherman: *Non-negative observables are squares.*

It is shown that the sum of squares of observables is the square of an observable. This makes the hypotheses of Theorem 4 and Corollary 4.1 of I. E. Segal, *Postulates for general quantum mechanics*, Ann. of Math. vol. 48 (1947) pp. 930–948 redundant, so that (1) in a system of observables a pure state of the subsystem coincides on that subsystem with a pure state of the system and (2) a spectral value of an observable equals the value of the observable in some pure state of the system. Thus part of the gap between systems of observables and the collection of self adjoint elements of a C^* algebra is closed. (Received October 27, 1949.)

159. Seymour Sherman: *On a theorem of Banach.*

S. Banach [Studia Math. vol. 10 (1948) pp. 159–179] has shown that if $\mathfrak{A}$ is a denumerably independent family $\{\mathfrak{B}^\gamma | \gamma \in \Gamma\}$ of Borel fields of subsets of Ω, where with each $\mathfrak{B}^\gamma$ there is a measure m^γ such that $m^\gamma(\Omega)=1$, then there exists a common extension measure m of each of the m^γ to $\mathfrak{B}$ the least Borel field containing the $\mathfrak{B}^\gamma$ with respect to which measure the $\mathfrak{B}^\gamma$ are stochastically independent. The set theoretical theorem that $\mathfrak{B}$ is isomorphic to the Borel field generated by the rectangles of a certain product space and standard results on the existence of an independent product measure yield a simple proof of Banach's theorem and clarify its relation to the theory of product measures. (Received September 28, 1949.)

160*t*. Seymour Sherman: *Order in operator algebras.*

The set $\mathfrak{S}$ of self-adjoint elements of C^* algebra $\mathcal{A}$ forms a real vector space which may be ordered by $U \geqq 0$ meaning the spectrum of U is real and non-negative. If the hermitian elements of a self-adjoint algebra (with or without unit, no topological assumptions) of bounded operators on a complex Hilbert space forms a lattice then the algebra is commutative. By example, the various possibilities for lattice ordered subspaces of $\mathfrak{S}$ are explored. (Received November 14, 1949.)

161. Fritz Steinhardt: *On equivalent gauge functions.*

Let $F(x)$, where (x) stands for $(x_1, \cdots, x_n)$, be of class C^1 and be the gauge function of a (smooth) convex body K in E^n. The gauge function $G(x)$ is called *equivalent* to $F(x)$ if the convex body K' represented by $G(x) \leqq 1$ is congruent to K under a translation, say by $(b) = (b_1, \cdots, b_n)$. Write $G(x) = F(x; b)$. For the class $\{F(x; b); -b \in \text{int. } K\}$ of all gauge functions equivalent to $F(x) = F(x; 0)$, a system of first-order partial differential equations is obtained (*not* involving the support function of K). $F(x; b)$ is convex in b also (even logarithmically so). Applications to the theory of polar convex bodies are considered, as well as to gauge functions as such. For example, the property $\int_\Omega (\partial F(\xi)/\partial x_i)/F^n(\xi) d\omega = 0$ $(i = 1, \cdots, n)$ of (not necessarily convex) gauge functions (where Ω = surface of n-dim. unit sphere; $|\xi| = 1$; $d\omega = d\omega(\xi)$) is an immediate consequence. (Received November 14, 1949.)

162*t*. Gabor Szegö: *On a generalization of Dirichlet's integral.*

Let B be a ring-shaped domain bounded by the curves K_0 and K_1 where K_0 is in the interior of K_1. Let K_0 contain the origin. The author denotes by $p(r)$ (r is the distance from the origin) a monotonically decreasing function for which $rp(r)$ is integrable in a neighborhood of $r=0$. Let $\mu = \mu(B)$ be the minimum of $\iint_B \{|\text{grad } u|^2 + p(r)u^2\} d\sigma - \iint_G p(r) d\sigma$ (G is the interior of K_1) where the admissible functions u satisfy the boundary conditions: $u=0$ on K_0, $u=1$ on K_1. Considering now all domains B whose boundary curves K_0 and K_1 have given areas, the quantity $\mu(B)$ will be a minimum if K_0 and K_1 are concentric circles about the origin. This is a generalization of a theorem of Carleman (Math. Zeit. vol. 1 (1918)) which corresponds to the special case $p(r) = 0$. (Received November 14, 1949.)

163*t*. W. J. Thron: *Singular points of functions defined by C-fractions.*

In the C-fraction (1) $1 + K(d_n x^{\alpha_n}/1)$ let $d_n \neq 0$ for all $n \geqq 1$, $\lim |d_n|^{1/\alpha_n} = 1$, and let $\{\alpha_n\}$ be a monotone nondecreasing sequence of positive integers with $\lim \alpha_n = \infty$. It is known that under these conditions (1) converges to a function $f(x)$ which is meromorphic for $|x|<1$. It is shown in this paper that $f(x)$ has at least one singular point (not a pole) on the circle $|x|=1$. From this it follows that $f(x)$ has the circle $|x|=1$ as a natural boundary provided that there exists a sequence of positive integers $\{m_k\}$ with $\lim m_k=\infty$ such that, for every k, m_k divides α_n for all $n>n_k$. This theorem contains the only previously known general results on this problem, due to Scott and Wall (Ann. of Math. (2) vol. 41 (1940) pp. 328–349) as special cases. (Received November 14, 1949.)

164. W. J. Trjitzinsky: *Multidimensional principal integrals, boundary value problems, and integral equations.*

In this work, a study is made of integrals in the sense of principal values, extended over surfaces with edges. Included are related boundary value problems of Hilbert-Riemann type and singular integral equations with principal kernels. (Received November 14, 1949.)

165. C. A. Truesdell: *A form of Green's transformation.*

Let the polyadic nth powers $b^{(n)}$ of a vector b be defined by $b^{(-1)}\equiv 0$, $b^{(0)}\equiv 1$, $b^{(1)}\equiv b$, $b^{(2)}\equiv bb$, $b^{(3)}\equiv bbb$, and so on. Let Σ be any polyadic, and let $\{\Sigma b^{(n)}\}\equiv b^{(n)}\,\Sigma+b^{(n-1)}\,\Sigma b+\cdots+\Sigma b^{(n)}$. It is shown that Green's transformation may be put into the form $\oint_S[dS\cdot(bc+cb)r^{(n)}-dSb\cdot cr^{(n)}]=\int_V[b\{cr^{(n-1)}\}+c\{br^{(n-1)}\}-b\cdot c\{r^{(n-1)}I\}+(\text{curl } c\times b+\text{curl } b\times c+b \text{ div } c+c \text{ div } b)r^{(n)}]dV$, where r is the radius vector. The case $n=0$ was given by Burgatti (Bollettino della Unione Matematica Italiana vol. 10 (1931) pp. 1–5). The cases $n=0$and $n=1$ yield as consequences several formulae useful in mechanics and physics. (Received November 14, 1949.)

166. J. L. Ullman: *A moment problem.*

Let $\alpha(t)$ be a nondecreasing function of total variation one, defined on the interval I_t $(-1\leqq t\leqq 1)$. The moments of the non-negative density function $d\alpha(t)$ are defined to be the numbers $m_k=\int_{-1}^{1}t^k d\alpha(t)$ $(k=0, 1, \cdots)$. For each positive integer n, the numbers $t_{n1}, \cdots, t_{nn}$, which may be complex, are uniquely determined by the conditions that $(1/n)\sum_1^n t_{ni}^k=m_k$ $(k=1, \cdots, n)$. Let Δ be the derived set of the t_{ni} $(n=1, 2, \cdots; i=1, \cdots, n)$. The answer to the question of when Δ lies entirely in I_t is as follows. Let M_j be the constant term in the expression $z^j(\sum_1^\infty m_k/z^k)\exp(-j\sum_1^\infty m_k/kz^k)$, and let $\tau=\liminf\int_{-1}^{1}\log|z-t|d\alpha(t)$, for $|z-1|+|z+1|>2$. The necessary and sufficient condition that Δ be contained entirely in I_t is that $\limsup(\log M_j/j)\leqq\tau$. This theorem is proved using methods developed in the author's *Studies on Faber polynomials*, being prepared for publication. (Received November 15, 1949.)

167*t*. M. A. Woodbury: *A decomposition theorem for finitely additive set functions.* Preliminary report.

Let m be a finitely additive measure defined for a field F of subsets of a base set W. Let $m_0^*(A)=\inf\sum_{i=1}^\infty m(A_i)$, where the infimum is over coverings of A by a denumerable family of subsets A_i in F. Then m_0^* is a Carathéodory outer measure and there eixsts a countably additive measure function m_0 defined for a Borel field of sets B which is shown to contain the field F. Since $m_0(A)\leqq m(A)$ for all A in F, the difference $m_1=m-m_0$ is a purely finitely additive measure. If F consists of all subsets of a denumerable set, then m_1 vanishes for finite sets and W is the union of countably many sets of m_1 measure zero. That the requirement of denumerability of W cannot be relaxed and conclusion retained is shown by an example of B. Jessen who developed the decomposition theorem separately. (Received November 14, 1949.)

168t. M. A. Woodbury: *Invariant functionals and measures.*

Let G be a commutative semi-group of linear transformations of a linear space L into itself leaving a subspace L_0 invariant. Let $p(x)$ be a Hahn functional (Banach, *Operations lineaires*, pp. 27–29) defined on L and invariant under G. Let $f(x) \leqq p(x)$ be a linear functional defined on L_0 and invariant under G. Then there exists $F(x)$ defined on L, coinciding with $f(x)$ on L_0 and such that $-p(-x) \leqq F(x) \leqq p(x)$ on L with the additional property that it is invariant under G. Also shown is a related result on bounded finitely additive measures defined over a quorum of subsets [Abstract 35, Ann. Math. Statist. vol. 20 (1949) pp. 141–142]. Specifically if $p(x)$ is such a measure which is invariant under a commutative semi-group of 1-1 transformations of the base set W into itself, then the measure can be extended to the field of all subsets of W and remain invariant under G. This implies a result of Banach (loc. cit. p. 231). (Received November 14, 1949.)

Applied Mathematics

169. Nashman Aronszajn: *Approximation methods for eigenvalues of differential problems.*

In recent years the methods of Rayleigh-Ritz and of A. Weinstein were greatly extended and made more precise. These methods present a great advantage as compared with other methods in that they give lower and upper bounds of eigenvalues. The author develops new methods presenting the same advantages, which can be applied in cases where the generalized Rayleigh-Ritz and Weinstein methods are not applicable. (Received December 21, 1949.)

170. J. R. Foote and C. C. Lin: *Some recent investigations in the theory of hydrodynamic stability.*

Two new elements in the theory of hydrodynamic stability of parallel flows are discussed. (1) When the main velocity distribution extends to infinity in both directions, the "viscous" solutions must be rejected. An example of this case has been calculated by Lessen and investigated by Chiarulli from a more general point of view. (2) When the distribution gives rise to two "critical" points the proper branch of the asymptotic solution must be re-examined. It is shown in the paper that for determining the proper asymptotic solution of the stability equation, the path in the plane of the (complex) independent variable must be taken below the critical point where the velocity is increasing and above it where the velocity is decreasing. Specific discussions are restricted to the inviscid case, for which general criteria of stability are worked out. The problem of the instability of zonal winds in the atmosphere is then considered. It is shown that, by a proper transformation of coordinates, this problem can be treated by the present theory. The gradient of "absolute" rotation is shown to be the crucial quantity. (Received November 14, 1949.)

171. A. E. Heins: *A note on a pair of dual integral equations.*

E. C. Titchmarsh [*Theory of fourier integral*, p. 334] solves a pair of dual integral equations by some rather involved analysis. It is shown here that this problem can be solved by the method of Wiener and Hopf [Paley and Wiener, *Fourier transforms* *in the complex domain*, chap. 4]. The special character of the kernels of the integral equations and the application of the Mellin transform to these equations reduces the problem to one of factoring of the Wiener-Hopf type. (Received November 14, 1949.)

172*t*. S. N. Karp: *Conical solutions of the time free wave equation.*

The conical flows, or homogeneous harmonics of degree zero, have been employed by Busemann in the linearized theory of supersonic flow, and by J. Keller in the theory of reflection of pulses in optics and acoustics. (The equation arising in those cases is $\Delta u=0$, one of the variables being pure imaginary.) These solutions are generalized for use with equations $u_{xx}+u_{yy}+u_{zz}+k^2u=0$ and conical surfaces of arbitrary cross section, excited by a point source at the vertex. The semi-infinite triangular lamina is a special case. A new derivation of the diffraction of a plane wave by a semi-infinite plane is obtained. Various applications are considered. (Received December 27, 1949.)

173*t*. S. N. Karp: *Diffraction at wedge shaped boundaries with cylindrical tips.*

A series solution is obtained for the problem of the acoustic or electromagnetic reflection or diffraction of an arbitrary time periodic incident wave by a perfectly reflecting wedge of arbitrary angle whose tip is surrounded by a cylinder of radius a and center at the vertex of the wedge, by a generalization of the method employed by H. M. Macdonald (*Electric waves*, S. 186, Cambridge, 1902) for the same problem in the absence of the cylindrical tip. There are various applications including the transmission of waves over a ridge, the reflection of a wave by a "bump" on a wall, and the comparison of scattered field in the present case with the edge effect for the pointed wedge. (Received December 27, 1949.)

174*t*. S. N. Karp: *Leading and trailing edge solutions for the oscillating airfoil in subsonic compressible flow.*

Solutions of the two-dimensional flutter problem valid near the leading and trailing edge of an airfoil were studied by treating the airfoil as a semi-infinite plane whose leading or trailing edge is in evidence. After Lorentz transformation the linearized theory of subsonic flow leads to the equation $u_{xx}+u_{yy}+k^2u=0$. The velocity potential is represented as a superposition of the singular solution $(e^{ik\rho}/\rho^{1/2})\cos(\phi/2)$, where ρ and ϕ are polar coodinates referred to a variable, origin $(x_0, 0)$. The origin x_0 is distributed along the airfoil cross section in the leading edge case. In the trailing edge case, a distribution over the trailing vortex sheet is added. The resulting integral equation is solved by inspection, for vertical oscillations. The leading edge solution is equivalent to the scattered field in diffraction of a plane wave by a semi-infinite plane. In the case of more general excitations the integral equation may be transformed to an Abel equation by change of variable, and the solution is then well known. (Received December 27, 1949.)

175*t*. S. N. Karp: *Refraction at a wedge shaped boundary.*

An integral equation is obtained for the refraction and reflection of electromagnetic or acoustic waves at a wedge shaped boundary between two media. The integrals involved initially represent single and double layers on the boundary, and mass distributions. An iterative solution is described, and the first approximation is calculated. (Received December 27, 1949.)

176. S. N. Karp: *Wiener-Hopf techniques and mixed boundary value problems.*

The duality between Green's function and eigenfunction procedures for exact solution in simple boundary value problems is shown to carry over the mixed b.v. problems resulting when the coordinate system is not perfectly suited to the boundaries. The Green's function procedure in the latter case has been introduced into diffraction problems by J. Schwinger and generalized by G. Carrier. The problems thus treated hitherto are shown to correspond to use of cartesian coordinates in separation, where the boundary cross section is one or more semi-infinite lines. These are characterized as two part b.v. problems. The three part boundary problem ($y=0$; $-\infty<x<0$; $0<x<1$; $1<x<\infty$) is reduced to a two part boundary in polars (r, θ), and solved in the case of Laplace's equation. (The eigenfunctions are $r^{\pm\nu}e^{\pm i\nu\theta}$, and this suggests Mellin transforms in the Green's function procedure.) A similar treatment may be employed for a circular disc, with Laplace's equation (using spherical coordinates). The question is raised of carrying out the solution for these two geometries when $\Delta u+k^2u=0$. (Received December 27, 1949.)

177. E. D. McCarthy: *The motion of a particle which changes mass when acted upon by a plane central force.* Preliminary report.

Let a large mass M attract a point mass m with a force $(-kv^2Mm/r^2)$ which changes m, the direction of the velocity, but not $|v|$. If this force is added to a gravitational force let $|v(r)|$ be the same as if the gravitational force acted alone. The equations of motion are $D_tmv_i=ma_i+v_iD_tm=f_i$ $(i=1, 2)$. If the added force is Newtonian, $m=m_0 \exp (kM/r)/\exp (kM/r_0)$. Results agree closely with general relativity for two problems of the solar system and have a removable disagreement for the motion of Mercury. The formula from physics $(h\nu=mc^2)$ is used in discussing the frequency (ν) of a light ray in a gravitational field. M is taken as $(2m_0-m)$ when a small particle of mass m is attracted by a second particle and the mass of each is m_0 at $r=r_0$. Gravitation is considered negligible and $|v|$ as constant. It is found that if km_0 is greater than r_0 and the ratio $v_r/|v|$ is not too large, the first particle can be restricted to a closed region. (Received November 28, 1949.)

178*t*. L. F. Meyers and Arthur Sard: *Best approximate integration formulas.*

Among the approximations $c_0x(0)+c_1x(1)+\cdots+c_mx(m)$ of $\int_0^m x(t)dt$ which are exact for degree n, those which are best in the sense of a previous paper (Amer. J. Math. vol. 71 (1949) pp. 80–91) are given for $n=1$, $1\leqq m\leqq 20$; $n=2$, $2\leqq m\leqq 12$; $n=3$, $2\leqq m\leqq 9$. Recursive relations are established which determine the best formulas for $n=1$, all m. (Received November 7, 1949.)

179. L. F. Meyers and Arthur Sard: *Best interpolation formulas.*

Best interpolation formulas in the sense of a previous paper (Amer. J. Math. vol. 71 (1949) particularly p. 90) are studied. The basic relationships are set forth in matrix form. Among formulas which are exact for degree 0 and use any number of ordinates, the conventional formulas (that is, those using fewest ordinates) are best. Likewise among formulas which are exact for degree 1 and use no more than 5 ordinates. Among formulas which are exact for degree 2, the conventional formulas are not best. The best formulas among those which are exact for degree 2 and use no more than 6 ordinates are given explicitly. (Received November 7, 1949.)

180t. K. S. Miller and R. J. Schwarz: *Analysis of a sampling servomechanism.*

An analysis of a sampling servomechanism with an error-clamping device and linear forward and return paths is given. The method leads to a determination of the continuous output of the system in terms of quadratures and gives explicitly the value of the output at the discrete sampling instants. Frequently this is all that is needed in order to study the output variation with time. The question of stability is discussed and a criterion for testing the stability of the system is given in terms of the system parameters. (Received December 15, 1949.)

181t. H. H. Mostafa: *On flows governed by simplified gas dynamical equations.*

The equation of state conidered is a polygonal approximation to the adiabatic equation of state. Flows governed by this equation have been considered before by others. The author obtains the prametric representation $z=A_\nu B_\nu f(\zeta) -4^{-1}(A_\nu/B_\nu)\overline{\int([G'(\zeta)]^2/f'(\zeta))d\zeta}$ of the complex coordinate z of the physical plane in terms of an arbitrary analytic function $f(\zeta)$. Here A_ν, B_ν are constants and $G(\zeta)$ is the complex potential of an incompressible flow around a circle in the ζ-plane. This is a generalization of Gelbart's representation [N.A.C.A., T.N. 1170] for a flow under the linearized equation of state. This polygonal approximation is equivalent to considering a step-function approximation to the function $K(t)$ in the equations $\phi_\theta=\psi_t$, $\phi_t=-K(t)\psi_0$, where $K(t)=(1-M^2)/\rho^2$; θ, t the independent variables in a modified hodograph plane, ρ density, M Mach number. In each region for which the step function is constant in t, the partial differential equations can be reduced to Cauchy-Riemann equations. In an unpublished paper, Bers and Gelbart have obtained a parametric representation in terms of the hodograph variables. The author uses their results to determine the coefficients A_ν, B_ν. A practical method is also given to obtain, flows round preassigned bodies by suitably computing $f(\zeta)$. (Received November 14, 1949.)

182. G. G. O'Brien, M. A. Hyman, and Sidney Kaplan: *On the numerical solution of partial differential equations.* I.

In a numerical solution one customarily attacks a partial difference equation approximating the partial differential equation to be solved. Thus an error of approximation is introduced. Additional errors enter during the process of solving the difference equation numerically. Here precise definitions are given of *truncation error* and *numerical error* as well as of the *convergence* and *stability* (*strong* and *weak*) of a given numerical procedure. It is shown that *a satisfactory numerical procedure for solving a partial differential problem (equation plus accessory conditions) must be both convergent and strongly stable.* Detailed investigation is restricted to hyperbolic and parabolic problems, whose numerical solution involves "stepping ahead." Strong stability is a necessary property of a satisfactory numerical procedure but weak stability is more readily analyzed. The gap is bridged here and weak stability is studied, using extensively the Fourier-analytic method outlined by von Neumann but never published. This method gives quantitative information about error growth and emphasizes the advantage to be obtained from use of "implicit" numerical procedures. A large number of calculations have been carried out for the parabolic equation $\phi_t = \phi_{xx}$. It has been found: (a) for a (weakly) stable numerical scheme, the truncation error is small and the numerical error negligible; (b) for a (weakly) unstable numerical scheme, the numerical error is appreciable but completely overshadowed by the truncation error. (Received October 21, 1949.)

183*t*. Eric Reissner: *On the theory of beams on an elastic foundation.*

The problem in question is formulated by means of the following system of equations: (1) $Bd^4w/dx^4 = q(x) - p(x)$, (2) $w(x) = \int_a^b k(|x-\xi|)p(\xi)d\xi$, with appropriate boundary conditions at a, b. The results obtained include (1) determination of a kernel function for the *three*-dimensional elastic foundation, (2) reduction of the problem to an integral equation of the first kind for the foundation pressure distribution p. In this reduction the boundary conditions are taken in the form $w''(a) = w''(b) = w'''(a) = w'''(b) = 0$. It is shown that for a class of kernels which include those for the two-dimensional and for the three-dimensional elastic half-space as foundation, the nature of the integral equation for p is analogous to certain equations arising in aerodynamics so that established techniques may be applied in the solution. The importance of the present reduction lies in the fact that with it the use of relations between divergent series is avoided. Such divergent series are encountered in a direct solution of equations (1) and (2). (Received November 14, 1949.)

184. E. K. Ritter: *Second differential effects on trajectories.*

An earlier paper (*Second differentials of functions in exterior ballistics*, Bull. Amer. Math. Soc. Abstract 55-11-537) established the existence of the second differential of the mapping defined by the equations of motion of the mass-center of a projectile. This second differential is expressible in terms of the second variations of the coordinates of the mass-center. The present paper contains: (1) formulas for the second variations in terms of double Stieltjes integrals; (2) formulas for the second differential effects of abnormal forces on trajectory elements (range, time-of-flight, and so on) in terms of the second variations of the mass-center coordinates; and (3) a theorem on the second differential analogous to that of McShane (*The differentials of certain functionals in exterior ballistics*, Bull. Amer. Math. Soc. Abstract 55-1-37) on the first differential. (Received November 14, 1949.)

185. J. B. Rosser: *A general iteration scheme for solving simultaneous equations.*

Solving simultaneous linear equations and making least squares fit are both special cases of the following general problem. Given a vector b and vectors c_i, to find a linear combination $q = b - \sum \alpha_i c_i$ of minimum length. The following simple procedure is studied. First subtract from b a multiple of c_1, choosing the factor so as to minimize the length. From the result, subtract a multiple of c_2, again minimizing the length. After going through all the c's, start over with c_1 and repeat the process. It is proved that the process converges to the desired q. It is shown that the process is improved if the c's are taken in reverse order on alternate cycles. The rate of convergence is studied, and it is shown that when slow convergence occurs, the process reduces to the summation of a slowly convergent geometric series; using this fact one can modify the process so as to get rapid convergence. The process is essentially free from round off errors, and is admirably suited for use with modern high speed computing machines. Several known methods of solution of equations by iteration are shown to be special cases of the general procedure outlined. (Received November 9, 1949.)

186t. J. B. Rosser: *Transformations to speed the convergence of series.*

Numerical instances are given of the speeding of the convergence of series by the Euler transformation. This is even applied advantageously to certain divergent series, and a rigorous justification is given. An example is given of a series for which use of the Euler transformation is not useful. Instances are given of several less widely known methods. Finally, the method of summation by transformation into a continued fraction is illustrated successfully in the case of certain divergent series. The possibility of applying two different methods in succession to a given series is exploited throughout the paper, in spite of the fact that this often requires summing a divergent series. (Received November 9, 1949.)

187. Charles Saltzer: *On the numerical determination of the conformal mapping function of a nearly circular region.*

Theodorsen and Garrick (T. Theodorsen, *Theory of wing sections of arbitrary shape*, N.A.C.A., T. R. No. 411; T. Theodorsen and I. E. Garrick, *General potential theory of arbitrary wing sections*, N.A.C.A., T. R. No. 452) reduced the problem of finding the function which maps a "nearly circular region" whose boundary is given in polar coordinates by $\rho=\rho(\theta)$ $(0\leqq\theta\leqq 2\pi)$ on the interior of the unit circle to the problem of solving the integral equation, $\theta(\phi)-\phi=L\{\log\rho[\theta(\phi)]\}=-(1/2\pi)\int_0^\pi\{\log\rho[\theta(\phi+t)]-\log\rho[\theta(\phi-t)]\}\cot(t/2)dt$. They proposed solving this equation by iteration, replacing the integrals involved by finite sums. Warschawski (S. E. Warschawski, *On Theodorsen's method of conformal mapping of nearly circular regions*, Quarterly of Applied Mathematics vol. 3 (1945) pp. 12–28) proved under certain hypotheses that the sequence of functions $\{\theta_n(\phi)\}_{n=0}^\infty$ defined by $\theta_0(\phi)=\phi$, $\theta_{n+1}(\phi)-\phi=L\{\log\rho[\theta_n(\phi)]\}$ converges to the solution of this integral equation. The author shows that the procedure of Theodorsen and Garrick is equivalent to replacing the integral equation by a system of simultaneous equations $\bar\theta-\bar\Phi=H\{\log[\rho(\bar\theta)]\}$ where $\bar\theta=(\bar\theta^1,\bar\theta^2,\cdots,\bar\theta^m)$, $\bar\Phi=(2\pi/m, 4\pi/m,\cdots,2\pi)$ and approximating the solution of this system by the sequence $\bar\theta_0=\bar\Phi$, $\bar\theta_{n+1}-\bar\Phi=H\{\log\rho(\bar\theta_n)\}$. It is proved under certain conditions that this sequence converges to the solution $\bar\theta$ of the above system. Estimates are given for $|\bar\theta-\bar\theta_n|$ and for the accuracy of the approximation of the solution of the integral equation by the solution of the system of simultaneous equations. (Received November 14, 1949.)

188t. B. R. Seth: *Finite elasto-plastic deformation of a rotating disc.*

In the first paper on finite elasto-plastic deformation it was shown that, for a first approximation which corresponds to ordinary small strains, both the plastic flow and deformation theories give similar results for the torsion of a circular cylinder. A second approximation, obtained by using finite components of displacements and strains, showed that unlike the plastic flow theory, the deformation theory gives discontinuities in the stresses across the elasto-plastic boundary, and hence could not be expected to give satisfactory results. In the second paper the rotation of a circular cylinder was discussed and explicit forms for the components of displacements were obtained. In the present paper the method is extended to a rotating disc. (Received November 4, 1949.)

Geometry

189*t*. L. M. Blumenthal: *Generalized euclidean space in terms of a quasi inner product.*

An abstract quasi inner product space Q is formed by an abstract set S in which a binary operation (x, y) on SS to R (the set of real numbers) is defined (quasi inner product) such that (1) $(x, y)=(y, x)$, (2) $(x, x)\geqq 0$, and (3) if $(x, x)=(x, y)=(y, y)$ then $x=y$. The author characterizes generalized euclidean spaces (real, normed linear spaces with an inner product related to the norm in the customary way) among the class of spaces Q by means of three conditions on the quasi inner product and three (existence) conditions on the space. Addition, scalar multiplication, and norm are *defined in such a space Q entirely in terms of the quasi inner product*, and these operations are proved to possess the desired properties. It is noted that the problem solved in this paper is the reverse of the well known problem of defining an inner product in a normed linear space, solutions of which have been given by Fréchet, Jordan and von Neumann, Aronszajn, Birkhoff, and others. (Received December 5, 1949.)

190*t*. Edward Kasner: *The spherical sections of an arbitrary surface.*

The system of ∞^3 plane sections of any surface S has been extensively studied. In the present paper the system of ∞^4 spherical sections of S is investigated. The differential equation, in arbitrary coordinates, is of the form $y''''=Ay'''^2+By'''+C$. This form presents itself in many dynamical situations and in the calculus of variations. The fundamental property is that the ∞^1 curves determined by a given point, direction, and curvature have osculating conics whose centers lie on a conic. The spherical curves of S may be regarded as another analogue of circles, to be compared to the standard definitions of Gauss and Minding. (Received November 21, 1949.)

191*t*. Edward Kasner and John DeCicco: *General theorems of physical systems of curves.*

The five characteristic properties of a physical system S_k of ∞^3 curves connected with an arbitrary field of force in the plane are discussed. Additional properties of a system S_k are obtained. If the lineal element $E(x, y, \theta)$ is fixed and k varies, the corresponding focal circles form a hyperbolic pencil, all of which pass through the fixed point $P(x, y)$ of E and a point Q on the line of E. The foci of the osculating parabolas of the actual curves of a system S_k describe the circular arcs PQ. The directrices of the osculating parabolas all pass through a point D. As k varies, D describes the line DQ, which is perpendicular to PQ at Q. As θ changes, D describes a conic section, and the lines to which the osculating parabolas are tangent envelop an algebraic curve of class four. The envelope of the focal circles, where k is fixed and θ varies, is a bicircular quartic with a singular point at P. This reduces to a circle in the case S_0 of dynamical trajectories. The locus of the centers of the focal circles is a conic. Finally the point Q describes a conic which passes through P in the direction of the force vector at P. (Received September 29, 1949.)

192. Edward Kasner and John DeCicco: *The hyperosculating speed for physical systems of curves.*

For a given physical system S_k, there is a single trajectory through a lineal element E such that it has four point contact with its osculating circle at E. The corresponding hyperosculating speed v of the particle, and the curvature κ at E, depend on

(k, x, y, y'). The cases where v and κ are constants, or depend only on (k, x, y), are studied in detail. For the general case, some of the results follow. If v_1 and v_2 are the hyperosculating speeds of the systems S_{k_1} and S_{k_2} at E, then $v_1^2/v_2^2=(3+k_1)/(3+k_2)$. For a given S_k, there are two possible extremal speeds v_1 and v_2 at an ordinary point P, for which the directions λ_1' and y_2' are symmetrical with respect to the line of force through P. If, through P, λ_1 and λ_2 are the angles between a direction y' and the two etremal directions y_1' and y_2', then $(v^2-v_1^2)/v_1^2 \sin^2 \lambda_1=(v^2-v_2^2)/v_2^2 \sin^2 \lambda_2$. Finally the ratio of the hyperosculating curvature of a system S_k to that of a velocity system S_∞ at any direction through P is $(1+k)/(3+k)$. This is another extension of Kasner's theorem which states that the curvature of the rest trajectory $(k=0)$ to that of the line of force is 1/3. (September 29, 1949.)

193. T. S. Motzkin: *Initial sets in p-adic geometry*. Preliminary report.

Let $f(t)=\sum a_k t^k$, $k=0,1,\cdots$, with coefficients a_k in an algebraically closed field K of characteristic p, and $f(t+c)=\sum g_k(c)t^k$. The set of integers k for which $g_k(c)=0$ does not hold identically in c is an initial set, that is, together with k it contains every k' whose p-adic digits are not greater than the corresponding digits of k. For $p=0$ write $p=\infty$; the condition for k' becomes $k'\leqq k$. Let $f(x_0)=\sum a_k x_0^k$, $k=0, 1, \cdots, n$, be a form in $x_0, \cdots, x_d$ with coefficients in K. The initial set of f, considered as type of the point $(1,0,\cdots,0)$ with respect to the hypersurface $f=0$, provides a first classification of singular points on and outside $f=0$. Let $x_i(t)=\sum a_{ik}t^k$, $k\geqq k_i$, $0=k_0<k_1<\cdots<k_d$, be a branch in projective d-space over K. For the branch of an irreducible algebraic curve C at its general point ξ (after $K(\xi)$-reduction), $k_1=1$ and $(k_0, \cdots, k_d)$ is an initial set. If and only if $k_d<p$ (whence $k_i=i$ and $d<p$) does the dual of the dual of C coincide pointwise with C, which is tantamount to saying that the natural correspondence between C and its dual is birational. Case $d=2$ of this is the condition given in Bull. Amer. Math. Soc. Abstract 56-1-25. (Received November 14, 1949.)

194. T. S. Motzkin: *Main exponents and p-adic curve branches*. Preliminary report.

Let $f(t)=\sum a_k t^k$, $k\geqq -n$, with coefficients a_k in an algebraically closed field of characteristic p. Let k_e, $e=0, 1, \cdots$, be the smallest $k\not\equiv 0$ (p^{e+1}) with $a_k\neq 0$, and $k_\infty=0$ if $n=0$, $a_0\neq 0$; these are the main exponents. For unitary parameter transformations $t=u+\sum c_k u^k$, $k\geqq 2$, the general $f(t)$ with given main exponents has a finite number of independent invariants which equals the number of its terms from the first to the last main exponent, both included. Every $f(t)$ can be transformed into a polynomial with this number of terms, which thus furnishes a canonical expansion generalizing the Puiseux expansion with $f(t)=a_0+a_m t^m$ for $p=0$. The main exponents of the projective coordinates vanishing at the origin of a, not necessarily plane or algebraic, branch B define a finite set of projectively covariant linear manifolds belonging to B, including the tangent, quasitangent, and quasiorigin of Bull. Amer. Math. Soc. Abstract 56-1-25. (Received November 14, 1949.)

Logic and Foundations

195. A. H. Copeland: *Logic and Boolean rings*.

A more precise formulation and a deeper understanding of logic can be achieved by means of Stone's representation theorem and the theory of ideals in Boolean rings.

In the present paper the following topics in logic are studied from the point of view of ideal theory: proposition, meaning, truth, falsity, truth tables, strict implication, material implication, non-transitive implication, the nature of proof, propositional function, logical quantifiers, and scientific intuition. (Received November 14, 1949.)

196t. R. L. Goodstein: *The formal structure of a denumerable system.*

A strictly formal mathematical system comprising number and function signs, logical constants and operators, and employing the sentential calculus, is called *denumerable* if the application of the operators is restricted to variables whose range of values is a denumerable class. The system is not fully determined by the restriction on the operators but depends also upon the method of formalisation and the types of function definitions allowed. The denumerable system under consideration is a finitist superstructure built on an extension to a rational field of the Hilbert-Bernays formalisation Z_μ, function theoretic attributes being expressed by the method of proof schemata. All the foundation theorems of classical analysis are shown to have demonstrable analogues in the system. (Received September 23, 1949.)

197. Leon Henkin: *An algebraic characterization of quantifiers.*

A characterization of universal and existential quantifiers is given which is valid for functional calculi of the classical (two valued), intuitionistic, and modal systems of logic. This is accomplished by treating a subsystem H_f common to these three logics, in which the only propositional connective is an implication symbol. An algebraic structure called an *implicative model* is defined as a set of elements, including a special one, 0, on which is defined a binary operation $\dot{-}$ satisfying certain simple axioms. Writing $x \leqq y$ for $x \dot{-} y = 0$ it can be shown that a model is partially ordered with 0 as minimal element. Certain canonical mappings of the formulas of H_f into an arbitrary implicative model are described, in which quantifiers (x) and $(\exists x)$ correspond to operations sup and inf respectively. The principal theorem is to the effect that a formula is formally provable in H_f if and only if it is mapped into 0 by every canonical mapping. This provides an answer to a question posed by Mostowski (Journal of Symbolic Logic vol. 13 (1948) p. 207). (Received December 5, 1949.)

198. Ira Rosenbaum: *Ordered q-ads in an $\aleph_0$-membered universe.*

In a universe with $\aleph_0$ elements, denoted by the positive integers, one q-ad is to precede another if the sum of the q elements of the one is less than the corresponding sum for the other; q-ads in a set each of which has the same q-fold sum are to be ordered by Hausdorff's principle of last differences. In the resulting ordering, the order number, $N(x_1, \cdots, x_q)$, of the ordered q-ad, $(x_1, \cdots, x_q)$, is the sum of the *rank*, $R(x_1, \cdots, x_q)$, and the *height*, $h(x_1, \cdots, x_q)$, of the q-ad, where $R(x_1, \cdots, x_q)$ is the number of q-ads with sum less than $x_1+x_2+\cdots+x_q=s$, and $R(x_1, \cdots, x_q)$ equals the binomial coefficient ${}_{s-1}C_q$, while $(hx_1, \cdots, x_q)$ is the order number of the given q-ad among those with sum s, and is recursively determined by the relations, $h(x_1, \cdots, x_q)=\sum_{j=1}^{s-q+1} d(x_q, j)P(q-1, s-j)+(x_1, \cdots, x_{q-1})$, with $d(x_q, j)=0$ if $x_q \leqq j$, and $=1$ otherwise, $h(x_1, x_2)=x_2$, and $P(q-1, s-j)={}_{s-j-1}C_{q-2}=$ the number of $(q$-$1)$-ads with sum $s-j$. The formula $N(x_1, \cdots, x_q)=R(x_1, \cdots, x_q)+h(x_1, \cdots, x_q)$ is easily used and generalizes the familiar formula $N(x_1, x_2)=(x_1+x_2-1)(x_1+x_2-2)/2+x_2$ (Carathéodory, *Vorlesungen über reele Funktionen*, pp. 28–29) in which x_2 is the height, and the product the rank, of the dyad (x_1, x_2). These processes are reversible; given N, the Nth q-ad is computable. (Received November 14, 1949.)

Statistics and Probability

199t. H. D. Brunk: *Note on a theorem of Kakutani.*

A recent thorem of Kakutani (S. Kakutani, *On equivalence of infinite product measures*, Ann. of Math. (2) vol. 49 (1948) pp. 214–224), giving conditions necessary and sufficient for the equivalence of direct infinite product measures, is used to derive conditions of an apparently less restrictive nature sufficient in order that one of two sequences of independent random variables shall satisfy one of a large class of limit theorems, provided the other sequence satisfies the limit theorem. An example is given, illustrating the manner in which this result may be regarded as extending known sufficient conditions for limit theorems. (Received October 25, 1949.)

200. D. A. Darling: *On the limiting distribution of sums of random variables with infinite expectations.*

Let $X_1, X_2, \cdots$ be independent, identically distributed random variables which are non-negative and have the (common) absolutely continuous distribution function $F(x)=1-f(x)=\Pr\{X_k<x\}$, and let $S_n=X_1+X_2+\cdots+X_n$. If $E(x_k^\alpha)=\infty$ for all $\alpha>0$ it is well known that $T_n=(S_n-a_n)/b_n$ has no nondegenerate limiting distribution for any two sequences a_n and b_n. The author studies the limiting distribution for S_n under a different normalization, and proves that if $\lim_{x\to\infty} f(ax)/f(x)=1$ for all $a>0$ then $\lim_{n\to\infty}\Pr\{nf(S_n)<\alpha\}=1-\exp(-\alpha)$. He obtains, for example, for the example discussed by P. Levy, where $f(x)=(\log x)^{-1}$, $x>e$, the expression $\lim_{n\to\infty}\Pr\{S_n^{1/n}<\alpha\}=\exp(-(\log\alpha)^{-1})$. It is further shown that if $f(x)$ does not satisfy the above condition, no normalization yielding a nondegenerate limiting distribution for S_n exists. This result is an extension of the necessary and sufficient conditions given by Doeblin for S_n to have a semi-stable limiting distribution. If moments of all orders are infinite then S_n is essentially dominated by its largest term in the sense that if $X_n^*=\max(X_1, X_2, \cdots, X_n)$ then $S_n/X_n^*\to 1$ in the mean. Furthermore, if some moment is finite, but $E(X_k^{1-\delta})=\infty$, then S_n and X_n^* are of comparable magnitude in the sense that given any $\epsilon>0$ there exists an $M=M(\epsilon)$ such that $\Pr\{S_n/X_n^*>n\}\leqq\epsilon$ for all n sufficiently large. If $E(X_k)<\infty$, then $S_n/X^*\to\infty$ stochastically. (Received November 15, 1949.)

201. Einar Hille: *On continuous transition probabilities.*

The transition frequency $f(t;\xi,x)$ of a temporally homogeneous stochastic process normally satisfies the adjoint equations of Kolmogoroff: $b(\xi)S_{\xi\xi}+a(\xi)S_\xi=S_t$, $\{[b(x)T]_x-a(x)T\}_x=T_t$, $b(x)>0$, $-\infty<x<\infty$. The converse problem of deciding when such differential equations determine a unique frequency, leads to problems for semi-groups: C. When is $C[f]=b(\xi)f''+a(\xi)f'$ the generator of a semi-group $\{S(t)\}$, $t\geqq 0$, of positive contraction operators in $C[-\infty,\infty]$, leaving $g(x)\equiv 1$ invariant, and strongly continuous in t with $S(0)=I$? L. When is $L[f]=\{[b(x)f]'-a(x)f\}'$ the generator of a semi-group $\{T(t)\}$, $t\geqq 0$, of positive contraction operators in $L(-\infty,\infty)$, isometric on positive elements, and strongly continuous in t with $T(0)=I$? Necessary and sufficient conditions for the solvability of these problems are found in terms of integrability properties of $W(x)=\exp\{-\int_0^x[a(s)/b(s)]ds\}$ in the intervals $(-\infty, 0)$ and $(0,\infty)$. If L is solvable, so is C, but not vice versa. If both problems are solvable, then there is a unique transition frequency satisfying Kolmogoroff's equations. If C has a solution, but L does not, there is a transition frequency satisfying both equations, unique as far as the first equation is concerned. Cauchy's problem for the second equation, however, does not have a unique solution in $L(-\infty, \infty)$. (Received December 19, 1949.)

202t. R. B. Leipnik: *On the number of zeros of a random polynomial.*

By means of a result of Schur, upper bounds are obtained in terms of integrals for the probability that $a(0)z^n + \cdots + a(n)$ has p real zeros, where $a(0), \cdots a(n)$ are independent random variables with the same distribution F. These integrals are evaluated for several choices of F. (Received November 14, 1949.)

203t. Herman Rubin: *On the existence of nearly locally best unbiased estimates.*

For any family $\mathfrak{F}$ of distributions, and any distribution F_0 of $\mathfrak{F}$, there exists a bilinear function κ whose arguments are all parameters defined for all distributions of $\mathfrak{F}$ and for which there exist unbiased estimates which have finite variance if F_0 is the true distribution, and which has the following properties: (1) If θ is any parameter in the domain of κ, and t is any unbiased estimate of θ, then var $(t|F_0) \geqq \kappa(\theta, \theta)$. (2) This result is best possible, that is, for any θ there is an unbiased estimate t of θ whose variance differs from $\kappa(\theta, \theta)$ by less than any preassigned amount. (Received November 14, 1949.)

204t. Jacob Wolfowitz: *Minimax estimates of the mean of a normal distribution with known variance.*

The classical estimation procedures (point and interval) are proved to be minimax solutions of properly formulated estimation problems, sequential and non-sequential. For example, for the problem considered by Stein and Wald (Ann. Math. Statist. vol. 18 (1947) pp. 427–433), it is proved that, for any choice of parameters in the classical (fixed-sample) procedure C, there exists a positive constant c with the following property: Let G be the generic designation of the sequential estimation procedure, $\alpha(\xi, G)$ the probability under G that the estimating interval will contain the mean ξ, and $n(\xi, G)$ the expected number of observations. Then $1-\alpha(\xi, C)+cn(\xi, C) = \inf_G \sup_\xi [1-\alpha(\xi, G)+cn(\xi, G)]$. The result of Stein and Wald is an immediate consequence. Other such optimal properties are obtained. (Received September 12, 1949.)

205t. L. A. Zadeh and J. R. Ragazzini: *An extension of Wiener's theory of prediction.*

The authors give an extension of Wiener's theory of prediction which differs from the latter in the following respects: I. The signal (message) component of the given time series is assumed to consist of two parts: (a) a non-random function of time which is representable as a polynomial of degree not greater than a specified number n, and about which no information other than n is available; and (b) a stationary random function of time which is described statistically by a given correlation function $\psi_M(\tau)$. (In Wiener's theory, the signal may not contain a non-random part except when such a part is a known function of time.) II. The impulsive response of the predictor or, in other words, the weighting function $W(t)$ used in prediction, is assumed to vanish outside of a specified time interval $0 \leqq t \leqq T$. (In Wiener's theory T is assumed to be infinite.) The determination of $W(t)$ reduces to the solution of a modified Wiener-Hopf equation $\int_0^T w(\tau)\psi_{M+N}(t-\tau)d\tau = \lambda_0 + \lambda_1 t + \cdots + \lambda_n t^n$

$+\int_{-\infty}^{\infty} k(\tau)\psi_M(t-\tau)d\tau$, where the λ's are Lagrangian multipliers; $k(t)$ is the prescribed prediction operator (in the time domain), and $\psi_{M+N}(\tau)$ is the correlation function of the random part of the time series. An explicit solution of this equation is given. (Received November 11, 1949.)

Topology

206. R. D. Anderson: *Monotone interior dimension-raising mappings.*

The author proves two theorems: (I) There exist a one-dimensional continuous curve M in the plane and a monotone interior mapping of M onto the plane; and (II) For any positive integer n, there exist a one-dimensional continuum M_n in Euclidean three-space and a monotone interior mapping of M_n onto Euclidean n-space. (Received November 14, 1949.)

207. R. H. Bing: *A characterization of 3-space by partitionings.*

It is known that if S is a compact continuous curve, there exists a decreasing sequence of regular partitionings $H_1, H_2, \cdots$ such that (a) H_i is a finite collection of mutually exclusive connected open sets whose sum is dense in S, (b) each element of H_i equals the interior of its closure, (c) H_{i+1} is a refinement of H_i, and (d) as i increases without limit, the diameters of the elements of H_i approach 0. A necessary and sufficient condition that a compact continuous curve be a simple solid (set topologically equivalent to a 3-sphere in Euclidean 4-space) is that one of its decreasing sequences of regular partitionings have the following 4 properties: (1) The boundary of each element of G_i is a simple surface. (2) If the boundaries of two elements of $\sum G_i$ intersect, this intersection is a 2-cell. (3) The intersection of the closures of three elements of G_i is 1-dimensional at each of its points. (4) The elements of G_i may be ordered $[g_{i1}, g_{i2}, \cdots, g_{in_i}]$ such that if g is an element of $G_{i-1}(g=S$ if $i=1)$ and j is an integer less than n_i, then $\bar{g}_{ij+1}$ intersects $S-g+\bar{g}_{i1}+\bar{g}_{i2}+\cdots+\bar{g}_{ij}$ in a connected set. Unicoherence cannot be substituted for condition (4) because projective 3-space is unicoherent and has a decreasing sequence of regular partitionings satisfying conditions (1), (2), and (3). (Received November 14, 1949.)

208*t*. A. L. Blakers and W. S. Massey: *The homotopy groups of a covering.* Preliminary report.

Let $(X; A, B)$ be a triad (see A. L. Blakers and W. S. Massey, Proc. Nat. Acad. Sci. U.S.A. vol. 35 (1949) p. 323) with $x\in A\cap B$, $A\cup B=Y$. Let S^n be an n-sphere, $n>0$, E^n_+, W^n_- the "upper" and "lower" hemispheres, and p a point of the equator $E^n_+\cap E^n_-$. Denote the function space of maps f: $(S^n; E^n_+, E^n_-, p)\rightarrow(Y; A, B, x)$ by $G^n(A/B, x)$ and the homotopy classes of $G^n(A/B, x)$ by $\pi_n(A/B, x)$. If $n>1$, $\pi_n(A/B, x)$ can be made into a group, called the ***nth homotopy group of the covering*** A/B. These groups are abelian if $n>2$ and fit into exact sequences: $\cdots\rightarrow\pi_n(A, x)\rightarrow\pi_n(A/B, x)\rightarrow\pi_n(B, A\cap B, x)\rightarrow\pi_{n-1}(A, x)\rightarrow\cdots$, and $\cdots\rightarrow\pi_{n+1}(X; A, B)\rightarrow\pi_n(A/B)\rightarrow\pi_n(X)\rightarrow\pi_n(X; A, B)$. Denote the function space of maps $(S^n, p)\rightarrow(X, x)$ by $F^n(X, x)$; let $G^n_0(A/B, x)$ and $F^n_0(X, x)$ be the components of $G^n(A/B, x)$ and $F^n(X, x)$ respectively which contain the constant map. Then the homotopy sequence of the pair $[F^n_0(X, x), G^n_0(A/B, x)]$ is naturally isomorphic to that of the triad $(X; A, B)$; for example, $\pi_q[F^n_0(X, x), G^n_0(A/B, x)]\approx\pi_{n+q}(X; A, B)$. This isomorphism enables one to define a natural homomorphism of the homotopy sequence of the pair (R_{n-1}, R_{n-2}) (where

R_m = rotation group of S^m) into that of the triad $(S^n; E^n_+, E^n_-)$. This is useful in studying the higher homotopy groups of spheres. (Received November 29, 1949.)

209. L. W. Cohen and Casper Goffman: *On completeness and category in uniform space.*

A uniform space S, metrizable by an ordered abelian group, is called complete in the sense of Archimedes (a-complete) if certain associated spaces, analogous to factor groups, are topologically complete (t-complete). It is shown that an a-complete space is of the second category. The authors have shown elsewhere (*A theory of transfinite convergence*, Trans. Amer. Math. Soc. vol. 66 (1949) pp. 65–74) that a t-complete space may be of the first category. (Received October 10, 1949.)

210. E. E. Floyd: *The decomposition maps associated with 0-dimensional transformation groups.*

Let X be a locally compact metric space; let G be a compact 0-dimensional transformation group operating on X such that no element of G except the identity leaves any points of X fixed. Let Y be the orbit decomposition space, and $f: X \rightarrow Y$ the decomposition map. Analogues of theorems concerning covering maps are proved for f. For example, the conclusion of the covering homotopy theorem holds here. As an application, it is proved that if X is an n-manifold and G is infinite, then Y contains no $(n-1)$-cell. (Received November 14, 1949.)

211. David Gale: *Compact sets of functions and function rings.*

Let Y^X denote the space of continuous functions from the space X to the space Y in the compact-open topology. The author investigates conditions under which a subspace $F \subset Y^X$ is compact. The classical conditions when Y is a metric space involve equicontinuity and uniform boundedness of F. If Y is not metric equicontinuity may be replaced by a sort of mutual continuity requiring that if U is open in Y, then $\bigcap_{f \in F} f^{-1}(U)$ be open in X. Boundedness is replaced by the condition that $\bigcup_{f \in F} f(x)$ be compact. Using these ideas one obtains simple, necessary and sufficient conditions for the compactness of F. The result is then applied to obtain a duality theorem for the ring $R(X)$ of real valued continuous functions on a space X. Namely, if $R(X)$ is given the compact-open topology and H is the space of all continuous homomorphisms of $R(X)$ onto the real numbers with the same topology, then, with mild restrictions on x, it is shown that H is homeomorphic with X. (Received November 9, 1949.)

212. F. B. Jones: *An elementary two color problem.*

Consider a unit square in the number plane with the ends of a diagonal at (0, 0) and (1,1). Let M_x denote the set of all straight line arcs spanning the square parallel to the y-axis and let M_y denote a similar set parallel the x-axis. Color the intersection of a line in M_x with a line in M_y either red or green, except along the line $y=x$ which is left uncolored; furthermore the coloring is symmetrical with respect to $y=x$. THEOREM: *If H_x is the subset of M_x with rational abscissas and H_y is the corresponding subset of M_y, then H_x contains an infinite subset H_x' such that H_x' intersects H_y' (the image in H_x with respect to $y=x$) in only one color.* QUESTION: *Does M_x contain a subset M_x' of the same number of elements as M_x such that M_x' intersects M_y' in only one color?* (Received December 23, 1949.)

213. A. N. Milgram: *A solution of the frame problem for the three sphere.*

H. Rademacher raised the question as to whether every bounded subset of E_n (Euclidean n-space) can be circumscribed by an n-dimensional cube. A solution for subsets of E_3 was given by S. Kakutani (Ann. of Math. (2) vol. 43, pp. 739–741). This note provides an affirmative answer to the four-dimensional case. Specifically we prove that if f is a continuous function defined on the sphere S_3 with center O in E_4, there exist points $P_i \in S_3$, $i=1, 2, 3, 4$, such that the vectors OP_i are mutually orthogonal and $f(P_i)=f(P_j)$, $i, j=1, \cdots, 4$. The proof makes essential use of the lemma: If $f(x, y)$ is a doubly periodic function of the real variables x, y, with period 1, there exist x_0, y_0, z_0 such that $f(x_0, y_0)=f(x_0+1/4, y_0)=f(z_0, y_0+1/2)=f(z_0+1/4, y_0+1/2)$. (Received November 15, 1949.)

214. L. T. Ratner: *An extended theory of semi-continuity.*

X and Y are Hausdorff spaces satisfying the first axiom of countability; further, Y is simply ordered by a proper transitive relation $\prec$ such that $\{y' | y' \preceq y\}$ and $\{y' | y \preceq y'\}$, for any $y \in Y$, are closed subsets of Y. Let f be a single-valued transformation of X into Y. For U any open set containing x, denote by $l(x; f; U)$ the set of *lower bounds* for $f(U)$ and by $L(x; f; U)$ the set of *upper bounds* for $f(U)$; let $l(x; f)$ be the closure of the union of all $l(x; f; U)$, $L(x; f)$ the closure of the union of all $L(x; f; U)$. f is said to be *lower semi-continuous*, *upper semi-continuous*, or *continuous at* x provided that $f(x) \in l(x; f)$, $f(x) \in L(x; f)$, or $f(x) \in l(x; f) \cap L(x; f)$, respectively. These definitions provide a generalization of the notions of semi-continuity employed in real variable theory and elsewhere; continuity reduces to the normal concept. A sequence of standard theorems is carried over for this theory of semi-continuity. (Received November 15, 1949.)

215t. Russell Remage: *Invariance and periodic properties of non-alternating transformations in the large.* Preliminary report.

Extensions of some of the results of cyclic element theory to compact connected Hausdorff spaces are obtained by means of nodal set theory. If X is a non-vacuous subset of a continuum S, a D-chain, $D(X)$, is the set $C(X)$ defined by Wallace as the intersection of all nodal sets containing X. Let X in nondegenerate $Z(X)$ be the set of points separating a pair of points of X in S. A nondegenerate prime chain P is contained in $D(X)$ if and only if either $P \cap (X \cup \overline{Z(X)})$ is nondegenerate or $P \cap \overline{Z(X)}$ is a single point which fails to separate P from X. End-elements which are not cut points are prime chains not contained in $D(z_1 z_2)$ for any pair of cut points z_1, z_2. Any power of a n.a.l. transformation T of a continuum S onto itself is n.a.l. If PT is not a cut point unless P is, and either $P \subset PT$ or $PT \subset P$, P is called nonvariant, and contains an invariant set. If a nodal set meets its image, it contains a nonvariant P. If R is the union of all nonvariant prime chains, then $z \in Z(R)$ is either fixed or has no recurrence properties. Applications yield extensions and analogues of several theorems dealing with element-wise recurrence. (Received November 14, 1949.)

216t. Jane C. Rothe: *The topological degree of certain mappings in Euclidean n-space.*

Mappings in complex and real Euclidean n-space E^n which are defined by n polynomials in n variables, the polynomials homogeneous of degree k in the n variables, are studied. By using resultant theory, it is proved that if E^n is complex and the topological degree at zero of such a mapping is defined, the topological degree is k^n. By elementary methods, it is proved that if E^n is real and $n=2$, the topological degree is one of the following values: if k is odd, $\pm 1, \pm 3, \cdots, \pm k$; if k is even, $0, \pm 2, \pm 4, \cdots, \pm k$. The results are applied to obtain existence theorems for certain types of non-linear integral and differential equations. (Received November 4, 1949.)

217*t*. G. T. Whyburn: *Dimension raising on surface sets.*

A locally connected generalized continuum every true cyclic element of which is a 2-manifold (open or closed) is called a surfacoid. It is shown that if A is a locally connected generalized continuum on a surfacoid and $f(A)=B$ is a light open mapping, for any branch point y of B, every point of $f^{-1}(y)$ is an isolated point of $f^{-1}(y)$ and dim $B=$dim A. Further, if A is a locally connected continuum on a cactoid and $f(A)=B$ is a quasi-monotone mapping, then dim $B\leqq 2$. Also it is shown that if A and B are compact metric spaces of dimension greater than 0 and $f(A)=B$ is a light open mapping, there exist locally connected continua A' and B' topologically containing A and B respectively, with dim $A'=$dim A, dim $B'=$dim B, and a light open extension $f(A')=B'$ of f. (Received October 28, 1949.)

T. R. Hollcroft,
Associate Secretary

Appendix

Excerpts from Report of Treasurer

December 22, 1949

To the Board of Trustees of the
American Mathematical Society

Gentlemen:

I have the honor to submit herewith the report of the Treasurer for the fiscal year ended November 30, 1949, with certain pertinent comments.

Investment Portfolio and Income

On November 30, 1949, the market value of securities held for Invested Funds exceeded book value by $20,403, but the market value of securities held for Current Funds was less than book value by $4,333. On the whole portfolio, the market value therefore exceeded book value by $16,070. Reserves held in accounts "Reserve for Investment Losses" ($4,386) and "Profit on Sales of Securities" ($15,851) may still be considered adequate protection against contingent depreciation in market value.

The following is a summary of the changes in security holdings made during the year.

Acquired

10 shares	American Can Co. common
100 shares	Borg-Warner Corp. common
100 shares	Sterling Drug, Inc. common
$2,000	U. S. Treasury 2's 1954/52

Sold

100 shares	General Electric Co. common

Redeemed

$8,000	U. S. Savings Bonds 1949

The investment portfolio, valued at market November 30, 1949, now includes Government bonds 32.6 per cent, other bonds 7.6 per cent, preferred stock 15.3 per cent, common stock 39.6 per cent, cash in savings banks 4.9 per cent.

Income received during the year from investment of Current Funds amounted to $2,991. This represents a return of 3.1 per cent computed on average book value of investments. Income on Invested Funds amounted to $8,832, representing a return of 4.6 per cent. Total investment income from all sources was $11,823, representing a return of over 4.1 per cent.

Income from the Henderson Estate was $5,200; in 1948 it was $4,990.

The Year's Operations

The budget for 1949 adopted by the Trustees authorized an expenditure of $47,000 in excess of anticipated income. Actually, for a number of reasons it was not necessary to expend the full amount authorized. The principal reasons were the inability of the Society to appoint an Executive Director until November 1 and delay in the publication of certain material, particularly the Birkhoff papers.

Another factor which should be noted is that the Society has incurred obligations of approximately $25,000 for work in process for which bills have not been rendered or were rendered so late in the fiscal year that it was possible to defer payment until 1950. The Society could have paid these bills had they been rendered only by substantially liquidating invested current funds. In other words, while the assets of the Society decreased only approximately $3,000, had all of the obligations incurred during the year been paid during the year, assets would have declined by nearly $30,000.

The Society has incurred a new fixed obligation through the employment of an Executive Director and will in all probability have to incur additional fixed obligations in the near future to provide adequate office, working, and storage space. Unless additional sources of income are developed, it would appear to be financially impossible to expand the program of publication or even to maintain it at its present level unless the Society wishes to borrow from its invested funds or from banks. However, the sale of mathematical publications, while dependable, is so slow that it is unlikely that any commercial bank would regard such a loan as attractive.

Respectfully submitted,
ALBERT E. MEDER, JR.
Treasurer

BALANCE SHEET

	November 30, 1949	November 30, 1948
Assets		
CURRENT FUNDS:		
Cash	$ 12,057.26	$ 16,292.01
Due from Invested Funds	31.58	
Account Receivable from United States Government	3,440.49	480.21
Investments	76,292.42	77,500.19
	$ 91,821.75	$ 94,272.41
INVESTED FUNDS:		
Cash		$ 531.01
Investments	$192,153.46	192,102.21
	$192,153.46	$192,633.22
TOTAL ASSETS	$283,975.21	$286,905.63
Liabilities		
CURRENT FUNDS:		
Publications	$ 23,928.48	$ 41,167.18
International Congress	18,089.17	5,484.84
Policy Committee	(160.70)	97.64
Prize Funds and Other Special Funds Accumulated Income	9,553.82	8,238.39
Sinking Fund	1,244.97	1,156.05
Profit on Sales of Securities	2,032.98	2,032.98
Miscellaneous	986.90	746.46
Surplus	36,146.13	35,348.87
	$ 91,821.75	$ 94,272.41
INVESTED FUNDS:		
Endowment Fund Principal	$ 71,000.00	$ 71,000.00
Prize Funds and Other Special Funds	33,033.22	33,033.22
Life Membership and Subscription Reserve	2,851.70	3,073.09
Mathematical Reviews	65,000.00	65,000.00
Reserve for Investment Losses	4,385.89	4,385.89
Profit on Sales of Securities	15,851.07	16,141.02
Due to Current Funds	31.58	
	$192,153.46	$192,633.22
TOTAL LIABILITIES	$283,975.21	$286,905.63

Note: Inventories of books and periodicals are not included in the balance sheet. Books on hand may be expected to yield eventually a net income from sales of approximately $40,000. Back issues of periodicals are difficult to value; average annual sales over the last five year period were approximately $7,500.

SUMMARY STATEMENT OF INCOME AND EXPENDITURES

1948–1949

	1949	1948
GENERAL RECEIPTS		
Dues—Ordinary Memberships	$ 30,017	$ 29,120
Dues—Contributing Memberships	746	712
Dues—Institutional Memberships	9,408	9,582
Initiation Fees	1,091	1,880
Investment Income	12,526	11,953
Miscellaneous	122	191
	$ 53,910	$ 53,438
PUBLICATIONS RECEIPTS	73,775[1]	77,680[2]
OTHER RECEIPTS	1,884	4,470
TOTAL RECEIPTS	$129,569	$135,588
GENERAL EXPENSE	$ 25,507	$ 19,684
COST OF PUBLICATIONS AND SALES	118,837	110,144
OTHER EXPENSE	533	1,212
TOTAL EXPENSES	$144,877	$131,040
EXCESS OF RECEIPTS OVER EXPENSES		$ 4,548
EXCESS OF EXPENSES OVER RECEIPTS	$ 15,308	

[1] This includes $7,140 received under a contract with the Office of Naval Research for support of Mathematical Reviews.

[2] This includes $22,560 received under a contract with the Office of Naval Research for support of Mathematical Reviews.