THE FEBRUARY MEETING IN EAST LANSING

The four hundred fifty-fourth meeting of the American Mathematical Society was held at Michigan State College on Friday and Saturday, February 24–25, 1950. There was a joint session with the Industrial Mathematics Society on Saturday morning.

The total attendance was about 135, including the following 75 members of the Society:

J. H. Bell, Gerald Berman, R. A. Beth, H. L. Black, M. I. Blyth, Richard Brauer, R. W. Bryant, R. E. Carr, Joshua Chover, R. V. Churchill, Harvey Cohn, A. H. Copeland, Max Coral, M. L. Curtis, D. A. Darling, Ben Dushnik, Benjamin Epstein, J. S. Frame, Cornelius Groenewoud, V. G. Grove, H. A. Hanson, Frank Harary, Gerald Harrison, H. L. Harter, G. E. Hay, Fritz Herzog, T. H. Hildebrandt, J. D. Hill, W. E. Jenner, Leo Katz, L. M. Kelly, D. E. Kibbey, W. M. Kincaid, K. J. Klein, Leo Lapidus, H. D. Larsen, J. H. Levin, D. J. Lewis, A. J. Lohwater, O. T. McMillan, Saunders MacLane, A. M. Mark, Imanuel Marx, D. C. Morrow, A. L. Nelson, W. J. Nemerever, E. A. Nordhaus, J. I. Northam, J. D. Novak, R. S. Pate, M. H. Payne, Sam Perlis, George Piranian, J. E. Powell, R. P. Reid, R. A. Roberts, E. H. Rothe, J. C. Rothe, Hans Samelson, W. C. Sangren, M. F. Smiley, W. F. Smith, T. H. Southard, G. L. Spencer, M. D. Springer, R. G. Stanton, B. M. Stewart, P. C. Sweetland, R. M. Thrall, Leonard Tornheim, E. F. Trombley, C. P. Wells, G. S. Young, J. W. T. Youngs, J. W. Zimmer.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, three addresses were delivered on Friday afternoon at 2:30 P.M. The speakers were Professor J. S. Frame, Michigan State College; Professor Sam Perlis, Purdue University; and Professor M. F. Smiley, State University of Iowa. The titles to the addresses were, respectively: *Induced monomial representations of a finite group*; *The holomorph of an algebra*; and *Some questions concerning alternative rings*. The presiding officer was Professor Saunders MacLane of the University of Chicago, who led the discussion period at the completion of each of the half hour addresses.

Sections for contributed paper were held at 11 A.M. Friday, with Professor H. D. Larsen presiding, and at 10:30 A.M. Saturday, with Professor T. H. Southard in charge.

On Friday evening there was a dinner for the Society in the Union Building of Michigan State College. Professor T. H. Osgood, of Michigan State College, welcomed the Society to the campus and Professor H. F. Smiley responded on behalf of the Society.

Abstracts of all papers presented at the Meeting are given below. Papers given by title are indicated by the letter "*t*." Mr. Nims was introduced by Professor A. W. Jacobson, Dr. Elliott and Mr. Johnson by Professor Benjamin Epstein.

ALGEBRA AND THEORY OF NUMBERS

218. J. H. Bell: *A note on the solution of unilateral matrix equations.*

M. H. Ingraham has given (Bull. Amer. Math. Soc. vol. 47 (1941) pp. 68–70) an algorithm for the solution of the unilateral matrix equation when the coefficient matrices are square and have elements in a field of characteristic zero. In this note it is shown that the algorithm is applicable when the coefficient matrices are $m \times n$ matrices and $m \neq n$. If $m > n$, the problem reduces to one of solving a unilateral matrix equation with square coefficients of order n. In this case a corollary obtained by W. E. Roth (Trans. Amer. Math. Soc. vol. 32 (1926) p. 67) is immediately obtained. If $m < n$, it is shown that the equation has either no solutions or an infinite number of solutions. In particular, it is shown that if $m < n$ and a certain equation, with square coefficient matrices of order m, has a solution, then the original equation has an infinite number of solutions. (Received January 9, 1950.)

219t. R. C. Buck: *Factoring of homomorphisms.*

Let $A, B,$ and C be algebras having an addition $+$, and a zero 0, and with B simple. Let $\phi_1, \phi_2, \cdots, \phi_n$ be homomorphisms of A onto B, let ψ be a homomorphism of A into C, let $K_i = \mathrm{kern}\ (\phi_i)$, and $K = \mathrm{kern}\ (\psi)$. Then, if $\bigcap_i^n K_i \subset K$, there exist homomorphisms θ_i of B into C such that $\psi = \sum_1^n \theta_i \phi_i$. As an application, the Jacobson density theorem for irreducible rings of endomorphisms follows (Trans. Amer. Math. Soc. vol. 57 (1945) pp. 228–245). (Received December 8, 1949.)

220. Harvey Cohn: *Periodic algorithm for cubic forms.* Preliminary report.

The basis vectors x_1^i, x_2^i, x_3^i are called a reduced basis of a lattice in three-space if these vectors together with $x_4^i = -\sum x_j^i$ lie each in one octant of space or its negative octant. Then the tetrary cubic form $\prod \sum n_j x_j^i$ is likewise called reduced. If this form is the norm in a cubic module, then there are only a finite number of reduced forms under change of basis, and they can be arranged into chains by a suitable definition of neighbors. There are three to six neighbors according to which one of 40 sets of inequalities prevails. The chains have the structure of a cube (8 forms) for the integers of the field with $D = 49$, and are more complicated structures of 16 and 28 forms respectively for $D = 81$ and $D = 148$. The abelian or non-abelian character of these totally real cubic fields is reflected in symmetries in these chains. (Received January 6, 1950.)

221t. D. O. Ellis: *Autometrized Boolean algebras.* II. *The group of motions of B.*

Let B be a Boolean algebra with meet, join, and complement denoted by ab, $a+b$, and a', respectively. Define distance in B by $d(x, y) = xy' + x'y$. The paper is concerned with the group of motions of B (self-isometries of B) and it is shown that: 1. The group of motions of B is a subgroup of the group of complementation-preserving bi-uniform mappings of B onto itself and has only the identity in common with the group of automorphisms of B. 2. The group of motions of B is isomorphic to the additive group of the Boolean ring associated with B (cf. M. H. Stone, *Subsumption of Boolean algebras under the theory of rings*, Proc. Nat. Acad. Sci. U.S.A. vol. 20 (1934) pp. 197–202). The paper is part of a series of studies the first of which (David Ellis,

Autometrized Boolean algebras) is to appear in the Canadian Journal of Mathematics. (Received January 9, 1950.)

222. B. M. Stewart: *Class equation for row-equivalent matrices over a Galois field.*

This note is concerned with the determination and the development of the properties of functions $Q(r, x)$ and $P(r, n, x)$ in terms of which it is possible to write the class equation for row-equivalent matrices of order n with elements in a Galois field of order $x = p^m$ as follows: $x^{n^2} = \sum Q(r, x) P^2(r, n, x)$, summed from $r = 0$ to $r = n$. The function $P(n, x) = \sum P(r, n, x)$, which gives the total number of classes, is studied. Some of these functions are related to enumerative functions previously known for finite projective geometries, but their independent development from the standpoint of matric theory reveals in a natural way some of their recursive properties. For example, $Q(r, x)$ and $P(r, n, x)$ are shown to be interesting generalizations of the familiar factorial and combinatorial symbols, respectively. (Received January 11, 1950.)

ANALYSIS

223t. Evelyn Frank: *Convergence of C-fractions.*

This paper is concerned with parabolic and circular convergence regions of C-fractions $1 + a_1 z^{\alpha_1}/1 + a_2 z^{\alpha_2}/1 + \cdots$ (cf. Bull. Amer. Math. Soc. Abstract 55-3-142). Corresponding value regions are found, as well as the types of functions to which the C-fractions converge. Special convergence regions are obtained for special C-fractions. (Received January 10, 1950.)

224t. W. S. Gustin: *Asymptotic behavior of mean value functions.*

The mean value $M(t)$ of order t of n positive real numbers x_ν weighted by n positive real numbers a_ν with $\sum a_\nu = 1$ is, except for the three singular values $t = -\infty, 0, +\infty$, given by the formula: $M(t) = (\sum a_\nu x_\nu^t)^{t^{-1}}$. The exact behavior of $M(t)$ at its three singularities is obtained and used to give a simple proof of the result that both $M(t)$ and $\log M(t)$ as functions of t are convex near $t = -\infty$, and concave near $t = +\infty$ (see Shniad, Bull. Amer. Math. Soc. vol. 54 (1948) pp. 770–776). (Received January 27, 1950.)

225t. Josephine M. Mitchell: *A theorem in the geometry of matrices.*

Let Z be a complex $m \times n$ matrix, $\overline{Z}'$ its transposed conjugate, I the identity matrix, $\sigma(Z)$ the trace of Z, and dZ its differential. It is known that in the domain defined by $I - Z\overline{Z}' > 0$ the quadratic differential form $\sigma[(I - Z\overline{Z}')^{-1} dZ (I - \overline{Z}'Z)^{-1} d\overline{Z}]$ is invariant with respect to the conjunctive group of motions of signature (m, n) [Hua, Amer. J. Math. vol. 56 (1944)]. Using Hua's techniques we prove that $\sigma[(I - Z\overline{Z}')^{-1} dZ (I - \overline{Z}'Z)^{-1} d\overline{Z}'] = \sum_{i,k=1}^{m} \sum_{j,l=1}^{n} (\partial^2 \log [\det (I - Z\overline{Z}')]^{-1}/\partial z_{ij} \partial \bar{z}_{kl}) \cdot dz_{ij} d\bar{z}_{kl}$, and the latter invariant quadratic form may be compared with a quadratic form $dz_{ij} d\bar{z}_{kl}$, obtained by Bergman in the case of two complex variables, which is invariant with respect to pseudo-conformal transformations [Bergman, Mémorial des Sciences Mathématiques vol. 106 (1947)]. Further it has been shown in the case of 2×2 matrices that the function $\pi^{-2n} n! (n+1)! [\det (I - Z\overline{Z}')]^{-2-n}$ possesses the characterizing properties of the Bergman kernel function with respect to analytic functions of $2 + n$ complex variables. (Received January 13, 1950.)

226t. A. E. Ross: *Markoff chains associated with transformations of the circle and some related stability problems.*

Let $\psi(\alpha, t)$ be a one-parameter family of transformations of the circle, and let $\psi^k(\alpha_0, t)$ be the smallest power (iterate) of $\psi(\alpha_0, t)$ with fixed points. If τ_1 is a fixed point of $\psi^k(\alpha_0, t)$ such that the difference $d(\alpha_0, t) = \psi^k(\alpha_0, t) - t$ changes sign at τ_1, then $\psi^k(\alpha, t)$ has fixed points for all values of α in a neighborhood of α_0. Write A_0 for the largest such neighborhood of α_0. If $d(\alpha_0, t) > 0$ (<0) for $t < \tau_1$, then we call τ_1 a stable (unstable) equilibrium point. Every closed chain $\tau_1, \cdots, \tau_k[\tau_i = \psi(\alpha\tau_{i-1}), \tau_1 = \psi(\alpha\tau_k)]$ of k stable fixed points is separated by a closed chain $\tau_1^*, \cdots, \tau_k^*[\tau_i^* = \psi(\alpha\tau_{i-1}^*), \tau_1^* = \psi(\alpha\tau_k^*)]$ of unstable fixed points. Assume next that the mechanism which carries out the transformation $\psi(\alpha_0, t)$ is not precise, but is subject to small errors Δ of certain probability density $p(\Delta)$ in the interval $\Delta' < \Delta < \Delta''$. If points τ_i do not lie too close to the points τ_i^*, then under quite general conditions all but a finite number of members of the Markoff chain (1) t_0, $t_1 = \psi(\alpha, t_0) + \Delta_0$, $t_2 = \psi(\alpha, t_1) + \Delta_1, \cdots$ are trapped in the suitably determined k neighborhoods (τ_i) of τ_i. This usually occurs when α is near the middle of the interval A_0. If, however, every τ_i has in its proximity a point τ_i^*, then there occur (intermittent) precessions of the members of the chain (1) either in the clockwise or in the counterclockwise direction. The precessions take place for the values of the parameter α near the ends of the interval A_0. There exist no similar stability properties for the non periodic $\psi(\alpha, t)$. (Received January 13, 1950.)

227. W. C. Sangren: *Expansions over finite intervals when coefficients are discontinuous.*

The solution of boundary value problems involving interfaces or surfaces of separation of two media calls for expansions of an arbitrary function $f(x)$ which generalize the ordinary Sturm-Liouville series. These expansions and necessary conditions imposed upon $f(x)$ are found by using the standard method of the Laplace transform. The expansions are also found formally by the classical method of separation of variables. The expansions in the case of arbitrary coefficients which are discontinuous at the interfaces is shown to be equi-continuous with corresponding expansions where the coefficients are step functions. (Received January 9, 1950.)

228t. W. C. Sangren: *Expansions over semi-infinite and infinite intervals when coefficients are step functions.*

In the case of boundary value problems involving coefficients which due to interfaces are step functions, it is necessary to have expansions of an arbitrary function $f(x)$ which generalize the ordinary Fourier integrals. The expansions and necessary conditions imposed upon $f(x)$ are found by using the method of the Laplace transform. The expansions are also found formally by using a method of separation of variables. (Received January 9, 1950.)

229t. I. E. Segal: *Decompositions of operator algebras.* I.

An algebra of operators on a Hilbert space can be decomposed relative to a Boolean algebra of invariant subspaces as a kind of direct integral, similar to the decomposition as a direct sum of algebras of linear transformations on finite-dimensional spaces. This decomposition results from a decomposition formula for the "states" of operator algebras. If the Boolean algebra is maximal, and with a certain separability restric-

tion, the constituents in the integral are almost everywhere irreducible. It follows that in the case of a separable Hilbert space, a weakly closed self-adjoint algebra is a direct integral of factors. Any continuous unitary representation of a separable locally compact group G is a direct integral of irreducible such representations. If G is unimodular, then its two-sided regular representation is the direct integral of irreducible two-sided representations. Any measure on a compact metric space which is invariant under a group of homeomorphisms of the space is a direct integral of ergodic measures. Our basic results are closely related to theorems of von Neumann and Mautner, but there are important differences in both techniques and results which yield considerable simplification and facilitate the study of algebras derived from groups. (Received January 26, 1950.)

230t. J. E. Wilkins: *Neumann series of Bessel functions*. II.

Theorems on representation of a function $f(x)$, defined when $0 \leqq x$, by a Neumann series of the form $\sum a_{n\nu} J_{\nu+2n+1}(x)$, in which $a_{n\nu} = \int_0^\infty t^{-1} f(t) J_{\nu+2n+1}(t) dt$, which were proved earlier (Trans. Amer. Math. Soc. vol. 64 (1948) pp. 359–385) for a certain class of functions $f(x)$, are here extended to the (best possible) class of functions $f(x)$ for which all the coefficients $a_{n\nu}$ exist. (Received January 11, 1950.)

APPLIED MATHEMATICS

231t. Nathaniel Coburn: *A method for constructing correlation tensors in isotropic turbulence.*

In this paper, a tensor method for constructing correlation tensors in isotropic turbulence is developed. The method depends upon: (1) a decomposition of the turbulence velocity vector, at any point P, along three mutually orthogonal unit vector fields (these fields are defined by parallelism); (2) a requirement that the correlation tensors be invariant under the orthogonal group acting on vector fields perpendicular to the line joining the two points involved in the correlations; (3) a decomposition of the metric tensor with respect to the three mutually orthogonal vector fields introduced in (1). The method is such that other types of symmetry than those encountered in the usual isotropic turbulence can be exploited. As an example, the correlations transverse to a specified direction (which can be interpreted as the direction of the mean flow) are studied. Here, the previous condition (2) is reformulated. It is shown that: (1) a two-dimensional analogue of the continuity equation for isotropic turbulence exists; (2) the third order correlation tensor vanishes; (3) the equations of motion lead to the non-steady heat equation in four dimensions. (Received December 7, 1949.)

232. P. T. Nims: *Design criteria for automatic controls.*

The differential equations of a linear automatic controls system may be studied by means of the Laplace transformation and useful information about the behavior of the control obtained without the necessity for finding the inverse transformation or even finding the poles of the transform. Gardiner in *Transients in linear systems* has shown how to compute the final value of the inverse by a limiting process, and if this method is combined with the integration operation, we may compute the net final area A_{10} enclosed between a step function disturbance and the response to that disturbance. The well known theorem on derivatives of a transform permits calculation of the weighted area A_{11}; that is, the integral of t times the difference between the

step input and the response. Several examples are given of the use of these expressions for areas in the design of automatic control systems. (Received January 13, 1950.)

GEOMETRY

233. L. M. Kelly: *Distance sets.*

With each subset S of a distance space is associated a set of non-negative real numbers $d(S)$ called the distance set of S, a number being an element of $d(S)$ if and only if it is a distance in S. This mapping induces an inverse mapping $d^{-1}(N)$ where N is a set of non-negative numbers including zero. This paper is primarily concerned with this inverse mapping. Thus it establishes: (1) every set of non-negative numbers and zero is the distance set of some metric space; (2) every countable set of non-negative real numbers and zero is the distance set for some subset of Hilbert space; (3) every set of $n+1$ non-negative numbers including zero is a distance set of some subset of E_n. On the other hand, it is shown that: (4) there exist sets of non-negative real numbers including zero which are not distance sets for any separable metric space; (5) there exist countable such sets not "realizable" in any E_n; (6) there exist sets of $n+2$ such numbers not realizable as the distance set in E_n for all n. The work is inspired by, and overlaps to some extent, the study of S. Piccard (*Sur les ensembles de distances,* Mémoirs de l'Université de Neuchatel, 1940). (Received January 13, 1950.)

STATISTICS AND PROBABILITY

234. D. R. Elliot: *A location estimator for non-Gaussian distributions.*

An expression has been found for an estimator of the location parameter of a non-Gaussian distribution. The estimator g satisfies the conditions (1) $g(X_1, X_2, \cdots, X_n) = g(X_1+U, X_2+U, \cdots, X_n+U) - U$ and (2) variance of $E = a$ minimum, where $X_1, X_2, \cdots, X_n$ are the n sample variates ordered according to magnitude and not in general just any arbitrary ordering, and U is a parameter of location. Both the median and the mean satisfy equation (1) but not necessarily equation (2). g for a Gaussian distribution gives $g = \overline{X}$; g for a rectangular distribution gives $g = $ (smallest variate + largest variate)/2; more complicated expressions are obtained for the Cauchy and single "saw-tooth" distributions. For some distributions the use of the above estimator may be considerably more efficient than the average of the sample. (Received January 13, 1950.)

235. L. G. Johnson: *The median ranks of sample values in their population with an application to certain fatigue studies.*

Let $_0x_1, _0x_2, _0x_3, \cdots, _0x_n$ be a set of "n" ordered observations from a population whose cumulative distribution function is $F(x)$. The author determines in a nonparametric fashion the median of each of the values $F(_0x_1)$, $F(_0x_2)$, $F(_0x_3)$, $\cdots$, $F(_0x_n)$ for all possible samples of size "n" from the given population. The use of these medians is illustrated in a graphical method of estimating population parameters from ball bearing fatigue data. Because of the skewness of extreme values, it is pointed out that the mean of each of the quantities $F(_0x_1)$, $F(_0x_2)$, $F(_0x_3)$, $\cdots$, $F(_0x_n)$ is graphically misleading and hence not as useful as the median. (Received January 13, 1950.)

Topology

236. E. W. Rothe: *A relation between the type numbers of a critical point and the index of the corresponding field of gradient vectors.*

Let $I(x)$ be a real-valued function of the point x of the real Euclidean n-space E^n which has continuous second derivatives in some neighborhood of the origin o of E^n. Let o be an isolated critical point of I, and m^r $(r=0, 1, \cdots, n)$ the corresponding Morse type numbers. The vector field $g(x) = \mathrm{grad}\, I$ has then an isolated singularity at o. Let j be its index. The result of the paper is the relation $j = \sum_{i=0}^{n} (-1)^r m^r$ which is proved under the following assumption: there exists a neighborhood U of o such that for all $x \neq o$ of the intersection $U \cap \{x \mid I(x) = I(0)\}$ the vectors $\mathrm{grad}\, I$ and $x - o$ are not parallel. (Received January 11, 1950.)

237t. A. D. Wallace: *A theorem on endpoints.*

Let X be a compact Hausdorff space and X_0 a closed subset. We use the Alexander-Kolmogoroff cohomology theory as developed by Spanier (Ann. of Math. (1948)). A set A in X is p-connected (mod X_0) if the image of $H^p(A, A \cap X_0)$ in $H^p(B, B \cap X_0)$ is 0 for every compact subset B of A. A set is p-trivial (mod X_0) if each of its compact subsets is p-connected (mod X_0) (G. T. Whyburn). A point is a p-endpoint (mod X_0) if each of its nbhds contains a nbhd (of it) whose boundary is p-trivial (mod X_0). A set is totally p-disconnected (mod X_0) if each of its compact subsets which is p-connected (mod X_0) is also p-trivial (mod X_0). Theorem: If X is p-connected (mod X_0) and E is a set of its p-endpoints (mod X_0) then $X - E$ is p-connected (mod X_0) and E is totally p-disconnected (mod X_0). (Received January 3, 1950.)

238. G. S. Young: *Smooth interior transformations on 2-manifolds.*

Let $f: A \to B$ be an interior transformation of a compact 2-manifold A. If the set F of all points x of B such that some component of $f^{-1}(x)$ is not locally connected is an F_σ, then either f is light (and B is a 2-manifold), or B is a set obtained from a dendrite by a finite number of identifications. If F is empty, and f is not light, then B is an arc or a simple closed curve. Partial extensions of these to non-compact manifolds are given. (Received January 28, 1950.)

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