THE FEBRUARY MEETING IN NEW YORK

The four hundred fifty-fifth meeting of the American Mathematical Society was held at Columbia University on Saturday, February 25, 1950. The attendance was about one hundred forty, including the following one hundred thirty members of the Society:

M. I. Aissen, R. G. Archibald, P. T. Bateman, F. P. Beer, Stefan Bergman, Lipman Bers, Garrett Birkhoff, S. G. Bourne, A. B. Brown, F. H. Brownell, Jewell H. Bushey, P. A. Caris, P. G. Carlson, Alan Cathcart, Jeremiah Certaine, K. T. Chen, Y. W. Chen, W. L. Chow, Alonzo Church, A. B. Clarke, F. M. Clarke, J. A. Clarkson, L. W. Cohen, Philip Cooperman, Richard Courant, L. M. Court, H. B. Curry, C. W. Curtis, Martin Davis, Philip Davis, J. B. Diaz, C. H. Dowker, Y. N. Dowker, Aryeh Dvoretzky, Samuel Eilenberg, W. H. Fagerstrom, J. M. Feld, F. A. Ficken, Emanuel Fischer, A. D. Fleshler, J. B. Freier, Gerald Freilich, Bernard Friedman, K. O. Friedrichs, Orrin Frink, G. N. Garrison, R. E. Gilman, G. H. Gleissner, Sidney Glusman, W. H. Gottschalk, Emil Grosswald, V. H. Haefeli, Henry Helson, H. L. Herrick, Einar Hille, Banesh Hoffmann, T. R. Hollcroft, E. M. Hull, L. C. Hutchinson, Wenceslas Jardetzky, Bjarni Jónsson, Shizuo Kakutani, Edward Kasner, M. E. Kellar, J. R. Kline, Kunihiko Kodaira, Paul Koditschek, E. R. Kolchin, R. B. Leipnik, Benjamin Lepson, Howard Levi, M. A. Lipschutz, E. R. Lorch, Lee Lorch, L. A. MacColl, H. M. MacNeille, A. J. Maria, W. T. Martin, A. E. Meder, Jr., A. N. Milgram, K. S. Miller, W. H. Mills, Don Mittleman, D. J. Newman, P. B. Norman, I. L. Novak, A. F. O'Neill, J. C. Oxtoby, L. G. Peck, E. L. Post, Hans Rademacher, John Raleigh, K. G. Ramanathan, H. E. Rauch, M. S. Rees, K. W. Reidemeister, R. W. Rempfer, Moses Richardson, J. F. Ritt, M. S. Robertson, Robin Robinson, S. L. Robinson, C. A. Rogers, H. D. Ruderman, C. W. Saalfrank, Arthur Sard, A. H. Sarno, Samuel Schecter, Abraham Schwartz, Seymour Sherman, James Singer, P. A. Smith, Ernst Snapper, J. J. Sopka, E. J. Specht, C. F. Stephens, F. M. Stewart, R. C. Stewart, J. J. Stoker, D. L. Thomsen, C. A. Truesdell, Abraham Wald, H. V. Waldinger, J. L. Walsh, Alan Wayne, Alexander Weinstein, Louis Weisner, E. T. Welmers, P. M. Whitman, Albert Wilansky.

At 2:00 P.M. Professor C. L. Siegel of the Institute for Advanced Study gave an address on *Classes of analytic transformations*. President J. L. Walsh presided.

At 3:30 P.M. there were two sections for contributed papers, in which Professors Orrin Frink and R. E. Gilman presided.

Abstracts of the papers presented follow below. Abstracts whose numbers are followed by the letter "*t*" were presented by title.

ALGEBRA AND THEORY OF NUMBERS

239*t*. A. S. Amitsur and J. Levitzki: *Remarks on minimal identities for algebras.*

Let A_n denote the matric algebra of all n by n matrices over a field F, and $d(A_n)$ the degree of the minimal identities satisfied by A_n. Consider the standard identity $\sum \pm x_{i_1} x_{i_2} \cdots x_{i_{2n}} = 0$, where $i_1, i_2, \cdots, i_{2n}$ is a permutation of $2n$ letters $1, 2, \cdots,$

$2n$, and the sign is $+$ for even and $-$ for odd permutations. It has been shown (J. Levitzki, *A theorem on polynomial identities*, Bull. Amer. Math. Soc. Abstract 55-5-233) that $d(A_n) \geq 2n$. Later the present authors have obtained the following results (*Minimal identities for algebras*, to appear in Proceedings of the American Mathematical Society): (1) $d(A_n) = 2n$. (2) If either $n > 2$ for $F \neq P_2$ where P_2 is the prime field of characteristic 2, then each minimal identity of A_n is a linear combination of standard identities of degree $2n$. This does not hold for $n \leq 2$ and $F = P_2$. (3) Conversely, in all cases the standard identity of degree $2n$ and all linear combinations of such identities are minimal identities for A_n. In the present note the authors show that above theorems hold also for semi-simple algebras over F. They further determine all minimal identities also in the exceptional case ($n \leq 2$, $F = P_2$). Finally an upper and a lower bound for the degree of the minimal identities of an algebra with radical are given, and it is shown by examples that these estimates are in a certain respect the best possible ones. (Received December 30, 1949.)

240t. Bailey Brown: *An extension of the Jacobson radical.*

A part of N. Jacobson's theory of the radical of a ring (Amer. J. Math. vol. 67 (1945) pp. 300–320) is extended to an arbitrary cluster (R. A. Good, Trans. Amer. Math. Soc. vol. 63 (1948) pp. 482–513). An element a of a cluster R is called *quasi-regular* if and only if a is in the right ideal generated by the set of all elements of the form $ax - x$, x in R. The *radical N* of R is defined to be the set of all elements a of R such that each element of the ideal generated by a is quasi-regular. If M is a right ideal in R, let M' denote the greatest ideal in R contained in M. A right ideal M is *modular* if and only if $ex - x$ is in M for some e and all x in R. Then $N = \bigcap M'$, where M ranges over all modular maximal right ideals in R. Call R *primitive* if and only if $M' = 0$ for some modular maximal right ideal M in R. Then a cluster $R \neq 0$ is isomorphic to a subdirect sum of primitive clusters if and only if $N = 0$. Under analogous definitions for *left-radical N_1* and *left-primitive*, an example is given of an algebra, necessarily non-associative, for which $N \neq N_1$ and which is primitive but not left-primitive. (Received January 20, 1950.)

241. Emil Grosswald: *On the structure of some subgroups of the modular group.*

Let the square matrix with first row a, b and second row c, d, $ad - bc = 1$, be an element of the modular group Γ, and let p be any prime. The subgroups $\Gamma_0^0(p)$ of Γ are defined by the conditions $b \equiv c \equiv 0 \pmod{p}$ and the "principal subgroups" $\Gamma(p)$ by the conditions: $a \equiv d \equiv 1 \pmod{p}$ and $b \equiv c \equiv 0 \pmod{p}$. Using the methods of H. Rademacher (Abh. Math. Sem. Hamburgischen Univ. vol. 7 (1929) pp. 134–148) and results of K. Reidemeister (ibid. vol. 5 (1926) pp. 7–23) and O. Schreier (ibid. vol. 5 (1926) pp. 161–183) the structure of the subgroups $\Gamma_0^0(p)$ is completely determined. The number of independent generators, their defining relations, and the genus of the corresponding fundamental region are found as functions of p; also the explicit form of the independent generators as functions of the generators of Γ is found. By the same methods also the structure of the principal subgroups $\Gamma(p)$ is studied, but the results are less complete. (Received January 18, 1950.)

242t. Jakob Levitzki: *Prime ideals and the lower radical.*

It has been shown by McCoy (Amer. J. Math. vol. 71 (1949) pp. 823–833) that the intersection of all the prime ideals containing an ideal A of a ring R is an ideal B

such that B/A is a radical ideal of R/A. He raised the problem as to the connection of B/A to other radicals of R/A. In the present note this problem is solved by showing that B/A is actually the lower radical of R/A. McCoy's results combined with this theorem yield, for example, the following corollary: The intersection of all the prime ideals of a ring is equal to 0 if and only if the ring does not contain nilpotent ideals other than zero. Use is made of the concept of an m-sequence (+) $a_0, a_1, \cdots$, where $a_{n+1} = a_n b_n a_n$ with $b_n \in R$, which is a special type of McCoy's m-systems. This concept yields the following characterization of the lower radical L: For $a \in R$ we have $a \in L$ if and only if each m-sequence (+) with $a = a_0$ has at most a finite number of nonzero terms. (Received January 5, 1950.)

ANALYSIS

243. Michael Aissen: *A class of superadditive functions.*

Functionals are superadditive in the ordinary sense if their value for the sum of disjoint sets is not less than the sum of their values over the separate sets. They are superadditive S if for a finite number of sets E_i (not necessarily disjoint), included in a single set E, the value of the functional for E is not less than the Sylvester sum of the functional values for the sets E_i. A simple example shows that ordinary super-additivity does not imply superadditivity S. Admissible sets are defined as bounded open plane sets, whose components are simply connected. A generalized stress function of such an admissible set is defined and is shown to be superadditive S. The proof is by induction, with the case $n = 1$ corresponding to the ordinary superadditivity of the generalized stress functions, which is well known. As special cases the St. Venant stress function and the torsional rigidity are superadditive S. (Received January 14, 1950.)

244. F. H. Brownell: *On the spectrum of the second order differential operator over n-space.* Preliminary report.

The continuity requirements on the potential function, $V(x)$, for the Schrödinger equation over all euclidean n-space, E_n, are replaced by a single weak integrability condition. This still allows the construction of a modified Green function, which defines a bounded linear operator on $L_2(E_n)$. By taking Fourier transforms the spectrum of this operator can be analyzed, the equation becoming the formal Fourier transform of the Schrödinger equation. This analysis generalizes the result of Putnam, Amer. J. Math. (1949) p. 109, from the half-line to E_n, repeats the result of Friedrichs, Math. Ann. (1934) Theorem 7, p. 705, now without continuity requirements, and finally removes the point spectrum from $\lambda > l$ if $|V(x) - l|$ has a very strong exponential bound at ∞, which seems new for E_n although much more is known for the half-line. (Received February 6, 1950.)

245t. N. J. Fine: *Proof of a theorem of Jacobi.*

A short, self-contained proof is given of the following theorem: If $G(z)$ is defined in $[-1, 1]$, then $\int_0^\pi G(\cos x) \cos nx\,dx = K_n^{-1} \int_0^\pi G^{(n)}(\cos x) \sin^{2n} x\,dx$, where $K_n = 1 \cdot 3 \cdot 5 \cdots (2n-1)$. (Received January 16, 1950.)

246t. N. J. Fine: *The generalized Walsh functions.*

In his dissertation [*On the Walsh functions*, Trans. Amer. Math. Soc. vol. 65 (1949) pp. 372–414] the author discussed various properties of the Walsh functions $\{\psi_n(x)\}$

and attempted to exhibit the close analogy between them and the exponentials $\{\exp 2\pi i n x\}$. It is natural to ask whether the analogy can be extended to the system $\{\exp 2\pi i y x\}$, that is, whether the Walsh functions can be imbedded in a larger class $\{\psi_y(x)\}$ so as to preserve most of the properties of the exponential which are desirable and useful in analysis. This question is answered in the affirmative here, and again group-theoretic considerations are important. The generalized Walsh functions are defined (essentially) as the characters of the additive group of a certain topological field, and are proved to be the only nontrivial measurable solutions of a functional equation similar to $f(x+y)=f(x)f(y)$. It is shown that $\psi_y(x)$ is periodic if and only if y is a *dyadic* rational. Finally, the analogues of the Riemann-Lebesgue theorem, Fourier integral theorem, and Poisson formula are given. (Received January 16, 1950.)

247. Benjamin Lepson: *Series of analytic functions and polynomial Dirichlet series.*

In the first part of this paper, some results are obtained concerning sequences and series of analytic functions. One of these is the theorem that the set of boundary points of the region of convergence which belongs to this region forms a set of first category in the plane. An analog of Osgood's theorem is also obtained, which states that given a series of analytic functions which converges absolutely at every point of a domain, there is a subdomain for which the least upper bounds of the functions in that subdomain form a convergent series. These results are applied, in the second part, to "polynomial Dirichlet series," that is, series of the form $\sum_{n=0}^{\infty} P_n(z)e^{-\lambda_n z}$, where $P_n(z)$ is a polynomial of degree μ_n and $0<\lambda_0<\lambda_1<\cdots\rightarrow+\infty$. With the single assumption $\mu_n/\lambda_n\rightarrow 0$, properties of convergence and analyticity are found similar to those for Dirichlet series. Certain differences are observed, however, notably the fact that the Abel limit theorem may fail. These series occur naturally in the study of linear differential equations of infinite order. (Received January 10, 1950.)

248. Lee Lorch: *On the Lebesgue constants for $(E, 1)$ summation of Fourier series.*

K. Prachar and L. Schmetterer have shown that the Lebesgue constants arising from the application of the Euler summation methods (E, 1) and (E, 2) to Fourier series diverge logarithmically (Anzeiger der Akademie der Wissenschaften in Wien. Mathematisch-Naturwissenschaftliche Klasse vol. 85 (1948)). For $(E, 1)$ this was noticed by C. N. Moore, who discussed the effectiveness of Borel's method (which is equivalent to the Euler methods in the study of Fourier series), pointing out, among other things, that the Borel-Lebesgue constants are of logarithmic order (Proc. Nat. Acad. Sci. U.S.A. vol. 11 (1925)). The purpose of the author is to prove that the Lebesgue constants for $(E, 1)$ have the same asymptotic representation as that obtained for the Borel-Lebesgue constants (L. Lorch, Duke Math. J. vol. 11 (1944); Bulletin of the Calcutta Mathematical Society vol. 37 (1945)). The method is essentially the one used in the Duke Journal reference. (Received January 3, 1950.)

249t. M. S. Robertson: *Applications of a lemma of Fejér to typically-real functions.*

If $f(z)=z+\sum_2^{\infty} a_n z^n$, $g(z)=z+\sum_2^{\infty} b_n z^n$ are regular and typically-real for $|z|<1$, then $F(z)=z+\sum_2^{\infty} a_n b_n z^n$ is regular for $|z|<1$ and typically-real for $|z|\leq 2-3^{1/2}$, while $\int_0^z (F(z)/z)dz$ is typically-real for $|z|<1$. If, in addition to being typically-real, $f(z)$ and $g(z)$ are univalent and convex in the direction of the imaginary axis for $|z|<1$,

then so is $F(z)$. If $f(z)$ is typically-real for $|z|<1$, a specially weighted average of the cosine of the angle subtended at the origin by the points $f(z), f(ze^{i\theta})$ is found to be non-negative for an explicit range of values of θ which depends on $|z|<1$. (Received January 16, 1950.)

APPLIED MATHEMATICS

250. C. A. Truesdell: *Bernoulli's theorem for viscous fluids.*

By a Bernoulli theorem is meant a statement that a certain finite expression one of whose terms is the specific kinetic energy $v^2/2$, a second contains one or more dynamical variables, and none involves an integral, remains constant or a function of time only upon each member of a certain family of curves. In any motion of a viscous compressible fluid in which the vorticity is steady such a finite expression is given explicitly, and a construction for the curves along which it is a function of time only is indicated. In the special case of an incompressible fluid subject to conservative extraneous force the finite expression reduces to that of the classical Bernoulli theorem, and the curves are defined in terms of the velocity field only, so that just as in classical hydrodynamics the Bernoulli theorem yields a formula for the pressure in terms of purely kinematic observables. The non-uniform character of the limit $\mu\rightarrow0$ appears strikingly in the fact that these curves are independent of μ so long as $\mu\neq0$, but in the limit $\mu=0$ they spread out into Lamb's Bernoullian surfaces. (Received January 9, 1950.)

GEOMETRY

251. L. M. Court: *Envelopes of plane curves.*

The envelope e of a plane family of curves touches each member c. Defining the envelope as the locus of intersections of "consecutive" members of the family, the author gives a purely geometric proof of this familiar proposition (the usual proof is analytic). Setting up a direction of advance along e and assuming, among other things, that the point P in which c and e meet is not a multiple point of c, he defines (locally) rear and forward ends for c with respect to P and e's direction of advance. He then shows that two members meet locally in *un*like ends, this property characterizing the envelope (as against other curves) vis-a-vis the family. From here on, it is a simple matter to show that c and e touch. (Received February 16, 1950.)

252. J. M. Feld: *On the geometry of lineal elements on a sphere, euclidean kinematics and elliptic geometry.*

Slides and turns of oriented lineal elements on the plane were first studied by Kasner (Amer. J. Math. vol. 33 (1911)). The author has for his object the investigation of the geometry of slides and turns of oriented lineal elements on a sphere. Slides and turns on a sphere generate the three-parameter group of spherical whirls W_3, which combined with the group of sphere rotations M_3 yield a six-parameter group of whirl-rotations G_6. The author determines invariants of lineal elements, spherical turbines, and flat fields of lineal elements under W_3, M_3, and G_6; also sets of fundamental differential invariants of series of lineal elements on a sphere. Finally he shows how the geometry of these groups of transformations on a sphere can be interpreted (1) as a model for the geometry of elliptic three-space and (2) as the kinematic geometry of a unit sphere sliding over a concentric unit sphere. These interpretations are analogous to those given by the author (Bull. Amer. Math. Soc. vol. 48 (1942); Amer. J. Math. vol. 70 (1948)) of the geometry of whirl-motions on the plane (1) as

the geometry of quasi-elliptic space and (2) as the kinematic geometry of one plane sliding over another. (Received January 13, 1940.)

253. Edward Kasner: *Conformal symmetry and satellites.*

Conformal symmetry with respect to a curve C is also called Swarzian reflexion. If the curve C is algebraic of degree n, the symmetry is multivalued of degree n^2 (or lower). The image of C with respect to C consists of C and another algebraic curve S called the satellite. The satellite of a conic is another definite conic which sometimes degenerates. (The satellite of a cubic is usually of sixth degree.) If the conic is a circle, symmetry is merely inversion and no satellite exists. If the conic passes through one of the absolute points, symmetry is one-to-two. For the general conic, symmetry is one-to-four. Multivalued groups are considered. (Received January 13, 1950.)

254. Don Mittleman: *Areal-elements and areal-transformations.*

Let O be the pole of a polar coordinate system (ρ, θ), C a curve, and α a differentiable function defined for all points of C. At each point p of C there is defined a vector r whose initial point is p, positively directed toward O, and whose length r is defined by the equation $2d\alpha = (2\rho r - r^2)d\theta$. The point p on C and the value of the function α at p together define an *areal-element*. The ∞^1 areal-elements determined by C and α is called a *series*. The special series for which the terminal points of the vectors r is O is called a *union*. The necessary and sufficient condition that a series be a union is $\alpha(q) - \alpha(p) = \int_p^q 2^{-1}\rho^2 d\theta$ for all points p and q on C. The transformation (1) $P = P(\rho, \theta, \alpha)$, $\Theta = \Theta(\rho, \theta, \alpha)$, $A = A(\rho, \theta, \alpha)$ is called an *areal-transformation*. An areal-transformation which carries unions into unions is called an *area-transformation*. The necessary and sufficient conditions that the transformation (1) be an area-transformation are: $2A_\rho = P^2\Theta_\rho$ and $(2A_\theta + \rho^2 A_\alpha)\Theta_\rho = (2\Theta_\theta + \rho^2\Theta_\alpha)A_\rho$. (Received January 3, 1950.)

LOGIC AND FOUNDATIONS

255t. A. R. Schweitzer: *Selected topics from Sir A. S. Eddington's "Fundamental theory."* I.

The subjects selected are classified into two interdependent parts pertaining respectively to A. Mathematics, B. Cosmology. Under A are placed: I. Chirality (from $\chi\epsilon\iota\rho$, cheir, hand), II. Algebras: II_1. The quaternion calculus or ζ algebra, II_2. The "E" calculus or the square of the ζ algebra, II_3. The "EF" calculus or the fourth power of the ζ algebra, III. The vector calculus; the complete momentum vector; the wave vector, IV. The tensor calculus; the Riemann-Christoffel tensor; the energy tensor; the wave tensor calculus, V. The wave equation, VI. The electromagnetic field. (Received January 9, 1950.)

256t. A. R. Schweitzer: *Selected topics from Sir A. S. Eddington's "Fundamental theory."* II.

Continuing the preceding abstract, the author considers under B: I. Concept of physics. II. Space, time, space-time. III. Wave, particle, and wave-particle theories. IV. Energy, including radiant energy (radiation). V. Concepts of universe. VI. Special, intermediate and general relativity. VII. Fundamental theory as a combination of intermediate relativity and quantum theory. VIII. The constants of nature; the cosmical number. The author characterizes Eddington's *Fundamental theory* as an attempt to unify relativity and quantum theory from a mathematical point of view

as distinguished from a unification envisaged by L. L. Whyte from the standpoint of logic (*Critique of physics*, New York, 1931) and a unification suggested by the philosophy of A. N. Whitehead (*Process and reality*, New York, 1929; see in particular, p. 364, last paragraph, and p. 365). (Received January 19, 1950.)

TOPOLOGY

257t. R. H. Bing: *Chained hereditarily indecomposable continua are homeomorphic.*

A continuum is hereditarily indecomposable if each of its subcontinua is indecomposable. Knaster gave an example of such a continuum and more recently such continua have been studied by Moise, Bing, and Anderson. It is shown that the compact nondegenerate hereditarily indecomposable continua M and M' are homeomorphic if for each positive number ϵ, each of M and M' can be covered by an ϵ-chain. In fact, if p and q are points of different composants of M while p' and q' are points of different composants of M', there is a homeomorphism carrying M into M', p into p', and q into q'. Most compact continua in a Euclidean n-space $(n>1)$ E are pseudo-arcs; that is, if G is the space of bounded continua in E, the set of points of G which are topologically equivalent (in E) to M is a dense inner limiting (G_δ) subset of G. (Received January 13, 1950.)

258t. David Gale: *Separating algebras of continuous functions.*

It has been shown by Stone that if B is an abstract Boolean ring, then B has a perfect representation as the family of all open and closed subsets of a topological space S whose points are the maximal ideals of B. On the other hand starting with a perfect ring of subsets $B(X)$ on a space X and then going to the abstract ring B, the space S of maximal ideals will not in general be homeomorphic with X. It is shown here, however, that if $B(X)$ is considered as a set of continuous functions from X to the two element field, and given the compact-open topology, a perfect representation of the topological ring B may be constructed as continuous functions on a space S' whose elements are the *closed* maximal ideals of B. In this case the function ring $B(S')$ will be equivalent to the original ring $B(X)$. The analogue of the theorem is also proved for algebras $A(X)$ of continuous real-valued functions on a space X, provided that $A(X)$ is "separating," that is, contains enough functions to separate points. Use is made of the Stone-Weierstrass approximation theorem. (Received January 13, 1950.)

259t. S. T. Hu: *On generalizing the notion of fibre spaces to include the fibre bundles.*

The notion of fibre spaces is generalized by localizing the slicing function as follows. A topological space X is called a generalized fibre space over a topological space B relative to a map $\pi: X \to B$ of X onto B if, for each point $b \in B$, there exist an open set U of B containing b and a map $\phi_U: U \times \pi^{-1}(U) \to X$ such that: (i) $\pi \phi_U(b, x) = b$ $(b \in U, x \in \pi^{-1}(U))$; (ii) $\phi_U(\pi(x), x) = x$ $(x \in \pi^{-1}(U))$. It is proved that every fibre bundle is a generalized fibre space. Nearly all familiar properties of fibre spaces are generalized to the generalized fibre spaces; in particular, the covering homotopy theorem is proved for the generalized fibre spaces in the usual form for fibre bundles. (Received January 11, 1950.)

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Associate Secretary