R. G. D. RICHARDSON
1878–1949

Roland George Dwight Richardson served the American Mathematical Society for more than twenty-nine consecutive years: as vice-president, 1920; as the Society's third secretary, 1921–40; and as trustee, 1924–49. On 17 July 1949 he died, as the result of a coronary thrombosis, at South River Lake, Nova Scotia, his native province in Canada. He was born at Dartmouth on 14 May 1878, the elder son of George Josiah and Rebecca Archibald (Newcomb) Richardson. His mother was a direct descendant of the seventeenth century Simon Newcomb, from whom also sprang Simon Newcomb, the Nova Scotia-born astronomer, fourth president of the American Mathematical Society.

Entering Acadia College, Wolfville, N. S., in 1896 he graduated A.B. two years later. During 1895–96 and 1898–99 he was teacher of a school at the fishing village, Margaretsville, N. S., and then principal of the high school at Westport, N. S. from 1899 to 1902, when he started on his brilliant course at Yale University, A.B. 1903, A.M. 1904; instructor in mathematics 1904–07; Ph.D. 1906.

In 1907 Richardson accepted a call to Brown University as assistant professor of mathematics, with the understanding that he might spend the following year in study and research at the University of Göttingen, where Hilbert and Klein were still active and attracting many disciples. Thus in June, 1908, on the day after his marriage in Montreal to Louise Janet MacHattie, a former pupil at Westport, he set sail for Europe. In 1912 he was promoted to an associate professorship at Brown, and in 1915, to a professorship and the chairmanship of his department. Before indicating Richardson's contributions to the intellectual life of Brown University, we shall pause to survey his scientific research, and the American Mathematical Society when he was secretary.

Scientific Research. Richardson's first publications [**1**] and [**3**],[1] appeared shortly after his arrival at Brown University, and were developments of his doctoral dissertation at Yale, where Pierpont was the chief source of his mathematical inspiration. Connected with the theory of integration these papers supplemented and extended work done by Pierpont in his *Theory of functions of a real variable* and in his published papers. In consequence the point of view is

[1] Numbers in brackets refer to the bibliography at the end of the paper.

essentially that of Riemann (content) rather than that of Lebesgue (measure). In [1] Richardson defines an improper integral on an n-dimensional set A having content after the manner of de la Vallée-Poussin in which the unbounded function $f(x)$ is approximated by the truncated functions $f(x, M_1, M_2)=f(x)$, M_1, or $-M_2$ according as $-M_2 \leqq f(x) \leqq M_1$, $M_1 \leqq f(x)$, or $f(x) \leqq -M_2$, so that $\int_A f = \lim_{M_1, M_2 \to \infty} \int_A f(x, M_1, M_2)dx$ whenever this limit exists, and he proves the equivalence of this integral to one defined by Pierpont.

The first half of paper [3] is concerned with a sequence of functions f_n converging to f on A, and convergence of $\int_A f_n$ to $\int_A f$ is proved when f_n is a monotone sequence and the set of functions f_n is uniformly bounded on A. In the second half of the paper a multiple integral and corresponding iterated integrals are considered. If $x_1, \cdots, x_n$ are the coordinates of the space containing A and the positive integer p is $<n$, C is the section of A obtained by holding $x_1, \cdots, x_p$ fixed and varying $x_{p+1}, \cdots, x_n$; B is the set over which $x_1, \cdots, x_p$ ranges; thus A is the product set $B \times C$. All sets involved being assumed to have content, the principal result is that if $\int_A f$ exists, the iterated integral $\int_B \int_C f$ exists and has the same value. The integrand f may be unbounded, and improper integrals are defined as in [1]. The results obtained are about as far as one can go without the notions of measure and Lebesgue integral.

The contact which Richardson had with Hilbert at Göttingen exerted a deep influence on all of his subsequent mathematical research. As the major part of this work dealt with oscillation properties of solutions of ordinary linear second order differential equations, it is in order to comment briefly on the development of certain aspects of this subject that had occurred in the preceding decade, and to which Richardson's work was most closely related. First, Bôcher[2] had given rigorous and accessible form to the work of Sturm and Klein on the real solutions of such differential equations. Secondly, in the development of his theory of integral equations, and in the application of this theory to self-adjoint boundary problems, Hilbert[3] had pointed out the interrelations with variational principles. In particular, in 1906 Hilbert[4] had conjectured that for Sturm-Liouville

[2] *The theorem of oscillation of Sturm and Klein*, Bull. Amer. Math. Soc. vol. 4 (1897–98) pp. 295–313, 365–376; vol. 5 (1898–99) pp. 22–43.

[3] *Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen*, Nachr. Ges. Wiss. Göttingen, 1. Mitt. (1904) pp. 49–91; 2. Mitt. (1904) pp. 213–259; 3. Mitt. (1905) pp. 307–338; 4. Mitt. (1906) pp. 157–227; 5. Mitt. (1906) pp. 439–480; 6. Mitt. (1910) pp. 355–417.

[4] See Jber. Deutschen Math. Verein. vol. 16 (1907) p. 77.

boundary problems the oscillation properties of the proper functions (characteristic functions, or eigenfunctions) were consequences of the Jacobi condition imposed on these functions by the fact that they afforded a minimum for certain associated problems of isoperimetric type in the calculus of variations. This question had been considered by Robert König in his Göttingen dissertation, 1907, but König's treatment contained errors. Hilbert proposed to Richardson the reconsideration of this matter, and in [5] Richardson established the validity of Hilbert's conjecture for the particular boundary problem

$$(1)\qquad \frac{d}{dx}\left[p(x)\frac{du}{dx}\right] + q(x)u + \lambda k(x)u = 0, \qquad 0 \leqq x \leqq 1,$$
$$u(0) = u(1) = 0,$$

where $p(x)$, $q(x)$, $k(x)$ are real-valued, analytic, and $p(x) > 0$, $q(x) \leqq 0$, $k(x) \not\equiv 0$ on $0 \leqq x \leqq 1$.

Richardson next turned his attention to oscillation theorems for solutions of linear second order differential equations containing two or more parameters. His paper [6] on the saddle point in the theory of maxima and minima in the calculus of variations was prefatory to [7], in which the methods of [5] were extended to obtain certain oscillation theorems for the solutions of two equations involving two parameters. From the standpoint of a Klein oscillation theorem, however, the results of this paper were incomplete. A brief statement on how the full Klein oscillation theorem might be derived by the methods of the paper, together with a reference to his forthcoming paper [8], appeared in a remark added in proof at the end of the paper. In [8] Richardson treated the Klein oscillation theorem for the system

$$\frac{d}{dx}\left(p_1(x)\frac{du}{dx}\right) + q_1(x)u + [\lambda A_{11}(x) + A_{12}(x)]u = 0, \qquad p_1(x) > 0,$$
$$(2)\qquad \frac{d}{dy}\left(p_2(y)\frac{dv}{dy}\right) + q_2(y)v + [\lambda A_{21}(y) + A_{22}(y)]v = 0, \qquad p_2(y) > 0,$$
$$u(a_1) = u(b_1) = 0, \qquad v(a_2) = v(b_2) = 0,$$

where the coefficients are analytic, real-valued functions on the respective intervals $a_1 \leqq x \leqq b_1$, $a_2 \leqq y \leqq b_2$. In [9] the methods introduced in [8] were extended to the corresponding problem for three equations involving three parameters. In [8] and [9] Richardson was concerned with the study of both necessary and sufficient conditions for the validity of a Klein oscillation theorem; in particular, the results of these papers were valuable additions to earlier work on such problems by Bôcher,[5] Hilbert[6] and Yoshikawa.[7]

In connection with papers [8] and [9], two relevant comments seem worthy of mention. First, in contrast to that of [5] and [7], the discussion of these papers was quite independent of methods of the calculus of variations, although in the introduction to [8] he stated that the results of that paper were first obtained by such methods. This shift in his method of attacking oscillation theorems was due to a suggestion of G. D. Birkhoff, as acknowledged in a footnote on p. 32 of [8]. Secondly, in [8] the discussion of the oscillation theorem for a system of the form (1) involving the single parameter λ was extended to a case not treated previously, namely, the case of $q(x)$ positive in at least a part of the interval and $k(x)$ taking on both signs.

Richardson's work on oscillation theorems culminated in paper [12], a comprehensive investigation of the conditions to be imposed on $G(x, \lambda)$, regarded as a function of λ, so that one may determine definite oscillation theorems for the solutions of the differential equation

$$\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + G(x, \lambda)y = 0, \qquad 0 \leqq x \leqq 1. \tag{3}$$

In this paper he established very general theorems for the existence of a unique solution of (3) vanishing at the end points of the interval and possessing a prescribed number of zeros, and also for the existence of two, and exactly two solutions of this nature. In addition, there were obtained general results on the oscillation of the proper functions of the boundary problem consisting of the differential equation (3) and arbitrary self-adjoint two-point boundary conditions.

Several years later, in paper [23], Richardson returned to the study of boundary problems of the form (1). The treatment therein given of the Jacobi condition for certain variational problems involving a finite number of isoperimetric conditions provided significant addenda to the results of [5], although the prime purpose of the paper was the study of properties enjoyed by the individual proper functions as extrema for variational problems involving an infinite number of isoperimetric conditions. The discussion of this latter problem was highly formal, and, in the words of the author, "omitting

[5] Loc. cit.

[6] See footnote 2, 6. Mitt.

[7] *Ein zweiparametriges Oscillationstheorem*, Nachr. Ges. Wiss. Göttingen (1910) pp. 586–594; *Dreiparametrige Randwertaufgaben*, ibid. pp. 563–585.

much in the way of justification of infinite processes." This deficiency was due to the author's attempt to carry over the machinery that sufficed for ordinary problems of the calculus of variations involving a finite number of isoperimetric conditions, instead of considering the problem in its proper setting as one in Hilbert space theory.

Paper [**11**] was the only one published by Richardson on partial differential equations. Using the classical method of approximating difference equations, he established in particular the existence of an infinite number of proper values for the first boundary problem over a square domain for a special type of self-adjoint linear second order partial differential equation of elliptic type. The paper also contained some partial results on the application of the method to a related boundary problem involving hyperbolic equations. A few years later the full value of the method of approximating difference equations was demonstrated by Courant and some of his associates.

The remainder of Richardson's research papers may be considered as secondary or auxiliary to his main interests. Paper [**4**], written jointly with W. A. Hurwitz at Göttingen in 1909, had points of contact with Richardson's paper [**5**]. In papers [**17**], [**19**], and [**20**] he presented an analytic discussion of certain algebraic properties of pairs of bilinear forms, and a related treatment of relative extrema for pairs of quadratic and hermitean forms.

In paper [**14**] Richardson gave a comprehensive and elegant review of Bôcher's *Leçons sur les méthodes de Sturm*. Although Richardson published no book on the subject, his mimeographed lecture notes on linear differential equations of the second order, listed as [**24**] of the bibliography, presented a well-formed introduction to the Sturmian theory and the significance of variational principles for the study of self-adjoint boundary problems.

Papers [**11**] and [**12**] were mainly the product of sabbatic leave spent at Harvard University during 1916–17. Scientific research was brought to an end in 1928, not only by demands made upon his time as secretary of the American Mathematical Society, but also by mounting obligations at Brown University, to which we shall presently refer.

The American Mathematical Society and its third secretary. The first period of the Society's history may be considered as terminating at about the time of its thirty-second birthday in December, 1920, when Cole withdrew from service as secretary. There were then 770 members; the total receipts for 1920 were $7167.24 and as yet there were no special funds, except that for Life Memberships, $4595.66.

By the time of Richardson's retirement from the secretaryship twenty years later enormous development of the Society's organization, resources, and activities had taken place. Some suggestions as to these may now be noted.

The membership had increased to 2314, and the total cash assets in December 1940 amounted to $189,226.65 in addition to productive assets from the sale of earlier publications. Besides the Life Membership fund, there were six other special funds (Bôcher, Frank Nelson Cole, Eliakim Hastings Moore, Marion Reilly, Ernest William Brown, Mathematical Reviews) and an Endowment Fund of $71,000. Reorganization of the Society's business with a New York office under the general direction of the secretary, incorporation, appointment of trustees to deal with financial matters, and life memberships based on actuarial principles, had been arranged long before. Greatly expanded activities including regular meetings of the Society in the East and Midwest, and on the Pacific Coast, necessitated the addition of four associate secretaries. In 1921–27 began Invited Addresses, Gibbs' Lectures, Visiting Lecturers, Reciprocity Memberships with Great Britain and Germany, and the Revolving Book Fund arrangement with the National Research Council. An elaborate Semi-Centennial Celebration was held in 1938. The Society collaborated in editorial representation on the Annals of Mathematics and the Duke Mathematical Journal, acquired editorial control of the American Journal of Mathematics, and established the new journal Mathematical Reviews (in part made possible by funds granted by the Carnegie Corporation of New York, the Rockefeller Foundation, and the American Philosophical Society). Successful campaigns for an Endowment Fund, for Institutional and Contributing Memberships, for Patrons, and for special grants had made possible large increases in publication in spite of mounting costs.

For the attainment of the Society's 1940 status—financial (how remarkable the achievement of Coolidge, Veblen, and Ingraham!) and scientific—very many people were responsible. By my attendance at Council meetings throughout the whole of Richardson's term of office as secretary, and through constant personal contacts, I became convinced that his contributions and influences towards this status were indeed great. He had a thorough grasp of every topic which he presented, in excellent form, for the Council's consideration; he was endowed with infinite capacity for taking pains, and was always forward-looking, receptive to new ideas, while constantly mindful of the Society's finances. In personal relations he was remarkably kindly and cooperative, and inspired confidence and affection. No American mathematician was more widely known among his colleagues and the careers of scores of them were notably promoted by his time-consuming activities in their behalf. And such help was being freely rendered up to the time of his death.

Richardson was deeply appreciative of the Council's action not only in presenting to him a beautifully hand-illuminated copy of the resolution of appreciation adopted by the Council upon his retirement as secretary, together with a silver coffee set, and for Mrs. Richardson a handbag, but also in dedicating to him the 1941 volume of this Bulletin.

Achievements at Brown. Richardson arrived at Brown shortly after a "Graduate Department," with its own Dean, had been organized, and when only one member of the mathematics department was offering advanced courses. The number of such courses had been appreciably increased when he became chairman of the department. Soon he was a member of more than one important administrative committee including the Graduate Council. In 1926 he was appointed the successor to the Dean of the Graduate Department which became the Graduate School in the following year. He held this position until 1948, when he became listed among "Officers Emeriti." Both within and without the University the School became recognized as a really vitalizing intellectual force. This development, for which the Dean was almost wholly responsible, led to Brown's election in 1933 as an institutional member of the Association of American Universities.

Even though by 1928 Richardson's duties as dean and secretary, in addition to demands upon him as chairman of a department, had forced him to give up his own scientific research, he had through the years been laying sure foundations for the establishment of a strong department, for attracting not only candidates for the doctorate in mathematical analysis, but also those seeking post-doctoral inspiration. From 1929 until his retirement as chairman in 1942, 28 Ph.D. degrees in mathematics had been awarded at Brown.

In 1938 (after more than a decade of earlier consideration) when the American Mathematical Society was active in the establishment of Mathematical Reviews, Richardson was mainly instrumental in arranging not only that Brown should provide housing for the project, but also that its chief editor should be brought from Europe as a professor, adding strength to his department.

As early as 1930 the Council of the American Mathematical Society had given earnest consideration to the important problem of providing a means of publication for research memoirs in the field of applied mathematics. The seed thus sown in the secretary's thought, fertilized and greatly expanded by needs connected with World War II, led to a splendidly conceived pioneer undertaking. The following descriptive extract is taken from the Brown University Faculty Meeting records for October, 1949: "In the summer of 1941, through his [Richardson's] efforts and with financial aid from foundations and government, there was established at Brown a Program of Advanced Instruction and Research in Applied Mechanics. Able men, including many who had come to this country from abroad, were brought here to teach, and substantial numbers of students (at one time exceeding a hundred) were recruited. Under war conditions the original educational aim of the Program, to train a succession of able young doctors in applied mathematics, especially in the various fields of mechanics, was of necessity modified. The demand by government and industry for the services of young men trained even for the short space of a year provided sufficient evidence of the usefulness of this Program for war purposes. At the same time substantial amounts of investigation connected with the war were carried on by the staff. To supplement the Program, Brown University in 1943 founded the *Quarterly of Applied Mathematics*, a journal which renders international service in its field. In 1946 the original educational aim of this Program was restored and the Program reorganized under the name Graduate Division of Applied Mathematics." During 1943–46 the dean was also a member of the applied mathematics panel of the National Defense Research Committee.

Richardson was acting vice-president of Brown during the first semester of 1928–29, and, at the conclusion of his 41 years of service in the University, the degree of LL.D. was conferred upon him. A similar degree had been granted by Lehigh University in 1941, and in 1931, he received a D.C.L. from his alma mater, Acadia University. In 1919 he was elected a vice-president of the Mathematical Association of America, and in 1932–34 became a member of the Council of the American Academy of Arts and Sciences, and again in 1945–49 when he was a vice-president. He was a lecturer at the University of Chicago in 1918 and at the University of California, Berkeley, in 1920. Having become a naturalized citizen of the United States in May 1932, he was eligible for membership in the Division of Physical Sciences of the National Research Council, 1933–36. By election of the policy holders, he served as a trustee of the Teachers Insurance and Annuity Association, 1926–30, and 1931–35. Long a member of the First Baptist Church in Providence, from time to time he served on some of its most important committees.

The range of our friend's interests, apart from his recreations of golf and fishing, have thus been indicated, and his record of enduring worthy achievements is varied and rich. In connection with every project that he undertook he found no rest until the goal sought was won, and ever was his motto: "Whatsoever thy hand findeth to do, do it with thy might." He had insight and remarkable organizing abilities, associated with exceptional mental powers. His complete honesty of character, natural wisdom, and selfless devotion to high educational ideals, commanded the admiration of his associates. He was endowed with a spirit of unaffected friendliness, the desire to play a part in helping and encouraging others to achieve the best of which they were capable, and as a result he was blest with a host of friends. There was never-ending hospitality at his home. Indirectly Mrs. Richardson contributed in no small measure to the happy solution of many of her distinguished husband's problems.

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[8] It may be noted that this volume 9 of the *Transactions* also contains the doctoral dissertations of G. D. Birkhoff, A. Dresden, F. L. Griffin and R. L. Moore.

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[9] In the preceding Memorial the comments on Richardson's scientific research were made by specialists in the fields, namely: Professor T. H. Hildebrandt on nos. [1] and [3]; Professor W. T. Reid on nos. [4]–[9], [11], [12], [14], [19], [20], [23], [24].