in each of these fields did the realization dawn upon me that essentially the same inequalities and theorems appeared again and again, and that I was simply proving the same theorems a wasteful number of times." He insists also that the theorems he proves be "operationally meaningful" in order that they can be treated as hypotheses about empirical data that could conceivably be refuted.

The unified treatment of the theory of consumer's behaviour (Chap. V) is a convincing sample of success with synthesis in operationally meaningful terms. It is the general notion that a consumer has an ordinal preference field $U=F[\phi(X)]$ where ϕ is some one cardinal index of utility such that preferences between combinations of goods X and Y are in accordance with $\phi(X)>\phi(Y)$, $\phi(X)=\phi(Y)$, or $\phi(X)<\phi(Y)$, where $\phi(X)>\phi(Y)$ implies that X is preferred to Y. (It is assumed that ϕ is continuous and differentiable and that $F'(\phi)>0$.) Now if total income is I and if the price of good i is p_i, the main problem is to derive the demand function $X_i(p_1, p_2, \cdots, p_n, I)$ for the quantity x_i of good i that the consumer would purchase, subject to the restriction $I=\sum p_ix_i$, if he wished to maximize U. It is shown that all restrictions on the demand function can be derived from the single condition that the form $K_{ij}=\delta x_i/\delta p_j+x_j\delta x_i/\delta I$ be symmetric and negative semi-definite. A constructive proof is sketched also to show that when this condition is satisfied there exists a $\phi(X)$ that satisfies the properties of a preference field. Professor Samuelson concludes that: "Despite its lofty beginnings, the pure theory of consumer's behavior, when its empirical meaning is finally distilled from it, turns out to be one simple hypothesis on price and quantity behavior." Here, then, is a concise hypothesis that provides the basis for an imposing theoretical economic structure that should soon be put to trial by some clever experimental economist.

The pure mathematician who wishes to sample an important segment of economic theory, written competently in his favorite language, will find the *Foundations* pleasant reading. He may well also be stimulated, especially by the discussion of dynamic economic theory in Part II, to extend some of the results reported by Professor Samuelson and this would constitute a well-deserved widening of the sphere of influence of this splendid book.

MERRILL M. FLOOD

Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability. Ed. by J. Neyman. Berkeley, University of California Press, 1947. 12+447 pp. $7.50.

The thirty papers which are collected in the volume cover such a wide range of subjects that a comprehensive review is impossible. The topics range from *Philosophical foundations of probability* (Reichenbach) to *Biological association of insects* (Holloway) and *Wheat-bunt field trials* (Baker and Briggs). Of particular interest to pure mathematicians are survey papers by Doob, *Time series and harmonic analysis*, and by Feller, *On the theory of stochastic processes, with particular reference to applications.*

Although the preponderance of papers is devoted to statistics (general, descriptive, and mathematical) mention should be made of a paper on *Statistical mechanics and its applications to physics* (Lenzen) and on *Statistical study of the galactic star system* (Trumpler).

There is even a paper on *The place of statistics in the university* (Hotelling) followed by a discussion by five other authors.

In spite of the somewhat chaotic arrangement and an overwhelming battery of topics this volume is a tribute to the great vitality of statistics and statistical methods. It should prove a valuable addition to the rapidly growing statistical literature.

M. KAC

An essay toward a unified theory of special functions. By C. Truesdell. (Annals of Mathematics Studies, no. 18.) Princeton University Press, 1948. 4+182 pp. $3.00.

It is very difficult to draw the line between mathematical physics and applied mathematics but this book shows that there does exist a definite and important difference between them. In mathematical physics many special functions such as the Legendre, Hermite, or Laguerre polynomials, Bessel or hypergeometric functions are used as tools to solve particular problems. As a consequence many properties of these functions and connections between them have been established. The author, as an applied mathematician, has posed the question of finding a unified approach to these different special functions so that from it most of the known properties could be found directly. The monograph under review gives an answer to this question.

The author found that many of the special functions, not only those previously mentioned but also such as the generalized Riemann zeta function, the incomplete gamma function, or the Poisson-Charlier polynomials, can be transformed into solutions of the equation

$$\frac{\partial F(z, \alpha)}{\partial z} = F(z, \alpha + 1). \tag{1}$$

A study of this equation shows that by simple techniques any solution