THE APRIL MEETING IN BERKELEY

The four hundred fifty-ninth meeting of the American Mathematical Society was held at the University of California, Berkeley, California, on Friday and Saturday, April 28–29, 1950. Over 100 persons attended, including the following 97 members of the society:

F. C. Andrews, Richard Arens, W. G. Bade, E. M. Beesley, Richard Bellman, H. F. Bohnenblust, F. A. Butter, E. H. Clarke, Paolo Comba, K. L. Cooke, F. G. Cresse, P. H. Daus, D. B. Dekker, J. E. Denby-Wilks, C. R. De Prima, Douglas Derry, R. C. H. Dieckmann, S. P. Diliberto, R. P. Dilworth, Worthie Doyle, Roy Dubisch, L. K. Durst, Arthur Erdélyi, E. A. Fay, J. M. G. Fell, Werner Fenchel, Harley Flanders, G. E. Forsythe, A. L. Foster, Marianne R. Freundlich, R. A. Fuchs, Bent Fuglede, P. R. Garabedian, M. A. Girshick, G. E. Gourrich, J. W. Green, K. D. C. Haley, G. J. Haltiner, J. G. Herriot, M. R. Hestenes, J. L. Hodges, D. W. Hullinghorst, R. C. James, S. L. Jamison, Irving Kaplansky, Samuel Karlin, William Karush, J. L. Kelley, S. H. Lachenbruch, R. M. Lakness, Cornelius Lanczos, D. H. Lehmer, Hans Lewy, Charles Loewner, A. T. Lonseth, J. E. McLaughlin, Rhoda Manning, W. A. Mersman, A. B. Mewborn, E. D. Miller, C. B. Morrey, F. R. Morris, A. P. Morse, Ivan Niven, Andrewa R. Noble, Fritz Oberhettinger, T. K. Pan, R. S. Phillips, George Pólya, F. P. Pu, Edris P. Rahn, W. C. Randels, Hugo Ribeiro, Julia B. Robinson, R. M. Robinson, J. B. Rosser, Herman Rubin, H. M. Schaerf, Abraham Seidenberg, Max Shiffman, W. H. Simons, W. M. Stone, Irving Sussman, Otto Szász, Gabor Szegö, Alfred Tarski, F. B. Thompson, F. G. Tricomi, A. W. Tucker, R. L. Vaught, R. K. Wakerling, S. S. Walters, W. R. Wasow, P. A. White, A. L. Whiteman, A. R. Williams, František Wolf.

The meeting opened Friday afternoon with an invited address by Professor Richard Arens of the University of California, Los Angeles, on *Representation of rings and functionals in spectral theory.* Professor J. L. Kelley presided. Following the invited address there was a session for contributed papers in analysis and applied mathematics, Professor C. B. Morrey presiding. A second invited address, *On the differential geometry of closed space curves* by Professor Werner Fenchel of the Technical University of Denmark and the University of Southern California was delivered on Saturday morning, Professor Hans Lewy presiding. On Saturday afternoon there were three sections: algebra and number theory, geometry and topology, and late papers. These were presided over by Professor Ivan Niven, Abraham Seidenberg, and George Pólya, respectively.

Abstracts of the papers presented at the meeting follow. Abstracts whose titles are followed by the letter "*t*" were presented by title. Paper 386 was presented by Mr. Fell, paper 388 by Mr. Jackson and paper 390 by Professor Karlin.

Algebra and Theory of Numbers

380. R. P. Dilworth: *Infinite distributivity in function lattices.*

A lattice L is said to be *infinitely distributive* if whenever $\cup S$ exists for a subset S of L, $\cup(a\cap S)$ also exists and $a\cap\cup(S)=\cup(a\cap S)$. A distributive lattice need not be infinitely distributive. Nevertheless, as first noticed by Tarski and von Neumann, every complemented distributive lattice (Boolean algebra) is infinitely distributive. In this note it is proved that the lattice of all bounded continuous real functions on a topological space is infinitely distributive. (Received March 15, 1950.)

381. Worthie Doyle: *An arithmetical theorem for partially ordered sets.*

Let S be a partially ordered set satisfying the ascending chain condition. An element $q\in S$ is *irreducible if* $q=$ g.l.b. (X) implies $q\in X$ for all finite subsets X of S for which g.l.b. (X) exists. The ascending chain condition implies that every element of S is representable as a meet of irreducibles. A subset A of S is an *ideal* if (1) $x\in A$ and $y\geqq x$ imply $y\in A$, (2) $x=$ g.l.b. (X) where X is a finite subset of A implies $x\in A$. The ideals form a complete lattice L. If $a\in S$, the $(a)=(x\mid x\geqq a)$ is a *principal* ideal. S is *upper semimodular* if B covers (a), $C\supseteq(a)$, and $C\not\supseteq B$ imply $B\cup C$ covers C for all $a\in S$ and B, $C\in L$. Let U_a denote the union of ideals covering (a) and let L_a denote the quotient lattice $U_a/(a)$. The following theorem is proved: If S is a partially ordered set satisfying the ascending chain condition, then every element of S is uniquely representable as a reduced meet of irreducibles if and only if S is upper semimodular and each L_a is a Boolean algebra. This result extends to partially ordered sets a known theorem for lattices. (Received March 15, 1950.)

382. Harley Flanders: *Algebraic field extensions.*

The purpose of this paper is to prove systematically certain basic theorems of the theory of algebraic extensions of algebraic fields. These theorems include, for example, the transitivity of norm, discriminant, and separability, the existence of a maximal separable subfield, and the equivalence of p-independence to separability. The proofs depend strongly on the use of the regular representation, trace, and discriminant and almost completely avoid the use of conjugate elements and decomposition into towers of simply generated extensions. (Received March 15, 1950.)

383. J. E. McLaughlin: *Simple quotient lattices in relatively complemented lattices.*

In a simple complemented modular lattice every quotient lattice is simple. While this is not true for arbitrary relatively complemented lattices, the following theorem gives a large class of simple quotient lattices: Let L be a simple relatively complemented lattice of dimension $n>1$. If p is any point and k is any integer such that $k<[(n+1)/2]$, let C_p^k be the class of points projective to p is not more than $2k$ transposes, and let $a_p^k=\cup(C_p^k)$. Then a_p^k/z is a simple lattice of dimension at least $2k+1$. (Received March 16, 1950.)

384. R. M. Robinson: *Arithmetical definitions in the ring of integers.*

It is shown that the set of natural numbers cannot be defined arithmetically in the ring of integers by a formula containing just one quantifier. There is however a suitable formula involving two quantifiers. (Received March 15, 1950.)

385. J. B. Rosser: *Real zeros of real Dirichlet L-series.*

A long standing conjecture is that if X is real then $L(s, X)$ has no positive real zeros. By a certain computational procedure the conjecture has been verified for each individual $k \leqq 67$. In the present paper, this computational procedure was tried for each $k \leqq 227$ and failed for $k=163$. An improved computational procedure is given in the present paper, but even by the new procedure the case $k=163$ presents difficulty. Finally, a new formula for $L(s, X)$ is presented which makes it possible to treat many values of k simultaneously. By means of the new formula, the case $k=163$ is readily handled. (Received March 8, 1950.)

386. Alfred Tarski and J. M. G. Fell: *On algebras whose factor algebras are Boolean.*

Consider algebras $\mathfrak{A}=\langle A, +\rangle$ having one binary operation with zero element 0. Two subalgebras B and C are called complementary factors if $B\times C=A$, where $\times$ is the operation of direct multiplication. A subalgebra B is a factor of $\mathfrak{A}$ if for some subalgebra C, $B\times C=A$. The factor algebra of $\mathfrak{A}$ is called Boolean if the algebra of all factors of $\mathfrak{A}$, with the operation $\times$, is a disjunctive Boolean algebra. (For the notions involved here see Tarski, *Cardinal algebras.*) The following three conditions are shown to be equivalent: (i) the factor algebra of $\mathfrak{A}$ is Boolean, (ii) every factor for $\mathfrak{A}$ has precisely one complementary factor, (iii) if B and C are any complementary factors, B is not homomorphic to any subalgebra of the center of C containing elements different from 0. The results extend to algebras with many operations. Further, if the factor algebra of $\mathfrak{A}$ is Boolean, the same applies to any algebra obtained from $\mathfrak{A}$ by adding new operations (result of Jónsson and Tarski). The results obtained show directly that the factor algebras of various special classes of algebras—namely, cyclic groups, centerless algebras, rings with unit, and the so-called zero-equivalent algebras—are Boolean. The class of zero-equivalent algebras includes groups identical with their commutator subgroups and algebras with an infinity element ($x+\infty=\infty+x=\infty$). (Received March 22, 1950.)

387. A. L. Whiteman: *Cyclotomy and Jacobstahl sums.*

Let g be a fixed primitive root of a prime p. Let e be a divisor of $p-1$ and write $p-1=ef$. The Jacobstahl sum $\phi_e(n)$ is defined by $\sum_{h=1}^{p-1}(h/p)((h^e+n)/p)$, where (h/p) denotes the quadratic character of h with respect to p. Of the theorems established in this paper, the following is typical. If e is odd, then $\sum_{k=1}^{e-1}\phi_e(g^k)\phi_e(g^{j+k})=e^2p-e(p-1)$ or $-e(p-1)$ according as $j=0$ or $j\neq 0$, $0\leqq j\leqq e-1$. Among the applications is the result: if $e=4$ and f is even, then $p=(\phi_4(1)/4)^2+2(\phi_4(g)/4)^2$. The paper also contains a detailed study of the connection between Jacobstahl sums and the theory of cyclotomy. (Received March 7, 1950.)

Analysis

388. E. F. Beckenbach and L. K. Jackson: *A generalization of subharmonic functions.*

The idea of a subharmonic function is generalized as follows, in analogy with E. F. Beckenbach, *Generalized convex functions*, Bull. Amer. Math. Soc. vol. 43 (1937) pp. 363–371. Let there be given a family of functions $\{F(x, y)\}$ which are (a) continuous in a domain D, (b) uniquely determined by continuous boundary values on simple closed contours Γ lying in D, (c) such that, if $F_1(x, y) \leqq F_2(x, y)$ on a simple closed contour Γ bounding a subdomain G, then $F_1(x, y) \leqq F_2(x, y)$ in G, the strict inequality holding throughout G if it holds at any point of Γ, and (d) such that if $F_1(x, y) = F_2(x, y) + K$ on Γ then $F_1(x, y) \leqq F_2(x, y) + K$ throughout G for $K \geqq 0$. Then a function $f(x, y)$ may be said to be a sub-$\{F\}$ function provided (1) $f(x, y)$ is continuous in D, and (2) if $f(x, y) \leqq F(x, y)$ on Γ, then $f(x, y) \leqq F(x, y)$ in G. Results of Littlewood and others concerning subharmonic functions are generalized to sub-$\{F\}$ functions. Also, results of the following type are obtained, if $\{F(x, y)\}$ consists of the solutions of $L(F) \equiv \Delta F + a(x, y)F_x + b(x, y)F_y + c(x, y)F + e(x, y) = 0$, $c(x, y) \leqq 0$ in D, then a function $f(x, y)$ is sub-$\{F\}$ if and only if $L(f) \geqq 0$. (Received February 15, 1950.)

389t. Richard Bellman: *On the asymptotic behavior of solutions of linear differential equations.*

A method is given for obtaining the asymptotic behavior of the solutions of $u'' - (1 + a(t))u = 0$ under the assumptions that $a(t) \to 0$ as $t \to \infty$ and that there exists an n for which $\int_0^\infty |a(t)|^n dt < \infty$. The case $1 \leqq n \leqq 2$ has recently been treated by Hartman, Trans. Amer. Math. Soc. (1948). The method is a combination of a device of Poincaré and the standard technique of Poincaré-Liapounoff stability theory. The general nth order equation $u^{(n)} + a_1(t)u^{(n-1)} + \cdots + a_n(t)u = 0$ is amenable to the same treatment and results similar to those of Levinson, Duke Math. J. (1948), may be derived under weaker assumptions. (Received March 10, 1950.)

390. H. F. Bohnenblust and Samuel Karlin: *On positive operators.*

Let E be a Banach space in which a closed cone K (x, y in K and λ, $\mu \geqq 0$ imply $\lambda x + \mu y \in K$; x and $-x$ in K imply $x = 0$) is given. It is assumed that K spans E. A bounded linear operator T is said to be positive ($T > 0$) if $T(K) \subset K$. Let $\sigma(T)$ denote the spectrum of T and let $\lambda_0 = \sup |\lambda|$ for λ in $\sigma(T)$. By extending results on power series with non-negative coefficients to abstract-valued functions, the following result are established: (1) λ_0 is in the spectrum, (2) If λ_0 is a pole of $R(\lambda, T)$ of order k then $n^{-k}\Gamma(k+1)\sum_1^n \lambda_0^{-i} T^i$ tends to $(\lambda_0 I - T)^{k-1}E$ uniformly, where E denotes a projection on the manifold of characteristic vectors of λ_0, (3) If $n^{-1}\sum_1^n \lambda_0^{-i} T^i$ tends to E uniformly, then λ_0 is a pole of order 1 of $R(\lambda, T)$, (4) the value λ_0 can be characterized as the sup of all λ ($\lambda \geqq 0$) for which there exists an x in K with $\lambda x \leqq Tx$. This holds for completely continuous positive operators and also for any strictly positive operators, that is, those which map K into the interior of K. (Received March 17, 1950.)

391. J. E. Denby-Wilkes: *Structure of commutative normed rings.*

Let A be a complex normed ring and S_u its unit sphere about the unit element u. It is shown that the origin θ is a vertex of S_u in the sense that there exists a total family of functionals which are all supporting planes of S_u at θ. In case A is finite-dimensional it is also shown that the cone formed by the radical R of A has only the point θ in common with S_u. If A is finite-dimensional and has a star operation, under suitable restrictions a representation of A as a direct sum of three rings $A = A_1 \oplus A_2 \oplus A_3$ is obtained in which the product in A splits up into three parts: in A_1 scalar, in A_2 convolution (or Cauchy), in A_3 void (x_3, $y_3 \in A_3 \rightarrow x_3 y_3 = \theta$). This decomposition yields in particular for A a "natural" norm with a polyhedral unit sphere ($\|x\|=1\rightarrow\exists\epsilon$, $|\epsilon|=1$ such that $\|(x+\epsilon u)/2\|=1$); it also gives a product in A^* making A^* a normed ring and such that $A^{**}=A$. (Received March 22, 1950.)

392. Bent Fuglede: *A commutativity theorem for normal operators in Hilbert space.*

The following theorem on linear operators in Hilbert space is established: "If a bounded operator B commutes with a bounded, normal operator N: $BN=NB$, then B commutes also with the adjoint operator N^* of N: BN^*NB." Therefore, it follows that B commutes with all projectors in the canonical spectral representation of N. The theorem holds also if N is nonbounded. It is not known whether it holds for a bounded, normal operator N and a closed, but nonbounded, operator B. If B and N are both nonbounded the concept of commutativity is not generally defined. As a conversion of the above theorem we find that a *bounded* operator N must necessarily be normal if N and N^* commute with exactly the same bounded operators B. This follows at once by the choice $B=N$. For nonbounded, even closed, operators N this conversion does not hold in general. In the author's note in Proc. Nat. Acad. Sci. U.S.A. vol. 36 where the above theorem is proved is also given an example illustrating the last remark by defining a closed operator which does not commute with any bounded operator at all. This operator is, of course, not normal. (Received March 20, 1950.)

393*t*. Edwin Hewitt and H. S. Zuckerman: *A group-theoretic method in approximation theory.*

The following general theorem is proved. Let G be an Abelian group, and let Σ be any group of characters of G. Let x be any character of Σ, let $\sigma_1, \cdots, \sigma_m$ be any finite subset of Σ, and let ϵ be any positive real number. Then there exists an element $g\in G$ such that $|x(\sigma_i)-\sigma_i(g)|<\epsilon$ $(i=1, 2, \cdots, m)$. This result is used to give new proofs of Kronecker's approximation theorems, and to prove a number of new approximation theorems, of which the following is typical. Let $\omega_1, \cdots, \omega_m$ be real-valued functions of bounded variation on $(-\infty, +\infty)$ which are rationally independent. Let $\alpha_1, \cdots, \alpha_m$ be any real numbers and let ϵ be any positive real number. Then there exists a real-valued continuous function f on $(-\infty, +\infty)$ such that $\lim_{x\to\pm\infty} f(x)=0$ and such that $|\alpha_i-\int_{-\infty}^{\infty} f(x)d\omega_i(x)|<\epsilon$ $(i=1, 2, \cdots, m)$. (Received January 31, 1950.)

394. R. S. Phillips: *On Fourier-Stieltjes integrals.*

A function $\phi(\tau)$ on $(-\infty, \infty)$ satisfies condition (A) if for some constant C, $|\sum a_n\phi(\tau_n)|\leqq C\|\sum a_n \exp(i\tau_n s)\|$ for all finite sets (τ_n) and (a_n) where $\|f(s)\| = \text{Lub } |f(s)|$. If $\phi(\tau)$ is measurable and satisfies (A), then there exists a unique function of bounded variation $y(s)$ continuous on the right with $y(-\infty)=0$ such that $\phi(\tau)=\int_{-\infty}^{\infty} \exp(i\tau s)dy(s)$ a.e. One obtains the Bochner, Riesz result for positive definite functions from this theorem by means of the lemma: Any non-negative trigonometric polynomial can be expressed as the sum of absolute value squares of periodic trigonometric polynomials. As a corollary one shows that a function satisfying the positive definiteness condition only for finite commensurate sets of τ's will be positive definite. An analogous result is obtained for condition (A) by means of a decomposition of the space of almost periodic functions into the direct product of a nondenumerable set of limited periodic spaces. The paper also contains generalized

Fourier-Stieltjes integral representation theorems for arbitrary complex valued functions satisfying (A) and for measurable Banach-space valued functions satisfying (A) and a weak compactness condition. (Received March 3, 1950.)

395*t*. R. S. Phillips: *On one-parameter semi-groups of linear transformations.*

Let X be a Banach space and let $E(X)$ be the Banach algebra of bounded linear transformations on X to itself. A one-parameter semi-group of operators in $E(X)$ is defined to be a function $T(\xi)$ on $(0, \infty)$ to $E(X)$ such that $T(\xi_1+\xi_2)x=T(\xi_1)T(\xi_2)x$ for $0<\xi_1, \xi_2<\infty$, and $x\in X$. $T(\xi)$ is said to be weakly measurable if the numerically valued function $f[T(\xi)x]$ is measurable for each $x\in X$ and $f\in\overline{X}$. $T(\xi)$ is said to be strongly measurable if, for each x, $T(\xi)x$ is the limit almost everywhere of a sequence of step functions. It is shown that if $T(\xi)$ is strongly measurable, then $\|T(\xi)\|$ is bounded in each interval $[\delta, 1/\delta]$ where $\delta>0$. An example is given of a one-parameter semi-group of transformations $T(\xi)$ which is weakly measurable but such that $\|T(\xi)\|$ is unbounded in every finite subinterval of $(0, \infty)$. (Received January 30, 1950.)

396. Max Shiffman: *Minimal surfaces, analytic functions, and symmetrization.*

In connection with boundary curves bounding several different minimal surfaces, the following problem was posed by T. Rado in the session on *Analysis in the large* of the Princeton Bicentennial Conferences on the Problems of Mathematics: show that a minimal surface bounded by two parallel circles, with the line of centers perpendicular to the planes of the circles, is a surface of revolution. This, and a more general result, is established by showing that a certain angle defined at each point of the minimal surface is a harmonic function on the surface, and that the curvature at each point of a family of plane curves on the minimal surface satisfies a certain partial differential equation of elliptic type. A discussion of the second eigenvalue of an associated elliptic equation yields the result. These theorems are generalizations and analogues of theorems in the theory of analytic functions. In fact, minimal surface theory can be put in the form analogous to the Cauchy-Riemann equations. The particular case of Rado can also be done by an extension of the notion of symmetrization, but the more general case treated leads to an open geometric question in this domain. (Received March 16, 1950.)

397. Otto Szász: *Gibbs phenomenon for Hausdorff means.*

We discuss here the Gibbs phenomenon for the Hausdorff means of Fourier series. These means for a sequence s_n are defined by $h_n=\sum_0^n C_{n,\nu}s_\nu\int_0^1 r^\nu(1-r)^{n-\nu}d\psi(r)$, where $\psi(r)$ is of bounded variation. The main result is: The series $\sum\sin nt/n=(\pi-t)/2$ presents a Gibbs phenomenon at $t=0$ if and only if $\max_{\tau>0}\int_0^1\{1-\psi(r)\}(\sin r\tau/r)dr>\pi/2$. It includes known results for Cesàro and for Euler means. Applied to Hölder means of order $p>0$, it is found that there is a constant γ so that Hölder means present a Gibbs phenomenon for $p<\gamma$, but not for $p\geqq\gamma$. Numerical calculation yields $\gamma=0.58\cdots$. For Cesàro means the corresponding constant is, according to Gronwall, $0.439\cdots$. (Received March 8, 1950.)

398. F. G. Tricomi: *On the finite Hilbert transformation.*

The finite Hilbert transformation $\mathfrak{T}[\phi] \equiv \pi^{-1}\int_{-1}^{1}(y-x)^{-1}\phi(y)dy$ (Cauchy's principal value of the integral) furnishes the necessary basis for the rigorous study of the singular integral equations of *Carleman's type*: $a(x)\phi(x)-\lambda\mathfrak{T}[\phi]=f(x)$. Some, but not all, of the properties of the $\mathfrak{T}$-transformation can be deduced from those of the corresponding *infinite* transformation treated carefully in Titchmarsh's *Theory of Fourier integrals*. In particular to these properties is added a kind of *convolution theorem* which asserts that $\mathfrak{T}\{\phi_1\cdot\mathfrak{T}[\phi_2]+\phi_2\cdot\mathfrak{T}[\phi_1]\}=\mathfrak{T}[\phi_1]\cdot\mathfrak{T}[\phi_2]-\phi_1\cdot\phi_2$ provided that the functions ϕ_1, ϕ_2 belong to suitable L^p classes. Precise conditions are also given for the *inversion* of the transformation $\mathfrak{T}$, that is, for the solution of the *airfoil equation* $\mathfrak{T}[\phi]=f$. (Received March 1, 1950.)

399. S. S. Walters: *Locally bounded linear topological spaces of analytic functions.*

A method for norming the conjugate space of a locally bounded linear topological space (LBLTS) is presented. Let $\Delta=E_z[|z|<1, z$ a complex number$]$, and $\mathfrak{A}$ be all complex valued f analytic on Δ. Then a subspace $\mathfrak{B}$ of $\mathfrak{A}$ is said to be of type I provided $\mathfrak{B}$ is a LBLTS$\ni\gamma_{0,z}\in\mathfrak{B}^*$ (conjugate space of $\mathfrak{B}$) for all $z\in\Delta$ and provided $\exists$ a real-valued function $N(r)\ni\|\gamma_{0,z}\|\leqq N(r)$ for $|z|\leqq r<1$. Here one defines $\gamma_{n,z}(f)=f^{(n)}(z)/n!$, $n=0, 1, \cdots, f\in\mathfrak{A}$, $z\in\Delta$. Then, in a space $\mathfrak{B}$ of type I, weak convergence is meaningful since $\mathfrak{B}^*$ distinguishes elements in $\mathfrak{B}$. It is shown that $f_n\to^w f$ (f_n converges weakly to f) implies $f_n(z)\to f(z)$ uniformly on compact subsets of Δ. For arbitrary $f\in\mathfrak{A}$ let $T_rf: T_rf(z)=f(rz)$, $0\leqq r<1$, $z\in\Delta$, and let $U_n: U_n(z)=z^n$, $n=0, 1, \cdots$. Then $\mathfrak{B}$, a space of type I, is of type II provided: (a) $\{U_n\}$ is a bounded subset of $\mathfrak{B}$, (b) $\sum_{n=0}^{\infty}\gamma_{n,0}(f)W^nU_n$ converges in $\mathfrak{B}$ for all $f\in\mathfrak{B}$, $w\in\Delta$, (c) $T_rf\to^w f$ as $r\to 1-$ for all $f\in\mathfrak{B}$, and (d) $\mathfrak{B}$ is complete. Let $\mathfrak{D}$ denote all $G\in\mathfrak{A}\ni\lim_{r=1}\int_0^{2\pi}f(\rho e^{i\theta})G((r/\rho)e^{-i\theta})d\theta$, $r<\rho<1$, exists for all $f\in\mathfrak{B}$. It is then shown that $\mathfrak{B}^*$ and $\mathfrak{D}$ are algebraically isomorphic, and if $\gamma\to G_\gamma$ is the isomorphism, then $\gamma(f)=\lim_{r=1}\int_0^{2\pi}f(\rho e^{i\theta})G_\gamma((r/\rho)e^{-i\theta})d\theta$, $r<p<1$, for every $f\in\mathfrak{B}$. An example of a space of type II is H^p, $0<p<1$, where the topology of H^p is that determined by the metric $d(f,g)=\sup_{0\leqq r<1}\int_0^{2\pi}|f(re^{i\theta})-g(re^{i\theta})|^pd\theta$. (Received May 22, 1950.)

400t. W. R. Wasow: *A study of the solutions of the differential equation $y^{(4)}+\lambda^2(xy''+y)=0$ for large values of λ.*

The differential equation $y^{(4)}+\lambda^2(xy''+y)=0$ belongs to a type studied in a previous paper by the author (Ann. of Math. vol. 49 (1948) p. 852). By means of Laplace contour integrals and the method of steepest descent the character of the solutions for large λ is studied more completely than is possible with the methods of the former paper. The results obtained include, in particular: (1) An asymptotic description of the solutions in a full neighborhood of the origin. This requires three sets of formulas, one valid when $|x|\geqq$const.>0, the second when $|x|\leqq$const., but $\lambda^{2/3}x$ very large, the third when $|\lambda^{2/3}x|\leqq$const. (2) Asymptotic expressions for the solutions called "balanced" in the paper mentioned above that are valid in the sector where these solutions diverge. They show that these solutions are there of the order of $\lambda^{-3/2}\exp[\lambda(2/3)(-x)^{3/2}]$. (3) Asymptotic formulas valid on the "Stokes lines." (Received March 10, 1950.)

Applied Mathematics

401. William Karush: *An iterative method for finding characteristic vectors of a symmetric matrix.*

The following iteration procedure for finding a characteristic vector of a real symmetric matrix A is treated. Let $s>1$ be a fixed positive integer. Given the vector x^i, construct the space $\mathcal{A}^1$ spanned by $x^i, Ax^i, \cdots, A^{s-1}x^i$. Then there is a unique vector x^{i+1} of the form $x^i+\eta$, $(x^i, \eta)=0$, which minimizes (maximizes) the Rayleigh quotient $(x, Ax)\cdot|x|^{-2}$ for x in $\mathcal{A}^1$. The initial vector x^0 is chosen arbitrarily. Let $\lambda_{\min}$ $(\lambda_{\max})$ be the least (greatest) characteristic value of characteristic vectors y for which $(x^0, y)\neq 0$. Then $x^i\rightarrow y_{\min}$ $(y_{\max})$, where $y_{\min}$ $(y_{\max})$ has characteristic value $\lambda_{\min}$ $(\lambda_{\max})$. The proof is based upon an investigation of the polynomials associated with an orthogonalization of the spaces $\mathcal{A}^1$. The iteration scheme is essentially a gradient method. For $s=2$ the vector η is a multiple of the gradient of the Rayleigh quotient; for $s>2$ the vector η contains higher correction terms. (Received March 7, 1950.)

402. Cornelius Lanczos: *Inversion of the Laplace transform.*

The customary method of inverting the Laplace transform makes use of integration along the imaginary axis from $-i\infty$ to $+i\infty$, closing the path of integration through the negative infinite semi-circle. The integration is then reduced to loops around the singular points or lines. This method becomes cumbersome if the singularities of the transform are not of a simple type. The present procedure maps the infinite imaginary axis into the unit circle, with the help of reciprocal radii. Inside of this circle the transform is regular and allows expansion into a Taylor series. In the inversion the series can be integrated term by term. The resultant $f(x)$ appears as an infinite expansion into modified Laguerre functions, orthogonal in the range between 0 and ∞ with respect to the weight factor x. These expansions combine the simplicity of the Taylor coefficients, obtained by successive recursions, with the advantages of an approximation in the large, usually obtained by cumbersome integrations. The method yields also a simple algorithm for designing an electric network of definite order which approximates a prescribed pulse response as closely as possible. (Received March 17, 1950.)

403. W. R. Wasow: *On random walks and eigenvalues of elliptic difference equations.*

Let B be a finite domain bounded by a polygon C whose vertices are points of a square lattice of mesh length h. A particle starting with mass one from a point P in B performs a random walk in the lattice with equal transition probabilities for the four possible directions, until it is absorbed by the boundary. At each step the mass is multiplied by the value of a given positive function $k(x, y)$ of the position. Let $E(P, R)$ be the expected value of the mass absorbed by a preassigned boundary point R. If $E(P, R)$ is finite, it is a solution of the difference equation $\Delta u+(4/h^2)(1-1/k)u=0$, where Δu is the usual finite difference analogue of the Laplace operator. It is shown that $E(P, R)$ is finite if and only if the difference expression above is positive definite in B. With appropriate modifications a similar result holds for infinite domains. This random walk procedure can be used for the approximate numerical solution by means of sampling methods of boundary and eigenvalue problems for the corresponding partial differential equations. (Received March 10, 1950.)

Geometry

404. D. B. Dekker: *Some generalizations of hypergeodesics.*

In a previous paper (*Hypergeodesic curvature and torsion*, Bull. Amer. Math. Soc.

vol. 55 (1949) pp. 1151–1168) the author studied an arbitrary family of hypergeodesics on a general surface in ordinary euclidean space. In the present paper a more general class of curves is obtained by replacing in the usual tensor differential equation of a family of hypergeodesics the homogeneous cubic form in the first derivatives of the surface coordinates with respect to arc length by a rational function homogeneous of degree three in the derivatives. This more general class of curves contains the hypergeodesics as a subclass and possesses most of their properties. It is found that a curve of a family is a plane curve if and only if it is a curve of a special related intersector net of a certain complex of lines determined by the family. If n and $n-3$ are the degrees of the numerator and denominator of the rational homogeneous function then in general the special intersector net consists of $3n-1$ one-parameter families of curves on the surface. (Received March 8, 1950.)

405. Douglas Derry: *The duality theorem for curves of order n in n-space.*

Let C_n be a curve in real projective n-space for which (1) all osculating r-subspaces, $1 \leqq r < n$, are defined for each curve point and (2) no hyperplane cuts the curve in more than n points. P. Scherk showed (*Über differenzierbar Kurven und Bögen*, Časopis pro pěstování, matematiky a fysiky vol. 66 (1937) pp. 172–191) by a fixed point theorem applied to a closed C_n that the dual curve of C_n possesses properties (1) and (2). The present note gives another proof of this result which is based on a variation of Rolle's theorem. (Received March 15, 1950.)

406. T. K. Pan: *Hypergeodesics and dual hypergeodesics.*

A system of curves on a general proper analytic surface in ordinary metric space called dual hypergeodesics is defined, which includes a system of dual union curves as a special case. First, an extended relation R with respect to a general net is defined and an equation of a line L in extended relation R to L' is derived. Then the envelope of L as L' generates the osc-cone at a point of a family of hypergeodesics is found, which leads from the definition to the intrinsic differential equations of dual hypergeodesics. The unicity and the reciprocity properties of hypergeodesics and dual hypergeodesics lead to the definition of reciprocal families of hypergeodesics, which is followed by the pairing of hypergeodesics of one or two reciprocal families into a conjugate net and the relationships between the associated cones and cubics. Torsal curves associated with a family of hypergeodesics are defined and the non-asymptotic torsal curves are shown to be the nonlinear plane hypergeodesics contained in the family. (Received March 14, 1950.)

Topology

407. P. A. White: *Regular convergence in terms of Čech cycles.*

In a compact Hausdorff space closed subsets $\{A_i\}$ converging to A are said to converge n-regularly if for each finite covering $\mathcal{U}$ of S by open sets, there is a finite refinement $\mathcal{V}$ of U and an integer N such that if z^r is a Čech cycle (with field coefficients) on A_i for $i > N$ and $r \leqq n$ with diameter less than $\mathcal{V}$, then z^r bounds a chain on A_i with diameter less than U. This differs from the original definition of G. T. Whyburn in that Čech cycles without assuming a metric are used instead of Vietoris cycles with a metric. It is proved that under n-regular convergence the limit set A is an lc^n, the Betti numbers $p^r(A_i) = p^r(A)$ for all $r \leqq n$ and almost all i, and if each A_i is an orientable $(n+1)$-dim generalized closed manifold (see R. L. Wilder, *Topology of manifolds*, Amer. Math. Soc. Colloquium Publications) so also is A. These results were previously known only in the metric case. In proving the above results it was shown that corresponding to any finite covering U of S there exists a finite refinement $\mathcal{V}$ of U which is normal relative to cycles on A_i for all i. The existence of a normal refinement for one set was proved in Wilder's Colloquium Lectures, but the above result seems to be the first case of the existence of one for infinitely many sets simultaneously. (Received February 2, 1950.)

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