THE APRIL MEETING IN NEW ORLEANS

The four hundred sixty-seventh meeting of the American Mathematical Society was held at Tulane University, New Orleans, Louisiana, on Friday and Saturday, April 20–21, 1951. The total attendance was about 100 including the following 56 members of the Society:

H. P. Beard, R. G. Blake, R. C. Bullock, Eugenio Calabi, W. L. Duren, J. C. Eaves, D. O. Ellis, Trevor Evans, J. K. Feibleman, J. G. Horne, S. T. Hu, G. B. Huff, L. H. Kanter, H. T. Karnes, J. L. Kelley, R. J. Koch, G. B. Lang, F. A. Lewis, J. L. Loflin, C. W. McArthur, C. C. MacDuffee, J. D. Mancill, M. H. Martin, Benjamin Ernest Mitchell, D. R. Morrison, C. G. Mumford, H. M. Nahikian, H. V. Park, W. V. Parker, P. B. Patterson, W. D. Peeples, B. J. Pettis, A. S. Rayl, P. K. Rees, F. A. Rickey, J. K. Riess, N. E. Rutt, Judson Sanderson, W. C. Sangren, E. V. Schenkman, C. L. Seebeck, D. C. Spencer, J. M. Thomas, J. F. Thomson, R. M. Thrall, H. S. Thurston, G. J. Trammell, F. E. Ulrich, H. S. Vandiver, L. I. Wade, A. D. Wallace, H. C. Wang, L. E. Ward, Jr., M. J. Weiss, W. M. Whyburn, L. S. Winton.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, Professor G. B. Huff of the University of Georgia and Professor M. H. Martin of the University of Maryland addressed the Society. Professor Huff's address, entitled *On the existence of plane curves with prescribed singularities*, was delivered at 11:00 a.m. on Saturday, with Professor W. V. Parker presiding. Professor Martin's address, entitled *Riemann's method and the problem of Cauchy*, was delivered at 8:30 P.M. Friday, with Professor W. L. Duren presiding.

Sessions for the presentation of contributed papers were held on Friday afternoon and Saturday morning. Presiding officers for these sessions were Professors F. A. Lewis and L. I. Wade. An informal business meeting was held just preceding the invited address on Saturday morning, at which the group voted to recommend to the Council that the invitation of the Alabama Polytechnic Institute to hold a meeting on its campus on November 23–24, 1951, be accepted.

Following the hour address on Friday evening, the members of the Department of Mathematics of Tulane University and their wives were hosts at a reception for visiting members and guests.

Abstracts of the papers presented follow below. Abstracts whose numbers are followed by the letter "*t*" were presented by title. Paper number 246 was read by Mr. Burton, number 251 by Professor Calabi, number 256 by Professor Lang, and number 257 by Mr. McArthur. Mr. Novosad was introduced by Professor Pettis, Mr. Chehata by Dr. Neumann, Mr. Spencer by Professor Carlitz, Mr. Burton and Mr. Young by Professor Whyburn.

ALGEBRA AND THEORY OF NUMBERS

237t. A. T. Brauer: *On the irreducibility of polynomials with large third coefficient.* II.

Let $f(x) = x^n + a_1 x^{n-1} + \cdots + a_n$ be a polynomial with integral rational coefficients, $a_n \neq 0$, and $t = |1 + a_4| + |a_1 + a_3| + |a_5| + \cdots + |a_n|$. Improving a theorem of Perron (J. Reine Angew. Math. vol. 132 (1907) pp. 288–307) it was shown in the first part of this paper (Amer. J. Math. vol. 70 (1948) pp. 423–432) that $f(x)$ is irreducible in the field of rational numbers if a_2 is sufficiently large. It is proved in this paper that $f(x)$ is irreducible if $a_2 > t$ and one of the other coefficients is sufficiently large. (Received March 7, 1951.)

238t. C. G. Chehata: *An algebraically simple ordered group.*

Let F be a fully ordered field, and $G(F)$ the set of all monotone increasing one-valued functions $f(x)$ defined for $x \in F$ and taking their values in F with the property that for each f there exists an interval $[\lambda, \mu]$ in F on which f is piecewise linear and outside which $f(x) = x$. Then $G(F)$ is a group under substitution and can be linearly ordered so as to be ordinally simple in the sense that it has no proper convex normal subgroup. Also those elements of $G(F)$ for which λ, μ lie in a fixed closed interval I or a fixed open interval I' form a subgroup $G(I)$ or $G(I')$ respectively. All $G(I)$, for different closed intervals, are order-isomorphic. The derived group of $G([\alpha, \beta])$ is $G((\alpha, \beta))$. When F is the field of real numbers, then all $G(I')$ are order-isomorphic to $G(F)$. In this case the group $G([0, 1])$ was first considered by K. Iwasawa, *On linearly ordered groups,* Journal of the Mathematical Society of Japan vol. 1 (1948) pp. 1–9, example 8. The principal result is that $G(I')$ for any open interval I', and thus $G(F)$, is algebraically simple in the sense that it has no proper normal subgroup. (Received March 5, 1951.)

239. D. R. Morrison: *Quotient semigroups.*

A subset N of a semigroup S is called *normal* with respect to $M \subset S$ if $mN = Nm$ for each $m \in M$. If $N \subset S$ is a semigroup normal with respect to S, a *quotient semigroup* S/N is defined. If a semigroup $B \subset S$ has elements cancellable in S and if S is normal with respect to B, then B is contained in a unique minimal semigroup $C \subset S$ such that C is normal with respect to B. Then elements of C are cancellable in S, and C and S are normal with respect to C. A *cross product* $S \times C$ is defined, with diagonal D. The quotient $S \times C/D$ contains an isomorphic image $\theta(S)$ of S, and elements of $\theta(C)$ have inverses in $S \times C/D$. If S is a semigroup and $B \subset S$ is a semigroup whose elements are cancellable in S, then S can be embedded in a semigroup $G \supset \theta_2(S)$ in which every element of $\theta_2(B)$ has an inverse if and only if S can be embedded in a semigroup $T \supset \theta_1(S)$ in which elements of $\theta_1(B)$ are cancellable and T is normal with respect to $\theta_1(B)$. (Received March 15, 1951.)

240. W. V. Parker: *On quadratic matric equations.*

The matric equation $XAX + BX + XC + D = 0$ is considered in connection with the unilateral equation $X^2 P + X(Q - M) - N = 0$ where the matrix with first column C, D and second column $-A, -B$ and the matrix with first column M, N and second column P, Q are commutative. The results obtained are extensions of results given by Roth in his study of the matric equation $X^2 + AX + XB + C = 0$ (Proceedings of the American Mathematical Society vol. 1 (1950) pp. 586–589). It is shown that when A

and P are nonsingular, any solution U of one of the equations which is such that both A and B are commutative with $UP+Q$ is also a solution of the other equation. (Received February 26, 1951.)

241. W. D. Peeples: *On elliptic ternary cubics with an infinite number of rational solutions.* Preliminary report.

Although Mordell and others have studied the fundamental properties of the configuration of rational points on an elliptic ternary cubic, there exist no general conditions sufficient to assure that a given curve have an infinite number of rational points. In this paper the problem is studied from the point of view of valuation theory. It is shown that simple sufficient conditions exist and that these results include as special cases results given in some previous papers. (Received March 7, 1951.)

242t. J. A. Silva: *Representations of arithmetic functions in* $GF[p^n, x]$ *with values in an arbitrary field.*

By an arithmetic function is meant a single-valued function $f(A)$ defined for polynomials A over $GF[p^n, x]$ for which $\deg A \leqq r$ (where r is a fixed positive integer); the values of $f(A)$ lie in an arbitrary field F. (See Carlitz, *Representations of arithmetic functions in* $GF[p^n, x]$.) In this paper a set of p^{nr} linearly independent arithmetic functions is given by means of which an arbitrary arithmetic function $f(A)$ may be represented with coefficients from the field F. The results obtained for F of characteristic zero or $q \neq p$ are quite similar but differ markedly from the results for F of characteristic p. (Received February 19, 1951.)

243t. S. M. Spencer: *Transcendental numbers over certain function fields.*

The first part of the paper contains a proof of an analogue of a theorem due to G. Faber (Math. Ann. vol. 58 (1904) pp. 545–557). It is shown that certain entire functions $f(t)$ have the property that $f(\alpha)$ is transcendental for all algebraic nonzero α. The second part of the paper contains proofs of several theorems over fields of characteristic p, all of which are generalizations of or suggested by certain theorems due to L. I. Wade (Duke Math. J. vol. 8 (1941) pp. 701–720; vol. 11 (1944) pp. 755–758). A typical example of such a theorem is: The series $\sum_{k=0}^{\infty} G_k^{-e_k^0}$ is transcendental, where the G_k are polynomials such that $G_k | G_{k+1}$; $\deg G_0 \geqq 1$; $e_0 < e_1 < e_2 < \cdots$; $g \geqq 2$; and $p \nmid e_k$. The underlying field is $F(x_1, \cdots, x_w)$, where F is a field of arbitrary characteristic in the first part but $F = GF(p^n)$ in the second part. (Received February 24, 1951.)

244. H. S. Thurston: *Automorphism and anti-automorphisms of a quaternion algebra.* Preliminary report.

Under a biunique correspondence, $\alpha \leftrightarrow \alpha'$, of a quaternion algebra over a formally real field, let $i \leftrightarrow i'$, $j \leftrightarrow j'$, $k \leftrightarrow k'$. If v denote the vector $(1, i, j, k)$ and v' the vector $(1, i', j', k')$, let P be the matrix such that $vP = v'$. R and R' being the first regular matric representations of α and α', and S and S' their second regular representations, the following relations are proved: (a) if the correspondence is an automorphism, then $PRP^{-1} = R'$, $PSP^{-1} = S'$, (b) if the correspondence is an anti-automorphism, then $PRP^{-1} = S'^T$, the transpose of S'. (Received March 7, 1951.)

245t. A. D. Wallace: *Isomorphism theorems in group theory and cohomology.*

Fix a non-negative integer p. If the pair X, A is a group and normal subgroup, let (X, A) be X/A. If the pair X, A is a fully normal space and a closed set, let (X, A) be $H^p(X, A)$. For groups $A \cup B$ will mean the smallest group containing both A and B. Then (tacitly assuming the proper normality and closure conditions) the second isomorphism theorem and the excision theorem can both be written as $(A \cup B, B) \approx (A, A \cap B)$. Let $(X_1, A_1) \cap (X_2, A_2)$ be $(X_1 \cap X_2, (A_1 \cap X_2) \cup (X_1 \cap A_2))$. The Zassenhaus isomorphism theorem asserts that $(X_1, A_1) \cap (A_1 \cup X_2, A_1 \cup A_2) \approx (X_1, A_1) \cap (X_2, A_2)$. With suitable closure conditions, this is a valid proposition for cohomology. It is interesting to observe that the cohomology group $(X_1, A_1) \cap (X_2, A_2)$ has arisen recently in a paper by N. E. Steenrod. (Received March 7, 1951.)

ANALYSIS

246. L. P. Burton and W. M. Whyburn. *Minimax solutions of ordinary differential systems.*

A system of n, nonlinear, differential equations of the first order is considered under conditions which insure the existence but not the uniqueness of a solution through a given point in $(n+1)$-dimensional real space. Perron, Kamke, and others have studied this system under hypotheses that insure the existence of solutions of maximum and minimum types through each point. Systems which do not have solutions of these types may have critical solutions of intermediate or "minimax" type provided they exhibit the monotonic character developed in the present paper. The paper establishes the existence and uniqueness of such solutions. It also shows that these solutions can be uniformly approximated by mixed over and under functions, these functions having the nature of the over and under functions used in the Perron theory of integration. (Received April 20, 1951.)

247t. J. D. Mancill: *On a proof of the Jacobi condition in the calculus of variations.* Preliminary report.

The proof of the Jacobi condition for simple problems of the calculus of variations by means of an envelope of a one-parameter family of extremals does not necessarily apply to all extremal arcs of the minimizing curve in the case when the minimizing curve may have arcs in common with the boundary of the region of admissible curves. So far as the author knows no use has been made of the second variation in this connection. The author has given a proof of this condition in two dimensions which is applicable to such problems (*The Jacobi condition for unilateral variations*, Duke Math. J. vol. 6 (1940) pp. 341–344). In the present paper this proof is extended to problems in space of three dimensions for the nonsingular case of the enveloping surface of a two-parameter family of extremals. (Received March 7, 1951.)

248t. K. S. Miller: *Self-adjoint differential systems.*

D. Jackson (Trans. Amer. Math. Soc. vol. 17 (1916) pp. 418–424) has given a criterion for an ordinary linear differential system to be self-adjoint. His results were later written in more explicit form by V. V. Latshaw (Bull. Amer. Math. Soc. vol. 39 (1933) pp. 969–978). In this paper a different criterion is derived which is more convenient to use in certain problems. If L is a linear differential operator of order n and $U_\alpha(u) = V_\alpha(u) + Z_\alpha(u)$, $\alpha = 1, 2, \cdots, n$, are two point boundary conditions in which all the terms involving the left end point of the interval are included in $V_\alpha(u)$ and all those involving the right end point are grouped in $Z_\alpha(u)$, then the differential system is self-adjoint if $L = (-1)^n L^+$ and the matrix $\Delta CD'$ is symmetric (skew-sym-

metric) if n is odd (even). Here L^+ is the formal adjoint of L. If V and Z are the matrices of the linear forms $V_\alpha(u)$, $Z_\alpha(u)$, then $D = V + Z$, $\Delta = V - Z$, and C is the matrix relating n linearly independent solutions of $Lu = 0$ to the corresponding ones for $L^+u = 0$. The proof makes use of the Green's function. (Received February 14, 1951.)

249t. B. J. Pettis: *On the continuity of parametric linear operations.*

Let H be an open semigroup in an additive second category topological group G, and let T_h be additive on H to the space $B(X)$ of all linear operations in a complex Banach space X. For each x_0 in X the function $\phi(h) = T_h(x_0)$ will be continuous on H to X if the following is true; for each x_0 there is a set $R(x_0)$ residual in H and a set $\Gamma(x_0)$ of complex linear functionals on X such that $\phi(R(x_0))$ is separable in X, $\Gamma(x_0)$ is total for $\phi(H)$, and $\gamma(\phi(h))$ is continuous for each γ in $\Gamma(x_0)$. (Received February 13, 1951.)

250. J. M. Thomas: *Equations equivalent to a linear differential equation.*

If $c_1y_1 + \cdots + c_ny_n$ is a general solution of a linear homogeneous ordinary differential equation $L(y) = 0$ and $F(y_1, \cdots, y_n)$, where F is sufficiently often differentiable, is a general solution of an ordinary differential equation E, then E is to be called equivalent to $L(y) = 0$. Any E equivalent to $L(y) = 0$ can be given the form $L(y) = G$, where G is a rational function of $y, y_1, \cdots, y_n$, F, and their derivatives. If the solution of $L(y) = 0$ is known, an equivalent equation can be had by arbitrarily assigning F. If merely the coefficients of $L(y) = 0$ and the solutions of certain equations of lower order formed with them are assumed known, then F must satisfy a system of partial differential equations, particular solutions of which lead to categories of equations solvable in terms of linear equations. For $n = 2$ there is found in this manner a category including the equation discussed by Pinney (Proceedings of the American Mathematical Society vol. 1 (1950) p. 681). (Received March 6, 1951).

Geometry

251. Eugenio Calabi and D. C. Spencer: *Completely integrable almost complex manifolds.*

An almost complex manifold is defined as a real, differentiable manifold with a differentiable transformation H on its covariant vector bundle satisfying $H^2 = -1$. Such a manifold is always even-dimensional. An almost complex manifold is called *completely integrable* if there exists in a neighborhood of each of its points an allowable coordinate system $(x^1, \cdots, x^{2k})$, satisfying (1) $Hd(x^\alpha + (-1)^{1/2}x^{k+\alpha})$ $= (-1)^{1/2}d(x^\alpha + (-1)^{1/2}x^{k+\alpha})$ $(1 \leq \alpha \leq k)$. Any two systems of complex-valued coordinates, $z^\alpha = x^\alpha + (-1)^{1/2}x^{k+\alpha}$ and $z'^\alpha = x'^\alpha + (-1)^{1/2}x'^{k+\alpha}$, associated with two systems satisfying (1) in a common domain, are complex analytically related, so that every completely integrable almost complex manifold corresponds to a complex manifold in a natural way. In an almost complex manifold one defines a tensor T^l_{ij} called the *pseudo-conformal curvature tensor*, from the tensor h^i_j representing the transformation H by $T^l_{ij} = (1/4)(\delta^{ab}_{ij} - \delta^{rs}_{ij}\ h^a_r h^b_s)\partial h^l_b/\partial x^a$, which leads to the main theorem. *An almost complex manifold is completely integrable if and only if its pseudo-conformal curvature vanishes identically.* The necessity is proved by a simple computation of exterior differential forms; the sufficiency is proved by constructing locally a Rie-

mannian metric satisfying Kähler's condition, from which one obtains the complex analytic coordinates by a variational method. (Received March 7, 1951.)

252t. H. S. M. Coxeter: *Group invariants and Poincaré polynomials.*

The product of the generators of a finite group generated by reflections is an n-dimensional orthogonal transformation whose period, h, has several interesting properties (Coxeter, Ann. of Math. vol. 35 (1934) pp. 606–617). In terms of ω, a primitive hth root of unity, the characteristic roots of this transformation are ω^m for n special integers m, which may be taken to lie between 0 and h. These are computed by means of a general formula (*Lösung der Aufgabe* 245, Jber. Deutschen Math. Verein. vol. 49 (1938) pp. 4–6). The point of interest is that the same integers occur in a different connection. Continuing the work of Klein (*Lectures on the icosahedron*, pp. 237, 242) and Burnside (*Theory of groups*, 2d ed., p. 362), Racah has found a set of n independent invariant forms for each group (Rendiconti della Accademia Nazionale dei Lincei (8) vol. 8 (1950) p. 112). These n invariants are a basic set, since the product of their degrees is equal to the order of the group. Actually, the degrees are just the n numbers $m+1$. In the case of the crystallographic groups, the Poincaré polynomial for the group manifold of the corresponding simple Lie group (Chevalley, Proceedings of the International Congress of Mathematicians, 1950) is $\prod(1+t^{2m+1})$. (Received March 6, 1951.)

253. D. O. Ellis: *On the metric characterization of metric lattices.*

We consider in this note a metric lattice in its metric topology and show that *each compact metric lattice has a first (and last) element.* Thus the result, *a metric space is congruent to a metric lattice with first element if and only if it is almost ordered,* of Smiley and Transue (Bull. Amer. Math. Soc. vol. 49 (1943) pp. 280–287) yields: *A compactum is congruent to a metric lattice if and only if it is almost ordered.* We deduce from a study by Blumenthal and Ellis (Duke Math. J. vol. 16 (1949) pp. 585–590) that *no nonlinear subset of a Euclidean space or (real, separable) Hilbert space is congruent to a metric lattice,* and from a result in Birkhoff's book (Amer. Math. Soc. Colloquium Publications, vol. 25) that *any compact metric lattice is lattice complete.* (Received January 17, 1951.)

254. Benjamin Ernest Mitchell: *Vector in the complex plane, its representation.*

This paper endeavors to achieve an intuitive (real) conception of the imaginary vector in the complex plane. The result is an ordered vector pair in the real projective plane. An interpretation of the imaginary vector is revealed by representative vector pair including among the special cases the three mentioned by Eduard Study, to wit: The Kreuzlage, the Trapezlage, and the Isometrisch position. Equivalent representations in standard and normal positions (so called) are proposed. Also a planar representation of a real vector in a point space of four dimensions is offered. (Received March 5, 1951.)

Logic and Foundations

255. J. K. Feibleman: *The problem of the metapostulates.*

In setting up a mathematical system, there are assumptions which have never been brought out into the open. We do not start at the beginning when we set up undefined terms, called constants and variables, and unproved propositions, called

axioms, and then proceed to draw theorems and other deductions from them. For the undefined terms, the unproved propositions, and the process of deduction each has its own set of unacknowledged presuppositions. These are stated in the proper statement of the metapostulates for any mathematical system. A tentative suggestion in this direction is proposed. There is a further problem engendered by this one, which will be here briefly indicated. The metapostulate-set, m_1, has its own set of presuppositions, which we shall call the meta-metapostulates, m_2. There are reasons for believing that the meta-metapostulate-set is equivalent to the metapostulate-set. The author seeks to devise a postulate-set, m_3, which allows for the repetition involved in the equivalence of m_1 and m_2, and which also is reflexive. (Received February 5, 1951.)

TOPOLOGY

256. D. O. Ellis and G. B. Lang: *The space of groupoids over a compactum.*

Let M be a compactum with distance function $\delta(x, y)$. Let $\mathfrak{G}$ be the set of all groupoid operations over the point set of M with distance function $\delta(a, b) = \sup \delta(xay, xby)$. We show: (1) $\mathfrak{G}$ is complete. (2) $\mathfrak{G}$ is distancial to M. (3) $\mathfrak{G}$ is convex when M is convex. (4) The following subsets of $\mathfrak{G}$ are closed: the continuous groupoids, continuous semigroups, continuous commutative groupoids, topological quasigroups, topological loops, and the topological groups. (5) An element of $\mathfrak{G}$ is continuous if and only if the sublattice of closed elements of the closure algebra of M is a subgroupoid of the closure algebra of M under the natural extension to this closure algebra of the given element of $\mathfrak{G}$. (Received January 17, 1951).

257. W. L. Gordon and C. W. McArthur: *On uniform Cauchy points.*

Let X be any topological space and ϕ any cardinal. A subset E of X is a I_ϕ-set if it is the union of ϕ nowhere dense sets; otherwise it is a II_ϕ-set. It is ϕ-residual if its complement $X - E$ is a I_ϕ-set. Let Y be any completely regular space, and of all uniformizations of the topology in Y let $\{V_\alpha | \alpha \in A\}$ be one having least cardinal $|A|$. For each element λ of a fixed directed set Λ let f_λ be a function on X to Y. A point x_0 in X is a *Cauchy point* of $\{f_\lambda\}$ if given any $\alpha \in A$ there is some $\lambda_\alpha \in \Lambda$ such that $f_\lambda(x_0) \in V_\alpha(f_{\lambda_\alpha}(x_0))$ for all $\lambda \geq \lambda_\alpha$. x_0 is a *uniform Cauchy point* of $\{f_\lambda\}$ if for each $\alpha \in A$ there exists $\lambda_\alpha \in \Lambda$ and an open set G_α about x_0 such that $f_\lambda(x) \in V_\alpha(f_{\lambda_\alpha}(x))$ for all $\lambda \geq \lambda_\alpha$ and all $x \in G_\alpha$. Let C be the Cauchy points and C_u be the uniform Cauchy points and $|\Lambda|$ be the least of the cardinals belonging to cofinal subsets of Λ. If each f_λ is continuous and $\phi \geq \max(|A|, |\Lambda|)$, then $C - C_u$ is I_ϕ. Hence if C is a II_ϕ-set, C_u is also; if C is ϕ-residual, so is C_u. This includes some recent results of A. Alexiewicz, A. M. Ostrowski, and B. J. Pettis. (Received March 5, 1951.)

258t. S. T. Hu: *On products in homotopy groups.*

In 1947, a product analogous to those of J. H. C. Whitehead was introduced by the author (J. London Math. Soc. vol. 22) for the relative homotopy groups; namely, for any two elements $a \in \pi_{p+1}(X, A)$ and $b \in \pi_{q+1}(X, A)$, it associates with a unique element $[a, b] \in \pi_{p+q}(X, A)$. In the present paper, it is proved that $[a, b]$ depends only on the boundary elements $\partial a \in \pi_p(A)$ and $\partial b \in \pi_q(A)$. Let $\alpha \in \pi_p(A)$, $\beta \in \pi_q(A)$ be any two given elements such that $i_* \alpha = 0 = i_* \beta$ and the Whitehead product $[\alpha, \beta] = 0$, where i_* denotes the homomorphisms induced by the identity map $i: A \subset X$.

Choose two elements a and b such that $\partial a = \alpha$ and $\partial b = \beta$. Then $[a, b]$ is an element in the image $j_*\pi_{p+q}(X)$ under the homomorphism j_*: $\pi_{p+q}(X) \to \pi_{p+q}(X, A)$ induced by the identity map j: $X \subset (X, A)$. By the exactness of the homotopy sequence, j_* induces an onto isomorphism j_0: $\pi_{p+q}(X)/i_*\pi_{p+q}(A) \approx j_*\pi_{p+q}(X)$. We denote by $\alpha \otimes_i \beta$ the element $j_0^{-1}[a, b]$ of the quotient group $\pi_{p+q}(X)/i_*\pi_{p+q}(A)$. By passing to the mapping cylinder of any given map f: $A \to B$, functional products are defined in a natural way. To each pair of elements $\alpha \in \pi_p(A)$, $\beta \in \pi_q(A)$, such that $f_*(\alpha) = 0 = f_*(\beta)$ and $[\alpha, \beta] = 0$, we associate a unique element $\alpha \otimes_f \beta$ of the quotient $\pi_{p+q}(B)/f_*\pi_{p+q}(A)$, which is an invariant of the homotopy class of the map f and is zero whenever f is homotopic to a constant. (Received March 5, 1951.)

259. J. L. Kelley: *Remarks on a theorem of Keesee.*

Keesee has shown that if X is a compact Hausdorff space the cohomology theory of Alexander-Kolmogoroff-Spanier-Wallace is isomorphic to the same theory with the restriction that each co-chain have finite range. The homology theory corresponding to this restricted co-chain theory can be easily described in terms of those contents (that is, finitely additive set functions) on X^{p+1} to a compact coefficient group g which vanish for sets whose closures fail to intersect the diagonal Δ^{p+1}. The connection with Kolmogoroff's original theory is then easy to see, as well as the fact that the contents considered may be restricted to interval functions of a rather special sort. (Received March 7, 1951.)

260. R. S. Novosad: *Groups of unit homomorphisms.*

Given a pair (X, x) consisting of a topological space X and a point x of X, a space Z can be constructed from a set of 1-spheres which are indexed by the elements of the fundamental group of (X, x). Z maps into X in a natural manner. Homotopies with the constant path, of paths in X which are images of paths in Z, form a subspace of the space of mappings of the unit square into X. The arcwise connected components of this subspace can be multiplied, and form a group which is denoted by $W_2(X, x)$. $W_2(X, x)$ is a central group extension of the second homotopy group of (X, x) by the group relations of the fundamental group of (X, x). $W_2(X, x)$ maps homomorphically onto the second singular homology group in a way similar to the mapping of the fundamental group onto the first homology group. (Received March 7, 1951.)

261t. B. J. Pettis: *A note on everywhere dense subgroups.*

Recently H. C. Wang has shown that any locally compact separable nondiscrete metric group must contain an uncountable everywhere dense proper subgroup. This can be extended as follows; in any second category nondiscrete separated group any everywhere dense proper subgroup lies in some uncountable everywhere dense proper subgroup. This in turn is a special case of the following conjecture; any proper subgroup of a second category topological group X lies in a second category proper subgroup. For abelian X the conjecture is verified for these two cases: (1) X is second category and x^k maps non-null open sets into somewhere dense sets for each positive integer k; and (2) X is locally euclidean. (Received January 19, 1951.)

262. B. J. Pettis: *Comments on open homomorphisms.*

This paper is concerned with certain refinements and extensions of recent results of the author (Ann. of Math. vol. 52 (1950) pp. 293–308) and of J. Dieudonné (Proceedings of the American Mathematical Society vol. 1 (1950) pp. 54–59). (Received March 19, 1951.)

263. A. D. Wallace: *Pseudo retraction and deformation.*

In this note, we define pseudo retraction and deformation in such a way that the main results connecting retraction and deformation with cohomology carry over to a more general situation. In this way, we can extend to spaces that are not locally connected theorems that were previously known to hold only in the locally connected case. (Received February 26, 1951.)

264. H. C. Wang: *On locally compact two-point homogeneous spaces.*

Let E be a metric space with metric d. It is called two-point homogeneous (or in short, a (*)-space) if, for any 4 points a_1, a_2, b_1, b_2 with $d(a_1, a_2) = d(b_1, b_2)$, there exists an isometry of E carrying a_1, a_2 to b_1, b_2 respectively. In a recent paper, the author has determined all the compact, connected (*)-spaces. This note concerns the locally compact (*)-spaces which are S.L. (roughly, any two points can be joined by a unique geodesic). We shall prove that such a space E must be a manifold, and if dim E is odd, then E is either the elliptic space, the euclidean space, or the hyperbolic space. (Received March 9, 1951.)

265. L. E. Ward, Jr.: *Relations between character groups of locally compact abelian T_0-groups.*

Let G be a topological group, and G^* its character group. Numerous computational theorems concerning the relations between G^*, the annihilator subgroups of G^*, and the character groups of closed subgroups and continuous relatively open images of G have been established by Pontrjagen, van Kampen, Freudenthal, and others in the cases where G is compact or discrete, or locally compact and satisfying the first countability axiom. The author proves a number of these theorems as well as some new results for the case when G is a locally compact Abelian T_0-group, without countability restrictions. The purpose of these generalizations is to establish a set of computational theorems for homology theory. (Received March 7, 1951.)

266. C. T. Yang: *On Borsuk's problem.*

Borsuk has asked this question: If each of the topological spaces X and Y is homeomorphic with a retract of the other, are the cohomology groups of X and Y isomorphic? It is shown that this is so if X is compact Hausdorff and the coefficient group is of finite exponent. (Received April 20, 1951.)

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