preface it considers several topics whose interest seems technical and special; while other topics of modern logic, whose mathematical interest seems far greater, are ignored.

H. B. CURRY

Theory and applications of distance geometry. By L. M. Blumenthal. Oxford University Press, 1953. 12+348 pp. $10.00.

The first systematic development of geometric aspects of metric spaces is due to Karl Menger, who published a number of results in his *Untersuchungen über allgemeine Metrik* in 1928. Since then several authors have made contributions to this new domain. In 1938 Blumenthal gave a survey of the material available at that time in his book *Distance geometries*, one of the University of Missouri Studies. Considerable progress has been made since 1938 and with this new book the author aims to give a detailed introduction to the subject. About two-thirds of the text is devoted to imbedding and characterization problems. This shows clearly that the writer's own taste has played a large part in selecting the topics.

In the first chapter the author introduces all the principal notions which play a part in the theory of abstract metric spaces. Some have a topological character. It should be remarked that the definition given for compact spaces (p. 29) is different from the usual one, which may lead to some confusion. Chapter II is devoted to the notions of betweenness, convexity and metric segments. It contains Aronszajn's proof of one of Menger's theorems concerning segments: Each two points of a complete and convex space are joined by a metric segment. Chapter III gives in 30 pages a somewhat scanty treatment of metric curve theory. It contains several possible definitions of length and curvature and a survey of the results obtained so far, but for the proofs of many results the reader is referred to the original papers. A fuller treatment is given of the characterization problem. A subclass of metric spaces is characterized metrically whenever necessary and sufficient conditions (in terms of the metric) are obtained which have to be satisfied in order that a metric space be congruent with a member of this subclass. If the subclass consists of the set of subspaces of a metric space the problem is called the imbedding problem. An extensive treatment of this problem for the Euclidean spaces is given in Chapter IV. Most of these results are due to Menger but the author has succeeded in simplifying several of the proofs. If the subclass consists of one space only the problem is to characterize this space metrically among the metric spaces. Several characterizations are given for Euclidean spaces and Hilbert spaces. In a wide class of spaces the validity of the weak four-point property or of the Pythagorean theorem is shown to ensure the euclidean character of the metric. Important notions in the theory of imbedding are the notion of congruence order and of congruence indices of a metric space with respect to some class of metric spaces. A detailed study is made of the congruence order of some euclidean subsets, mostly with respect to the class of subspaces of a euclidean space (E_n). The congruence order of E_n with respect to metric spaces is shown to be $n+3$; this means that a metric space is congruently contained in E_n if every $n+3$ of its points have that property. This leads to the study of a pseudo-euclidean space S, for which each set of $n+2$ points is congruently imbeddable in E_n, without S being congruent with a subset of E_n.

Part III of the book is devoted to the imbedding and characterization theorems of spherical spaces, elliptic spaces, and hyperbolic spaces. Compared with the E_n no essentially different methods are needed for hyperbolic spaces and the theorems obtained are quite analogous. The theory of spherical spaces contains some new elements (diametrical points). One of the principal differences is the existence of pseudo-sets of more than $n+3$ points. These sets are completely investigated. The metric behaviour of elliptic spaces, as compared with euclidean spaces, is still more different and much more complicated. One of the peculiarities of elliptic space is the distinction between congruence and superposability. An intensive comparative study of these two notions is made. Another difference in metric character is shown by the theorems about the congruence order. The congruence order of the elliptic plane is 7 and not 5, whereas the congruence orders of elliptic spaces of more than two dimensions are not yet known.

The distance relations which enter into many metric characterizations of subsets of euclidean and non-euclidean spaces lead to a number of determinant theorems formulated in Chapter XIII as applications of distance geometry. As a second application the author presents a geometrization of the theory of linear inequalities. The final chapter deals with metric methods in the theory of normed lattices and autometricized Boolean algebras. Among the topics not included are the applications in the calculus of variations.

The book is clearly written and self-contained. The reader is well provided with exercises of various degrees of difficulty. The bibliography is almost complete, and the index adequate. In my opinion the author would have improved the usefulness of the book as a textbook if he had restricted the number of special problems treated in the text. On the other hand, this work is the only exposition of distance geometry available at this moment and it is certainly useful to have a survey of the results obtained so far in the geometric study of metric spaces.

J. HAANTJES

Exterior ballistics. By E. J. McShane, J. L. Kelley, and F. V. Reno. The University of Denver Press, 1953. 834 pp., 15 figures, 24 plates. $12.00.

This book forms a welcome addition to the very limited number of works on exterior ballistics; had there been many more competitors I suspect its welcome would have been equally warm.

The first chapter, roughly one-sixth of the book, is devoted to mathematical and physical preliminaries and is intended "to make the book intelligible to anyone who has had a reasonably good undergraduate course either in mathematics or physics." This chapter discusses vectors, the equations of rigid body motion, dimensional analysis (including an original proof of the Buckingham Π theorem) and appropriate parts of statistics. I feel that it is impossible in a single chapter, even a chapter as long as this, to provide an adequate background to exterior ballistics and am of the opinion that the authors were over-ambitious to attempt it. Any complete background picture must contain the elements of aerodynamics as well as of dynamics, and any such inclusion would no doubt have made the space required entirely prohibitive. Leaving aside the desirability of such a chapter and its sins of omission I found the presentation of the material selected very satisfying and feel that the authors are to be congratulated on it, save on one point. This point concerns the equations of motion of a rigid body which are proved on the sweeping assumption that the internal forces between any two points of the body lie in the line joining them (for a discussion of this point see, for example, Jeffreys and Jeffreys, *Methods of mathematical physics*, Cambridge University Press, pp. 76 and 294).

Coming now to exterior ballistics proper the treatment starts in Chapter II by a discussion of the aerodynamic forces acting on the projectile; two force coefficients (K_{XF} the Magnus cross force due to cross spin and K_S the cross force due to cross spin) and a moment coefficient (K_{XT} the Magnus cross torque due to cross spin) are added to "complete" the system considered in the classical treatment of Fowler, Gallop, Lock and Richmond (Philos. Trans. Roy. Soc. London vol. 221 (1921) p. 295). The equations of motion (both C.G. and yawing) are then derived relative to axes fixed in the projectile—one along the projectile axis; the equations in the two directions