THE APRIL MEETING IN CHICAGO

The five hundred second meeting of the American Mathematical Society included a Symposium on Applied Mathematics (sponsored by the Society and the Office of Ordnance Research) and was held at the University of Chicago on Thursday, Friday and Saturday, April 29, 30 and May 1.

There were a total of 421 registrations, including 308 members of the Society, representing almost twice the attendance at the 1953 meeting. (By order of the Council the detailed list of the names of members attending this and future meetings is deleted from the report.)

The Symposium was divided into three sessions which met respectively at 9:00 A.M. and 2:00 P.M. on Thursday, and at 9:00 A.M. on Friday. Major General L. E. Simon, Department of the Army, served as chairman of the first meeting at which the following papers were presented: *Operations research* by Professor P. M. Morse of Massachusetts Institute of Technology, *Problem of inductive inference*, by Professor Jerzy Neyman of the University of California, *Recent developments in analysis of variance*, by Professor H. O. Hartley of the University of London and Iowa State College. Dr. F. E. Grubbs of the Ballistic Research Laboratories served as the discussion leader.

The second session was presided over by Dr. T. J. Killian of the Office of Ordnance Research. He introduced Professor M. R. Hestenes of the University of California, Los Angeles, Mr. John Todd of the National Bureau of Standards, Professor A. A. Bennett of Brown University, and Professor J. E. Mayer of the University of Chicago, who spoke respectively on *Computational methods*, *Motivations for working in numerical analysis*, *Some numerical computations in ordnance problems*, and *Unsolved problems in statistical mechanics*. Mr. M. W. Leutert of the Ballistic Research Laboratories led the discussion.

The topics of the final session were *The simplest rate theory of pure elasticity*, *On the stability of mechanical systems*, *Problems associated with hyperbolic partial differential equations*, and *On the nature of differential operators and boundary value problems*, presented respectively by Professor C. A. Truesdell of Indiana University, Professor J. J. Stoker of New York University, Professor Florent Bureau of the University of Liege and the University of Chicago, and Professor William Feller of Princeton University. Mr. W. W. Leutert of the Ballistic Research Laboratories presided, and Professor Paul Germain of the Office National de l'Etude et des Recherches Aéronautiques and Brown University acted as the discussion leader.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor S. C. Kleene of the University of Wisconsin addressed the Society on the topic *Hierarchies of number-theoretic predicates* at 2:00 P.M. on Friday. Presiding officer at this session was Professor Saunders MacLane.

Sessions for the presentation of contributed papers were held at 3:15 P.M. on Friday and at 9:00 A.M., 10:30 A.M. and 2:00 P.M. on Saturday. Presiding officers at these sessions were Professors Zeev Nehari, J. J. Gergen, R. M. Thrall, G. de B. Robinson, Michael Golomb, L. M. Graves, Dr. Serge Lang, and Mr. John Todd.

The society is indebted to the ladies of the Department of Mathematics, who entertained the assembled mathematicians and their guests at tea on Thursday and Friday afternoons.

The Council met on Friday evening, April 30, 1954.

The Secretary announced the election of the following thirty-six persons to ordinary membership in the Society:

Mr. Louis Richard Bragg, University of Wisconsin;
Professor Martin Dudley Burrow, McGill University;
Mr. Angelo Anthony Caparaso, New York University;
Mr. Joseph Biggi Chiccarelli, Fordham University;
Mr. Bernard H. Chovitz, Army Map Service, Washington, D. C.;
Mr. John Francis Daly, St. Louis University;
Mother Charlotte Anne Dames, Barat College;
Mr. Jesus Gil de Lamadrid, University of Michigan;
Dr. Eduardo Hernán del Busto, Universidad Nacional de La Plata, Eva Perón, Argentina;
Mr. George Welva Fairchild, Bendix Computer Division, Los Angeles, California;
Mr. Herman Paul Friedman, Reeves Instrument Corporation, New York, New York;
Mr. James Scott Hanna, Jr., Southwestern Engineering and Equipment Company, Dallas, Texas;
Dr. Heinz G. Helfenstein, University of Alberta;
Mr. Manfred Kochen, Columbia University;
Mr. Arthur John Leino, University of California, Berkeley;
Mrs. Helen White Lindley, Tulane University;
Dr. Viktors Linis, University of Saskatchewan;
Mrs. Ruth Johnson MacKichan, University of North Dakota;
Mr. Henrik Herman Martens, Consolidated Edison Company of New York, Inc.;
Mr. Robert M. Meisel, New York University;
Professor Earle Frederick Myers, University of Pittsburgh;
Sister Miriam Patrick, St. Mary's College, Notre Dame, Indiana;
Dr. Aubrey Hampton Payne, Aberdeen Proving Ground, Aberdeen, Maryland;
Mr. George Owen Peters, Aircraft Armaments Incorporated, Baltimore, Maryland;
Mr. Samuel I. Plotnick, Mathematics Research Incorporated, State College, Pennsylvania;

Mr. Donald Clayton Rose, University of Kentucky;
Dr. Joseph William Siry, Naval Research Laboratory, Washington, D. C.;
Mr. Elmo J. Stewart, Bendix Computer, Bendix Aviation Corp., Los Angeles, California;
Professor Tsuneo Suguri, Kyusyu University, Fukuoka, Japan;
Professor Robert F. Tidd, Canisius College;
Professor Yasuro Tomonaga, Utsunomiya University, Utsunomiya, Japan;
Mr. Henry Snowden Valk, George Washington University;
Dr. Guido Leopold Weiss, University of Chicago;
Mr. John Sheldon Youtcheff, General Electric Advanced Electronic Center, Ithaca, New York;
Mr. Wilson Miles Zaring, University of Kentucky;
Mr. Jack Ira Zektzer, University of Wisconsin;

It was reported that the following forty-seven persons had been elected to membership on nomination of institutional members as indicated:

University of Florida: Mr. Thomas Roscoe Horton.

Harvard University: Mr. Marshall Leonard Freimer, Dr. Dieter Gaier, Professor Albert Haertlein, Mr. James Brown Herreshoff, Mr. Frank Albert Raymond, Mr. Eugene R. Rodemich, Mr. Richard Steven Varga, and Mr. Tai Tsun Wu.

State University of Iowa: Mr. Frank Wylie Anderson.

Kenyon College: Mr. David Ryeburn.

University of Maryland: Mr. James Hill, Mr. Robert Hanson Moore, and Mr. George Norman Trytten.

University of Michigan: Mr. William Sherwood Bicknell.

University of Missouri: Mr. Charles David Gorman.

Oklahoma Agricultural and Mechanical College: Miss Margaret Ann Reiff, Mr. Tetsundo Sekiguchi, and Mr. Albert William Wortham.

Oregon State College: Miss Elvy Lennea Fredrickson.

Princeton University: Mr. Harvey James Arnold, Mr. Julian Brody, Mr. Bradley Dean Bucher, Mr. I. Thomas Cundiff, Jr., Mr. Eric Yngve Domar, Mr. Nathaniel Roy Goodman, Mr. David Kent Harrison, Mr. Robert Hermann, Mr. Joseph John Kohn, Mr. Henry Pratt McKean, Jr., Mr. Pinchas Mendelson, Mr. Peter Gerald Moore, Mr. Lionel MacLean Noel, Mr. Paul Emery Thomas, Mr. Hale Freeman Trotter, Mr. Alan John Weir, and Mr. John William Woll, Jr.

University of Texas: Mr. Theodore Parker Higgins, Mr. William Andrew Holley, Mr. Charles Albert Nicol, Mr. G. P. Owen, Jr., Mr. Leon Bruce Treybig, and Mr. Don Harrell Tucker.

Tulane University: Mr. James H. Case and Mr. Erwin Stuart Krule.

Vanderbilt University: Mr. Howard Leroy Rolf.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: Deutsche Mathematiker Vereinigung: Mr. Erik Sparre Andersen, Institute of Copenhagen, Dr. Alexander Peyerimhoff, University of Cincinnati, and Professor Wolfgang B. Jurkat, University of Cincinnati; Finnish Mathematical Society: Dr. Olli Erkki Lehto, University of Helsinki; London Mathematical

Society: Dr. James Edmund Gibbs, Carleton College, Ottawa 1, Ontario, Canada, and Mr. Kenneth Povey, University of Liverpool; Société Mathématique de France: Professor Jacques Deny, University of Strasbourg, and Dr. Mark Wilmet, 1 Place Street, Thomas d'Aquin, Paris 7, France; Svenska Matematikersamfundet: Mr. Hans Ivar Riesel, Matematiska Institutet, Stockholm, Sweden, and Mr. Lars Erik Zachrisson, Research Institute of National Defense, Stockholm 5, Sweden.

Goucher College, Baltimore, Maryland, was elected to institutional membership.

The following appointments by the President were reported: as a Committee to Nominate Officers and Members of the Council for 1955: C. B. Morrey, Chairman, R. H. Bing, Richard Brauer, A. W. Tucker, and A. D. Wallace; as a Committee to Nominate an Executive Director: E. G. Begle, J. R. Kline, A. E. Meder, Jr., and G. T. Whyburn; as a Committee to Study the Programs of Society Meetings: W. M. Whyburn, Chairman, E. G. Begle, R. P. Dilworth, P. R. Halmos, and J. L. Kelley; as a Committee to Recommend Policy on Reprinting of Books: W. T. Martin, Chairman, Einar Hille, and R. J. Walker; as a Committee on Subsidies to Journals: Professor A. E. Meder, Jr., Chairman, E. G. Begle, Saunders MacLane, and A. W. Tucker; as a Committee on the Columbia University Bicentennial: E. G. Begle, L. W. Cohen, B. P. Gill, and H. M. MacNeille; as a Committee on Arrangements for the Meeting to be held at the University of Alabama, November 26–27, 1954: Professors F. A. Lewis, Chairman, J. H. Hornback, F. W. Kokomoor, J. H. Roberts, C. L. Seebeck, Jr., and H. S. Thurston; as a Committee on Arrangements for the Annual Meeting to be held at the University of Pittsburgh, December 27–29, 1954: J. C. Knipp, Chairman, I. Barsotti, J. L. Blumberg, A. M. Bryson, L. W. Cohen, H. M. Gehman, George Laush, Norman Levine, E. F. Myers, J. S. Taylor, and Jean Teats; as a Program Committee for the Summer Institute of 1954: Salomon Bochner, Chairman, D. C. Spencer, Alternate Chairman, Lipman Bers, N. S. Hawley, and Oscar Zariski.

The Secretary reported that the following persons have accepted invitations to deliver hour addresses during 1954: F. I. Mautner, New York City, April 23–24, 1954; Harry Pollard, New York City, April 23–24, 1954; S. C. Kleene, Chicago, Illinois, April 30–May 1, 1954; V. L. Klee, Portland, Oregon, June 19, 1954; Richard Bellman, Laramie, Wyoming, Summer Meeting 1954; Edwin Hewitt, Laramie, Wyoming, Summer Meeting 1954; R. D. James, Laramie, Wyoming, Summer Meeting 1954; Ralph Phillips, Laramie, Wyoming, Summer

Meeting 1954; Vaclav Hlavatý, Iowa City, Iowa, November 26–27, 1954; and Edmund Pinney, Los Angeles, November 27, 1954.

The following items were reported for the information of the Council: selection of W. T. Martin as Managing Editor of the Bulletin Editorial Committee for 1954; A. C. Schaeffer as Managing Editor of the Proceedings Editorial Committee for 1954; J. L. Doob as Managing Editor of the Transactions and Memoirs Editorial Committee for 1954; R. J. Walker as Chairman of the Mathematical Surveys Editorial Committee for 1954; Einar Hille as Chairman of the Mathematical Reviews Editorial Committee for 1954; Deane Montgomery as Chairman of the Colloquium Editorial Committee for 1954; C. J. Rees as Chairman of the Committee on Printing and Publishing for 1954.

It was reported to the Council that the Trustees had approved a recommendation of the Council that reduced dues be authorized for members of the Society serving as enlisted men in the armed forces of the United States or Canada. Dues have been fixed at one dollar per year for such individuals. Members taking advantage of these reduced dues will continue to receive all the privileges of membership in the Society, including the Bulletin, Proceedings, and Notices. In order to obtain this privilege of reduced dues, a member must be in good standing at the time he requests the privilege and, in particular, must have discharged any past dues and other financial obligations to the Society. This privilege is normally available for a period of two years, but extensions may be made at the discretion of the Secretary.

It was reported that Dr. Paul Erdös has accepted an invitation to be the Society's Visiting Lecturer during the academic year 1955–1956.

It was reported that the Society for Industrial and Applied Mathematics will meet in conjunction with the Mathematical Association of America, the Association for Symbolic Logic, and the Society at the time of the Annual Meeting of 1954 in Pittsburgh.

The following actions taken by mail vote of the Council were reported: The election of Professors J. L. Doob and G. B. Price to serve as members of the Executive Committee of the Council for a period of two years beginning January 1, 1954; and the election of Professor Leo Zippin to the Policy Committee for Mathematics for a period of four years beginning January 1, 1954.

The Council set meetings at the University of California, Los Angeles, November 27, 1954, and at Brooklyn Polytechnic Institute on April 15–16, 1955.

The Council voted to co-sponsor with the Office of Ordnance Research a Symposium in Applied Mathematics in conjunction with the April 1955 meeting in New York. The topic of this Symposium will be *Mathematical probability and its applications.*

The Council voted to approve the following substitutions on the Editorial Committee for the Transactions and Memoirs: Leo Zippin for Herbert Busemann for the academic year 1954–1955; M. M. Day for J. L. Doob for the period from June through November 1954; O. F. G. Schilling for Saunders MacLane for the summer of 1954.

The Council approved amendments to the by-laws to create a new class of members, namely corporate members.

The Council voted that lists of names of members attending Society meetings be omitted from the reports of meetings published in the Bulletin.

Abstracts of the papers presented follow below. Abstracts whose numbers are followed by the letter "*t*" were presented by title. In the case of joint papers, (p) following one of the authors' names indicates the one who actually read the paper. Mr. Marsh was introduced by Professor B. W. Jones and Dr. Livesay by Professor A. J. Lohwater.

ALGEBRA AND THEORY OF NUMBERS

455. J. L. Brenner: *A bound for a determinant with dominant principal diagonal.*

Let $A=(a_{ij})$ be an $n\times n$ matrix in which the relations $|a_{kk}|>\sum'|a_{kj}|$ hold $(k=1, 2, \cdots, n)$. The following bound is obtained by an elementary inductive proof: $|\det A|\geqq|a_{11}|\prod_{k=2}^{n}(|a_{kk}|-m_k+L_k)$. Here, m_k is $\sum_{j<k}|a_{kj}|$ and L_k is $R_k+|a_{k1}/a_{11}|\cdot\sum_{j>k}|a_{1j}|$, where R_k is a certain sum of absolute values of minor determinants of A; a corollary bound is obtained by replacing R_k by 0. The bound of Price (Proc. Amer. Math. Soc. vol. 2, p. 492) is the corollary bound obtained by replacing all of L_k by 0. (Received December 17, 1953.)

456*t*. J. L. Brenner: *Linear recurrence relations.*

Corresponding to the arbitrary linear recurrence relation (over rational integers) $u_{n+1}=a+\sum_0^{k-1}\alpha_t u_{n-t}$, homogeneous or not, a matrix B is defined such that B^{n+1} has $(0, 1, u_{n-1}, \cdots, u_{n+k-2})$ for a first row. A theorem is deduced concerning the order of the recurrence modulo a prime. (Received January 18, 1954.)

457*t*. J. L. Brenner: *The factorization of orthogonal matrices.*

A matrix A is orthogonal if the product AA^T is the identity. If the field of coefficients has more than two elements, such a matrix is a product of plane rotations T diag $(R, I)T^{-1}$, where T is a permutaton matrix and R has dimension 2, $RR^T=I$. (Received January 18, 1954.)

458. Sarvadaman Chowla and W. E. Briggs (p): *On discriminants of binary quadratic forms with a single class in each genus.*

Consider the classes of positive, primitive binary quadratic forms $ax^2+bxy+cy^2$ of discriminant $d=b^2-4ac<0$ and let $\Delta=-d$. There are 101 known values of Δ such that $d=-\Delta$ is a discriminant which has a single class in each genus. The largest of these values is 7392 and Swift (Bull. Amer. Math. Soc. vol. 54 (1948) pp. 560–561) has shown that there are no more up to 10^7. It is also known that there is only a finite number of discriminants of this type. Using the L-series $L_k(s)=\sum_1^\infty \chi(n)n^{-s}$, where χ is a real nonprincipal character modulo k and $R(s)>0$, and some facts about the number of genera into which the classes are divided, it is proved that: (1) There is at most one fundamental discriminant with a single class in each genus with $\Delta>10^{60}$. (2) If $L_k(53/54)\geqq 0$, $k>10^{14}$, then there are no such discriminants with $\Delta>10^{14}$. The first result is the counterpart of the result of Heilbronn and Linfoot (Quart. J. Math. vol. 5 (1934) pp. 293–301) which says that there is at most one fundamental discriminant with class-number one for $\Delta>5\cdot 10^8$. (Received February 22, 1954.)

459*t*. W. F. Darsow: *On certain division algebras.*

Let G_n be the group under symmetric difference of all subsets of the set $(1, \cdots, n)$ of the first n positive integers; and let A_n be the linear space of all functions on G_n to the field R of reals. For a function w on the cartesian product $G_n\times G_n$ to R a weighted convolution $f*g$ is defined for all f, g in A_n by $(f*g)(X)=\sum_Y f(Y)g(Y+X)w(Y, Y+X)$ with respect to which A_n is a linear algebra (not necessarily associative) over R of order 2^n. Two proper choices of w yield the algebra of Clifford numbers and Grassmann's algebra for each n. For $n=1, 2, 3$, appropriate choices of w yield the complex numbers, the quaternions, and the Cayley numbers; and a variety of other division algebras exist. However, as is not surprising, for $n>3$ no choice of w yields a division algebra A_n over any ordered field R. (Received February 17, 1954.)

460*t*. W. E. Deskins: *A radical for near-rings.*

In a recent paper (*Simple and semisimple near-rings*, Proc. Amer. Math. Soc. vol. 4 (1953) pp. 772–785) D. W. Blackett developed a structure theory for semisimple near-rings, analogous to the structure theory for semisimple rings (with DCC). Let N be a near-ring and let the right modules of N satisfy the DCC. If R is a minimal right module of N, define $r(R)$ to be the (two-sided) ideal of N consisting of all the elements of N which annihilate R from the right. Then K, the intersection of all $r(R)$, is an ideal of N such that $N-K$ is semisimple. If N is a ring, then K coincides with the usual nilpotent radical. (Received January 25, 1954.)

461. W. E. Deskins: *On the homomorphisms of an algebra onto a Frobenius algebra.*

A linear associative algebra A which possesses a nonsingular parastrophic matrix is called a Frobenius algebra. Such algebras have been studied by a number of mathematicians, notably R. Brauer, C. C. MacDuffee, T. Nakayama, and C. Nesbitt. Simple examples may be given to show that in general a singular parastrophic matrix of rank m does not determine a homomorphism of A onto a Frobenius algebra of order m. However, the following result is proved in this paper. Theorem. If Q is a parastrophic matrix of rank m of the algebra A of order n, if Q is congruent to a corner matrix, and if Q is associated with an element of a subalgebra of A which has either a left or right identity element, then Q determines a homomorphism of A onto a Frobenius algebra of order m. Furthermore, every homomorphism of A onto a Frobenius algebra is determined by such a parastrophic matrix. (Received January 25, 1954.)

462. Leonard Gillman (p) and Melvin Henriksen: *On rings of continuous functions in which every finitely generated ideal is principal.*

Let X be a completely regular Hausdorff space, $C=C(X)$ the ring of all real-valued continuous functions on X. X is (i) P, (ii) P', (iii) P'', if for every $p\in X$ and $f\in C, f(p)=0$ implies that there is a deleted neighborhood U of p such that (i) $f(U)=0$ (see Bull. Amer. Math. Soc. Abstract 59-4-445), (ii) $f(U)=0$ *or* $f(U)>0$ *or* $f(U)<0$, (iii) $f(U)\geqq 0$ *or* $f(U)\leqq 0$ (respectively). Clearly, P implies P' implies P''. Every extremally disconnected (ED) space (i.e., the closure of every open set is open) is P''. *Results.* (1) If (a) for all $f\in C$, f is a multiple of $|f|$, *or* (b) for all $f\in C$, the ideal $(f, |f|)$ is principal, *or* (c) every finitely generated ideal of C is principal, then X is P''. (2) If X is P' *or* ED *or* normal P'', then (a), (b), *and* (c) hold. (3) If X is P' *or* ED, then (d) for every $f\in C$, f is a *unit* multiple of $|f|$, *and* (e) for every $p\in C$, the ideal of all $f\in C$ that vanish on a neighborhood U_f of p is prime; if (e), then X is P''. (4) Examples are given of spaces that are P' but not P, ED but not P', P' but not ED. (Received March 10, 1954.)

463. Leonard Gillman and Melvin Henriksen (p): *Concerning adequate rings and elementary divisor rings.*

A commutative ring S with unit is *adequate* if (A_1) every finitely generated idea[1] is principal, *and* (A_2) for all a, b, with $a\neq 0$, there exist r, s, with $a=rs$, $(r, b)=1$, and $(s', b)\neq 1$ for every non-unit divisor s' of s (Helmer, Bull. Amer. Math. Soc. vol. 49 (1943) pp. 225–236; Kaplansky, Trans. Amer. Math. Soc. vol. 66 (1949) pp. 464–491). The problem has been open as to whether there exist S satisfying A_1 but not A_2. *Results* (see preceding abstract). (1) $C(X)$ satisfies A_2 *iff* $C(X)$ is adequate *iff* X is P (hence *iff* $C(X)$ is regular—see Bull. Amer. Math. Soc. Abstract 59-4-446). (2) If X is P' *or* ED *or* normal P'', but not P, then $C(X)$ satisfies A_1 (but not A_2). S is an *elementary divisor* ring if for every (finite) matrix M, there exist unimodular matrices K, L such that KML is diagonal. Generalizing a result of Helmer, Kaplansky has shown (loc. cit. Theorem 5.3) that if S is adequate and its zero-divisors are in the radical, then S is an elementary divisor ring. Clearly, for any X, the zero-divisors of $C(X)$ are not in the radical. (3) If X is P' *or* ED, but not P, then $C(X)$ is an elementary divisor ring (satisfying A_1 but not A_2). (Received March 10, 1954.)

464. H. E. Goheen: *The Wedderburn theorem.*

Using the lemma that any invariant division subring of a skew field must be in its center (Hua, Proc. Nat. Acad. Sci. U.S.A. vol. 35 (1949) p. 533) and the lemma that any finite group must be Abelian if the normalizers of all its Abelian subgroups are their centralisers (Zassenhaus, Proc. Glasgow Math. Assoc. vol. 1 (1952) p. 53), the author provides a simplified version of Zassenhaus' proof of the theorem of Wedderburn on the non-existence of finite skew fields. (Received March 8, 1954.)

465. Franklin Haimo: *Power-type endomorphisms of some class 2 groups.*

An endomorphism ϕ of a group G is said to be of power-type if $\phi(x)\equiv x^{n(\phi)}$ mod Q for every $x\in G$, where Q is the derivative of G. If G is non-abelian of class 2, and if G/Q is an elementary p-group, then N, the set of endomorphisms of G into Q, is a prime ideal in the ring R of all power-type endomorphisms of G for which $p\mid n(\phi)$. If p is odd, then such a group G has precisely p endomorphisms of the form $\phi(x)=x^n\in Z$, where Z is the center of G. (Received March 10, 1954.)

466. I. N. Herstein: *On the Lie ring of a simple ring.*

If A is any associative ring one can transform into a Lie ring by defining a new product, the Lie product, by means of $[a, b] = ab - ba$ for all $a, b \in A$. It has been conjectured by Kaplansky that if A is a simple associative ring of characteristic not 2, then the only Lie ideals of $[A, A]$ are in the center of A. This conjecture is proved in this paper as long as A is also of characteristic different from 3. (Received March 13, 1954.)

467*t*. W. G. Leavitt: *Finite-dimensional modules.*

This paper considers the problem of dimensionality for a finitely based module M over a ring K with unit. Such a module is said to be *finite-dimensional* if, for some integer n, (a) every basis has length n, and (b) M contains no independent set of length more than n. Examples are known of finitely based modules satisfying both conditions, condition (a) alone, or neither condition. The main results proved are: (1) a finitely based module containing no infinite independent set is finite-dimensional; (2) if a module contains two bases of different lengths, then no upper bound exists for the length of a basis; (3) a sufficient condition that every finitely based module over K be finite-dimensional is that there exist an integer m such that every set $\{a_1 \cdots a_m\} \subset K$ is related by $\sum_1^m a_i x_i = 0$, with either (i) at least one x_i is not a left zero divisor or (ii) at least one $a_i x_i \neq 0$; (4) If K is imbeddable in a ring admitting the descending chain condition, then any finitely based module over K satisfies condition (a). (Received March 1, 1954.)

468. D. C. B. Marsh: *An investigation of the number of classes in the genus of certain indefinite ternary quadratic forms.* Preliminary report.

The greatest common divisors of the two-rowed minors of a symmetric matrix and its reciprocal matrix are invariant under unimodular transformations; necessary and sufficient conditions that the removal of a factor from such an invariant leave the class number of the matrix's form unaltered are found in terms of automorphs of the matrix. Application of a simplified expression for these automorphs (Duke Math. J., forthcoming) and a study of sufficient conditions that integral automorphs exist with elements of specified congruential nature enabled one to replace the form by another of reduced invariants for consideration in determining the class number. Employing an early theorem by Adolf Meyer (J. Reine Angew. Math. vol. 108 (1891) p. 139) and restricting consideration to forms where neither invariant is divisible by four, one developed an algorithm whereby the invariants might be further reduced without altering the class number. Finally, group theoretic considerations showed precisely how the class number was altered by a final transition to a form in a genus of one class. (Received February 23, 1954.)

469*t*. H. T. Muhly: *A remark on a paper of P. Samuel.*

It is pointed out that in the case of the ideals of an integral domain, the notion of asymptotic equivalence introduced by P. Samuel (Ann. of Math. vol. 56 (1952) pp. 11–21) coincides with the notion of valuation equivalence (b-equivalence) introduced by W. Krull (Math. Zeit. vol. 41 (1936) pp. 545–577). (Received February 26, 1954.)

470t. R. J. Nunke: *A relation between the functors* Ext *and* Tor. Preliminary report.

Definitions of the functors Ext and Tor may be found in the *Foundations of algebraic topology*, Princeton University Press, 1952, by S. Eilenberg and N. Steenrod. For abelian groups A, B, C a natural isomorphism is given between Ext (Tor(A, B), C) and Ext (A, Ext (B, C)). The following theorems are consequences: (1) For any prime p, Ext (A, C) is divisible by p if and only if either A has no elements of order p or C is divisible by p; (2) if all elements of A are of finite order, then Ext (A, C) has no nontrivial divisible subgroups; (3) Ext (A, C) is the direct sum of Ext ($T(A)$, C) and a homomorphic image of Ext ($A/T(A)$, C), where $T(A)$ is the torsion subgroup of A; (4) Ext (A, C) is never free. (Received March 10, 1954.)

471t. Alex Rosenberg: *Finite-dimensional simple subalgebras of the ring of all continuous linear transformations.*

Let M, N be dual spaces over a division ring D with center Z. Let $A = L(M, N)$, the ring of all continuous linear transformations, and let $S = F(M, N)$ be its socle. Let B be a simple finite-dimensional subalgebra of A over Z containing the unit of A, and let C' denote the A-centralizer of a subring C of A. Then B' is again a ring of the form $L(R, T)$, R, T dual spaces over some division ring, and the socle, U, of $B' \subset S$ and $MU = M$, $U^*N = N$. In case $D = Z$, an algebraically closed field, it is further shown that $M = \sum_1^n M_i$, $N = \sum_1^n N_i$, $(M_i, N_j) = 0$, M_i, N_i dual spaces over Z and $B' \cong L(M_i, N_i)$. In this case $B'' = B$ and one gets the usual one-to-one correspondence between finite-dimensional simple subalgebras of A containing the unit and those subrings of A which are of the form $L(R, T)$ with socle $U \subset S$ and $MU = M$, $U^*N = N$. Furthermore if σ is an isomorphism of B into A, σ can be extended to an inner automorphism of A if and only if $B' \cong (B^\sigma)'$. These results generalize those obtained in the case dim M = dim $N \aleph_0$, Bull. Amer. Math. Soc. Abstract 59-5-532. (Received March 11, 1954.)

472t. A. R. Schweitzer: *Grassmann's extensive algebra and modern number theory.*

In this paper the quadratic form is interpreted as a link connecting Grassmann's algebra with Minkowski's *Geometrie der Zahlen* (Leipzig and Berlin, 1910). In Grassmann's theory a quadratic form is obtained as the inner square of a linear vector. Minkowski's theory is viewed primarily as an arithmetic of quadratic forms generalizing certain researches of Hermite. Minkowski succeeds in arriving at interesting results in number theory including algebraic numbers (loc. cit., p. 123) and references to Gauss (p. 166). In the course of his investigation Minkowski uses concepts pertaining to geometry (pp. v, vi), analysis (pp. 1, 37), and point set theory (pp. iv, 5, 18, 36, 201). (Received March 8, 1954.)

473. Leonard Tornheim: *Approximation to irrationals by classes of rational numbers.*

Let $(r, s, m) = 1$. Then for every irrational number x there exist infinitely many rational numbers a/b in lowest terms with $a \equiv rt$, $b \equiv st$ (mod m) such that $|x - a/b| < km/b^2$ if and only if when $m = p^e$ (p an odd prime) then $k \geqq 1/5^{1/2}$, or when $m = 2^e$ ($e \geqq 1$) then $k \geqq 1/2$, or when $m = p^e q^f$, where p, q are distinct primes and $e, f \geqq 1$, then

$k \geqq 1$. These results for the case $m=2$ have been proved by W. T. Scott, R. M. Robinson, A. Oppenheim, and L. Kuipers and B. Meulenbeld. (Received March 9, 1954.)

474. J. H. Walter: *Automorphisms of the projective unitary groups.*

The projective unitary group $PU_n(K, f)$ can be defined as the factor group $U_n(K, f)/C_n$, where $U_n(K, f)$ is the unitary group acting on a right vector space E of dimension n over a sfield K which possesses an involutive anti-automorphism J, C_n is its center, and f is a nondegenerate hermitian sesquilinear form defined on E with respect to J. If $n \geqq 7$ and $n \neq 8$ and K is not of characteristic 2 and is not the finite field $GF(3)$, it is shown that every automorphism of $PU_n(K, f)$ is induced by an automorphism of $U_n(K, f)$. The crucial step is to characterize the extremal involutions of $PU_n(K, f)$, which is done by using the commuting sets of involutions introduced by Rickart [Amer. J. Math. vol. 72 (1950) pp. 451–464 and vol. 73 (1951) pp. 697–716]. This work was sponsored in part by the OOR, Contract DA-20-018-ORD-13281. (Received March 10, 1954.)

475. L. M. Weiner: *J-simple algebras.*

An algebra A is called J-simple if the attached algebra $A^{(+)}$ is a simple Jordan algebra. This paper considers power associative algebras A which are both J-simple and Lie admissible. Albert has shown (Summa Braziliensis Mathematicae vol. 5 (1951) pp. 183–194) that when $A^{(+)}$ is a central simple Jordan algebra of matrices over an algebraically closed field, there exists a linear transformation T such that multiplication x, y in A is given in terms of the matric multiplication by $x, y = (xy+yx)/2+(xy-yx)T$. The present paper shows that when $A^{(+)}$ is the algebra of all matrices of order $t \geqq 3$ with multiplication given by $(x, y)=(xy+yx)/2$, and T is nonsingular, there exists a scalar α and a linear function $\lambda(B)$ such that $BT=\alpha B + \lambda(B)I$ for each matrix B of trace zero. If there exists a scalar extension K of the base field such that $A_K^{(+)}$ is the set of all t by t Hermitian matrices with elements in a central simple alternative algebra C of order $r=1$, 4, or 8, and $t \geqq 3$ for $r=1$, and $t \geqq 2$ for $r=4$ or 8, then T must be singular. (Received March 3, 1954.)

476. L. R. Wilcox (p) and R. J. Mihalek: *A generalization of independence in semi-modular lattices.*

Let L be a lattice in which modularity is symmetric, i.e., if $(b, c)M$ means $a \leqq c$ implies $(a \cup b) \cap c = a \cup (b \cap c)$, then $(b, c)M$ implies $(c, b)M$. If $a, b, c \in L$, define a relation θ so that $(a, b, c)\theta$ means $(a \cup b) \cap (b \cup c) = b$, $(a \cup b, b \cup c)M$, together with the two other conditions obtained by cyclic permutation of a, b, c. This definition may be extended in an obvious way to n terms. If $(a, b)M$, then $(a, b, c)\theta$ reduces to $c \cap (a \cup b) = a \cap b$, $(c, a \cup b)M$, whence $(a, b, c)\perp$ in the sublattice of all $x \geqq a \cap b$ (see L. R. Wilcox, *Modularity in the theory of lattices*, Ann. of Math. vol. 40, pp. 491–496, where the properties of $\perp$ are developed). Thus θ is a generalization of ordinary independence $\perp$. In the present paper, formal properties of θ are obtained, most of which are identical to or modifications of properties of $\perp$. The present generalization of the theory of independence has been developed by the authors as a tool in the imbedding of certain Birkhoff lattices into modular lattices. (Received March 11, 1954.)

477. Ti Yen: *Trace on AW*-algebras.*

An AW^*-algebra is [Kaplansky, Ann. of Math. vol. 53 (1951) pp. 235–249] a C^*-algebra satisfying the following two conditions: (a) In the set of projections any collection of orthogonal projections has a least upper bound. (b) Any maximal commutative self-adjoint subalgebra is generated by its projections. Let A be an AW^*-algebra of type II_1, Z its center. Denote by (P) and (P″) respectively the following conditions: (P): There exists a total set of completely additive positive functionals on A. (P″): There exists a completely additive positive linear transformation from A to Z which is identity on Z. (1) A has a trace if it satisfies (P) or (P″). (2) Any finite AW^*-algebra A satisfying (P) has a representation as an AW^*-subalgebra of some W^*-algebra. Its weak closure is also finite. A is already weakly closed if it is commutative. The underlying Hilbert space is one induced by the functionals in (P) and the trace. As a corollary of (2) and a theorem of Kaplansky [Ann. of Math. vol. 56 (1952) p. 470], an AW^*-subalgebra of type I of a W^*-algebra is weakly closed. (Received March 8, 1954.)

478*t*. Daniel Zelinsky: *Every linear transformation is a sum of nonsingular ones.*

Let α be a linear transformation on a vector space over a division ring. Then $\alpha=\beta+\gamma$ where β and γ are nonsingular linear transformations (automorphisms) of the same space—except when the vector space contains only two elements and $\alpha\neq 0$. This theorem is well known if the vector space is finite-dimensional; one possible proof constructs β as any isomorphism of the kernel of α onto a complement of the range of α, coupled with a suitable isomorphism of a complement of the kernel onto the range. The present proof provides the modifications necessitated in the infinite-dimensional case by the fact that a complement of the range need no longer be isomorphic to the kernel. (Received February 26, 1954.)

479. J. L. Zemmer: *Some remarks on p-rings and their Boolean geometry.*

The idempotents in a commutative ring R form a Boolean ring with multiplication that of R and addition defined by $x\oplus y=x+y-2xy$. The following results are obtained: (i) if B is a Boolean ring with identity, p a fixed prime, then there exists a unique p-ring R with identity, with B as its Boolean ring of idempotents; (ii) the automorphism group of a p-ring with identity is isomorphic to the automorphism group of its Boolean ring of idempotents. The remaining results in this paper extend some of the theorems of Ellis (*Autometrized Boolean algebras*, I, Canadian Journal of Mathematics vol. 3 (1951) pp. 87–93; and *Autometrized Boolean algebras*, II, ibid. pp. 145–147). Denote by $\mathfrak{B}$ the Boolean algebra associated with the Boolean ring of idempotents of the p-ring R with identity. It is shown that R is a "distance" space with respect to the function $d(x, y)=(x-y)^{p-1}$, defined on R to $\mathfrak{B}$. The group of isometries of this "distance" space is determined, and finally it is shown that the space has the property of free mobility if and only if $\mathfrak{B}$ is a complete Boolean algebra. (Received March 10, 1954.)

Analysis

480*t*. H. A. Antosiewicz: *On nonlinear differential equations of the second order with integrable forcing term.*

Differential equations of the form $\ddot{x}+\phi(x, \dot{x})\dot{x}+h(x)=e(t)$ are considered where $\int^\infty |e(t)|dt<\infty$ and $\phi(x, \dot{x})$, $h(x)$ are sufficiently smooth so as to guarantee the existence of a unique solution for all $t\geqq 0$. It is proved that if $\phi(x, \dot{x})\geqq 0$ for all x, $\dot{x}$, $H(x)=\int_0^x h(u)du>0$ for $x\neq 0$, $H(x)\rightarrow\infty$ with $|x|$, then every solution satisfies $|x(t)|<C_1$, $|\dot{x}(t)|<C_2$ as $t\rightarrow\infty$. If $\phi(x, \dot{x})=f(x)+g(x)\dot{x}$, the same boundedness result holds pro-

vided $a(x)=\exp\left(\int_0^x g(u)du\right)\leqq\alpha$ $(\alpha\geqq 1)$ for all x, $h(x)\int_0^x a(u)f(u)du\geqq 0$ and $H^*(x)=\int_0^x a^2(u)h(u)du>0$ for $x\neq 0$, $H^*(x)\rightarrow\infty$ with $|x|$. These results are a considerable generalization of a result by G. Sestini (Atti 4° Congresso Un. Mat. It., Messina, 1951). (Received March 4, 1954.)

481t. W. B. Caton: *On the zeros of certain Laplace integrals.* I.

Let $F(t)$ defined on $(0, \infty)$ be non-negative, non-increasing, and positive in a set of positive measure. Let $F(t)\in L(0, R)$, for every $R>0$. The Laplace transform $f=\mathcal{L}\{F\}$ exists and has no zeros in $R(s)>0$. If $f_i=\mathcal{L}\{F_i\}$, $i=1, 2, 3, \cdots$, where each F_i has the above properties and $c_i\geqq 0$ are real numbers, then $\sum_{i=1}^n c_if_i$ has no zeros in $R(S)>0$. Finally, $\sum_{i=0}^\infty c_if_i$ will have no zeros in $R(s)>0$ if the sequence $\{c_i\}$ of non-negative real numbers is chosen so that the series is uniformly convergent in any bounded closed domain in $R(s)>0$. Let F_i $(i=1, 2)$ have the properties mentioned above and assume that $F_i\in L^2(0, R)$ for every $R>0$. Then $\mathcal{L}\{F_1F_2\}$ exists and has no zeros in $R(s)>0$. In particular, $\mathcal{L}\{F_1F_2\}=f_1*f_2$ in case F_i $(i=1, 2)$ are also entire functions of order one and minimal type. (Received March 5, 1954.)

482. Lamberto Cesari: *Orbital stability.*

Given a system of ordinary differential equations $x_i'=f_i(x, t)$, $x=(x_1, \cdots, x_n)$, $i=1, 2, \cdots, n$, and a periodic solution (cycle) C: $x=x(t)$, $-\infty<t<+\infty$, in E_n, the question of the orbital stability of C can be discussed by using a function $V(x)$ of Liapounoff's type defined in a neighborhood U of C (tube). Then V could be for instance a definite positive quadratic form in a system of orthogonal local coordinates $y_2, y_3, \cdots, y_n$ in the hyperspace π normal to C at the point $x=x(t)$. For systems $x_i=f_i(x, t, \epsilon)$, weakly nonlinear (autonomous, or not) families $[C_\epsilon]$ of cycles C_ϵ can be determined, by using known methods, around some periodic solutions C_0 of the linear system $x_i=f_i(x, t, 0)$, $i=1, 2, \cdots, n$. Then functions V can be determined by means of various processes. Examples are discussed. (Received March 11, 1954.)

483t. W. L. Duren, Jr.: *Real measurement operators.*

In a set X which is linearly ordered, dense, and Dedekind complete, a second ordering, called a comparability relation, is introduced locally. For every point x in X it is assumed that there exists a neighborhood U_x of x and a comparability relation which is a total ordering of the pairs (y, z) in $U_x\times U_x$ and is related to the linear order of points x in a natural way. A complete measurement of a set $Y\subset X$ is a 1-1 mapping of Y onto itself which preserves comparability. It is proved that every complete measurement of an open interval in X consists of an element of a field of multiplication operators followed by an element in an abelian group of translations. Moreover the field of multiplication operators with its induced linear order is isomorphic to the real numbers. The comparability is a local property so that X does not necessarily possess the algebra of the reals. It is a linearly ordered real manifold. Applications in a theory of scientific measurement and particularly in econometrics are indicated. (Received March 11, 1954.)

484. W. F. Eberlein: *Theory of numerical integration.* I. Preliminary report.

Let $E(s)=(s+1)^{-1}\sum_1^n A_m e^{-\alpha_m s}$. It is shown that from the standpoint of operator theory the "best" asymmetric quadrature formulae for fixed n are obtained by choosing the $2n$ constants $(A_m)(\alpha_m)$ to minimize E in a suitable norm. For example, the "Laguerre procedure" for the interval (0, 1) amounts to setting $E^{(k)}(0)=0$ ($k=0, 1, \cdots, 2n-1$) and yields the "best one-sided" formula. (Received April 16, 1954.)

485. W. H. Fleming: *An example concerning the problem of least area with unrestricted topological type.*

The existence of a surface of least area with a given simple closed curve C in euclidean 3-space as boundary, and prescribed character of orientability and Euler characteristic, is well known (see R. Courant's book, *Dirichlet's principle*, and references cited there). The problem for unrestricted rather than fixed topological types is unsolved, however. An example is given of a simple closed rectifiable curve C in 3-space for which the solution of the problem with unrestricted types must be of infinite topological type. This paper was sponsored by the Office of Ordnance Research. (Received March 12, 1954.)

486. R. E. Fullerton: *An inequality for linear operators between L^p spaces.*

Let R, S be two sets with completely additive measures ϕ, γ defined over σ-rings $\mathfrak{R}(R)$, $\mathfrak{R}(S)$ respectively and assume that R and S are σ-finite. Let $L^p(R, \phi)$, $L^q(S, \gamma)$ be the L^p and L^q spaces defined over (R, ϕ), (S, γ), $1 \leqq p$, $q \leqq \infty$. A bounded linear operator T defined from L^p to L^q is known to be representable in the form $Tx = (d/de)\int_R K(e, t)x(t)d\phi$ where $K(e, t)$ is defined on $\mathfrak{R}(S)\times R$ and satisfies certain known conditions and where the symbol $d/de\ (F(e)=f(t))$ denotes the integrable function associated with the completely additive, absolutely continuous set function $F(e)$ by the Radon-Nikodym theorem. It is shown that the kernel $K(e, t)$ must satisfy the inequality $[q-\mathrm{var}_S(\int_R |K(e, t)|^{p'}d\phi)]^{1/p'}/\sup_{\{e_j\}}(\sum_{j=1}^{n} \gamma(e_j)^{(p'-1)(q-1)})^{1/qp'} \leqq \|T\|$, $(1/p)+(1/p')=1$, where $\{e_j\}$ varies over all finite disjoint families in $\mathfrak{R}(S)$ of finite nonzero measure. In particular, if $p\leqq q$, $\gamma(S)\leqq 1$, $[q-\mathrm{var}_S\ (\int_R |K(e, t)|^{p'}d\phi)]^{1/p'} \leqq \|T\|$.(Received March 10, 1954.)

487. R. A. Gambill and J. K. Hale (p): *Subharmonic and ultraharmonic solutions for weakly nonlinear systems.*

Consider the system of differential equations (1) $\dot{y}_j = \rho_j y_j + \epsilon q_j(y_1, \cdots, y_n, t; \epsilon)$, $j=1, 2, \cdots, n$, where $\rho_1, \cdots, \rho_n$ are complex numbers, ϵ is a small parameter, and each q_j is a holomorphic function of $y_1, \cdots, y_n, \epsilon$ for $|y_j| < A$, $|\epsilon| < \epsilon_0$, with coefficients periodic in t of period $2\pi/\omega$. This abstract refers to research toward the determination of subharmonics and ultraharmonics of system (1) by using a variant of the Poincaré method of successive approximations analogous to the variants considered for linear systems (1) [L. Cesari, Mem. Accad. Italia (6) vol. 11, pp. 634–692] and for autonomous nonlinear systems [J. K. Hale, Bull. Amer. Math. Soc. Abstract 60-1-114]. By using a convergent process which at the same time furnishes approximate solutions, the following conditions, among others, have been obtained. I. The numbers $\rho_j = ik_j\omega/m_j + O(\epsilon)$ can be determined so that there is a solution of (1) of the form (2) $y_j = y_{j0} + \epsilon w_j(t; \epsilon)$, $j=1, \cdots, n$, where $y_{j0} = a_j e^{ik_j\omega t/\omega m_j}$, k_j, m_j positive integers, a_j arbitrary complex constants, $|a_j| < A$, and $w_j(t; \epsilon)$ holomorphic in ϵ with coefficients of period $T = 2\pi m_1 \cdots m_n/\omega$. II. If $\rho_j = k_j\omega/m_j$ and if $S_j(a_1, \cdots, a_n) = \int_0^T q_j(y_{10}, \cdots, y_{n0}, t; 0)e^{-\rho_j t}dt$, then there is a periodic solution of (1) of the form (2) if $S_j(a_1^0, \cdots, a_n^0) = 0$, and the Jacobian $|\partial S_j(a_1^0, \cdots, a_n^0)/\partial a_k| \neq 0$. This condition on $S_j(a_1, \cdots, a_n)$ has been obtained [E. A. Coddington, N. Levinson, Annals of

Mathematics Studies, no. 29, pp. 19–35]. However, cases where the Jacobian is zero are discussed when system (1) arises from a real system of second order equations. (Received March 11, 1954.)

488t. R. P. Gosselin: *On the convergence behaviour of trigonometric interpolating polynomials.*

It is known that there exist continuous functions for which the sequence of trigonometric interpolating polynomials taken at various sets of fundamental points diverges almost everywhere or even everywhere (cf. e.g., J. Marcinkiewicz, *Sur la divergence des polynomes d'interpolation*, Acta Univ. Szeged. vol. 8 (1936–37) pp. 131–135). Fundamental points of the form $\alpha+2\pi i/(2n+1)$, $i=0, \pm 1, \pm 2, \cdots$, are considered. It is shown by refinements of the Marcinkiewicz example that for any number α_0, irrational with respect to π, there is a continuous function for which the sequence of interpolating polynomials corresponding to $\alpha=0$ diverges for $x\neq 0 \pmod{2\pi}$ while the sequence corresponding to $\alpha=\alpha_0$ converges uniformly. (Received March 8, 1954.)

489. L. M. Graves: *A generalization of the Riesz theory of completely continuous transformations.*

Let $T_c=E-cK$ be a family of linear transformations mapping a Banach space X into another such space Y, where E maps X onto the whole of Y (but *not* one-to-one), K is completely continuous, and c is a complex parameter. A *proper value* c is one for which $W_1=T_cX\neq Y$. The range W_1 of T_c is the orthogonal complement of the null space of the adjoint transformation T_c^*. As in the classical theory of Riesz, the set of proper values has no finite accumulation point. The spaces W_k defined inductively by $W_k=T_cE^{-1}W_{k-1}$ are closed linear subspaces of Y, and there exists a finite ν such that $W_{\nu+1}=W_\nu$. Each proper value c_0 has a positive integral order, defined in terms of the behavior of the adjoint transformation T_c^*, and this order is proved to equal the smallest ν for which $W_{\nu+1}=W_\nu$. A decomposition $K=K_1+K_2$ relative to a proper value c_0 is exhibited, where K_1 has only c_0 as a proper value, while K_2 has as proper values only the remaining proper values of K. Furthermore, $K_1=\sum K_{1j}$, where each K_{1j}^* has only one proper vector. (This work was done under a contract with the Office of Ordnance Research.) (Received March 9, 1954.)

490t. L. M. Graves: *Remarks on singular points of functional equations.*

Let $G(x, y)=0$ be an equation between two Banach space variables, having (0, 0) as an initial solution at which the partial differential $L(dx)=d_xG(0, 0; dx)$ (mapping X into X) fails to have an inverse. There may then be several continuous solutions $x=x(y)$ defined near $y=0$ and vanishing with y. Bartle (Trans. Amer. Math. Soc. vol. 75 (1953) pp. 366–384) was able to count the exact number of real solutions in cases when the null space of L is one-dimensional, with the help of Newton's polygon method. By an extension of the notion of Newton's polygon to equations involving several variables it is possible to count the exact number of real solutions when the null space of L is two-dimensional. As an illustration, the method is applied to a nonlinear elliptic partial differential equation. The theory is applicable to cases when the null space of L has more than two dimensions, but the computations become unmanageable. (This work was done under a contract with the Office of Ordnance Research.) (Received March 9, 1954.)

491t. I. I. Hirschman, Jr.: *Weighted quadratic norms for Legendre polynomials.*

Let $\omega_n(x)=[(n+1)/2]^{1/2}P_n(x)$, $n=0, 1, \cdots$, be the normalized Legendre polynomials and let $f(x)=\sum_0^\infty a_n\omega_n(x)$. Set $N_\alpha[f]^2=\int_{-1}^1(1-x^2)^\alpha|f(x)|^2dx$. It is established that if $0<\alpha<1/2$, then $A'(\alpha)\leqq N_\alpha[f]^{-2}\sum_{k>j}|a_j/(2j+1)^{1/2}-a_k/(2k+1)^{1/2}|^2 \cdot p(j, k)(2j+1)(2k+1)/|j-k|^{1+2\alpha}|j+k|\leqq A''(\alpha)$. Here $p(j, k)=1$ or 0 as j and k are of the same or different parity. Many properties of expansions in Legendre polynomials relative to the weighted quadratic norm $N_\alpha[f]$ can be deduced from this relation. (Received April 30, 1954.)

492. Meyer Jerison (p) and Gustave Rabson: *Induced homomorphisms of group algebras.*

Theorems announced earlier for compact zero-dimensional groups [Bull. Amer. Math. Soc. Abstract 59-4-467] are proved for a wider class of groups. Let G be a locally compact group which is the inverse limit of a sequence of groups $\{\Gamma_n\}$, and such that for each n, the kernel K_n of the homomorphism of G onto Γ_n is compact. If f is a locally integrable function on G, define $f_n(x)=\int_{K_n}f(xk)d\mu_n(k)$, where μ_n is the Haar measure in K_n, and $\mu_n(K_n)=1$. The sequence $\{f_n\}$ is a martingale, and consequently, $f_n(x)\rightarrow f(x)$ for almost every x, and if $f\in L^p$, $p\geqq 1$, then $f_n\in L^p$ and $f_n\rightarrow f$ in L^p. Moreover, if f is continuous at x_0, then $f_n(x_0)\rightarrow f(x_0)$, and if f is uniformly continuous on $S\subset G$, then $f_n\rightarrow f$ uniformly on S. As in the special case considered previously, f_n can be interpreted as a partial sum of the Fourier series of f. If G is a countable product of circles, then application of the results announced here leads to generalizations of some theorems of Jessen [Acta Math. vol. 63 (1934) pp. 249–323]. (Received March 9, 1954.)

493. E. R. Johnston: *Two theorems concerning the function spaces $H(C, p)$ and $\mathcal{L}^2_D$.* Preliminary report.

Let $D=I(C)$ denote the interior of a closed rectifiable Jordan curve C. Let $z=h(w)$ map $|w|<1$ conformally one to one onto D, $\zeta=h(0)$, $\Gamma_r\equiv\{z=h(re^{i\theta}), 0\leqq\theta\leqq 2\pi\}$, and $\Delta_r=I(\Gamma_r)$. In this paper the following two theorems are proved. (1) $\mathcal{L}^2_D\supset H(C, p)$ for $p\geqq 1$. If $f(z)$ is analytic in D, then $[\iint_{\Delta_r}|f|^2dxdy]^{1/2}\leqq(1/2^{1/2})\int_{\Gamma_r}|f||dz|$ for $0<r\leqq 1$ $(z=x+iy)$. (2) Let $\mathfrak{I}(\mathcal{L}^2_D)$ represent the class of all functions which are integrals of functions in $\mathcal{L}^2_D$. If $\int_0^{2\pi}|h(e^{i\theta})|^q d\theta<\infty$ for some $q>1$, then $H(C, p)\supset\mathfrak{I}(\mathcal{L}^2_D)$ for every $p>0$. If $F(z)=\int_\zeta^z f(t)dt$, $f(z)\in\mathcal{L}^2_D$, then $[\int_{\Gamma_r}|F|^p|dz|]^{1/p}\leqq K(C, p)[\iint_{\Delta_r}|f|^2dxdy]^{1/2}$ for $0<r\leqq 1$. In addition there exists $\omega>0$ (which depends on f) such that $\int_C\exp\omega|F||dz|<\infty$. (Received March 8, 1954.)

494. N. D. Kazarinoff and R. W. McKelvey (p): *Asymptotic solution of differential equations in a domain containing a regular singular point.*

This paper concerns the behavior of the solutions of the differential equation (A): $(z-z_0)^2d^2u/dz^2+\lambda(z-z_0)P_1(z, \lambda)du/dz+\lambda^2P_2(z, \lambda)u=0$, in a finite region R about z_0 in the complex z-plane and for large absolute values of the complex parameter λ. $P_1(z, \lambda)$ and $P_2(z, \lambda)$ are assumed to be regular in z throughout R and in λ at infinity. The theory developed is in some respects analogous to that known to apply to the equation (B): $d^2u/dz^2+\lambda P_1(z, \lambda)du/dz+\lambda^2P_2(z, \lambda)u=0$, under the assumption that $P_2(z, \infty)-P_1^2(z, \infty)/4$ be bounded from zero for z in R. With this hypothesis the theory for (A) develops along lines which are formally similar to those for (B) when considered under the same hypothesis. This analogy seems not to have been fully appreciated previously, although a paper by Cashwell [Pacific Journal of Mathematics vol. 1 (1951) pp. 337–352] in some degree anticipates the method used here. Cashwell dealt with an equation much less general than (A) and determined only one term of the asymptotic series for its solutions. Here, complete asymptotic series for solutions of (A) are found. Following the development of the general theory, an application is made to determine the structure of the Whittaker functions $M_{am+b,m}(x)$ for $|m|$ large, x bounded, and $R(m)\geqq 0$. (Received March 4, 1954.)

495. Jacob Korevaar: *Numerical Tauberian theorems for Dirichlet and Lambert series.*

In three papers published in Neder. Akad. Wetensch. vols. 56, 57, the author obtained various numerical Tauberian theorems for power series. The transformation $\sum a_n n^{-w}=\{\Gamma(w)\}^{-1}\int_0^\infty(\sum a_n e^{-nx})x^{w-1}dx$ and its inverse are used in this paper to derive the following result on Dirichlet series. Let the series $\sum a_n n^{-w}$ $(w=u+iv)$ have a half-plane of convergence. Let $F(w)$, the analytic continuation of the sum of the series, be regular for $u\geqq -a$ $(a>0)$, and let $F(u+iv)=O(e^{A|v|})$ $(u\geqq -a)$. Then under the Tauberian condition (1) $a_n\geqq -B/n$ one has the best possible estimate $|s_n-F(0)|\leqq C/\log (n+1)$ $(n=1, 2, \cdots)$. If $F(w)$ is known to be regular and $O(e^{A|v|})$ only in a region (2) $u\geqq -a/\{\log (2+|v|)\}^b$ $(a>0,\ b>0)$, then (1) implies the slightly weaker estimate (3) $|s_n-F(0)|\leqq C\{\log \log (n+e)\}^b/\log (n+1)$. It is known that $\zeta(w+1)$ is free of zeros in a region (2) $(b>3/4)$. Thus the transformation $(\sum a_n n^{-w})\zeta(w+1)\Gamma(w+1)=\int_0^\infty\{\sum a_n nx/(e^{nx}-1)\}x^{w-1}dx$ enables one to prove the following numerical Tauberian theorem for Lambert series. Let the Lambert series $\sum a_n nx/(e^{nx}-1)$ be convergent for $x>0$, and let its sum $f(x)$ for a certain s and positive D, α satisfy the inequality $|f(x)-s|<Dx^\alpha$ $(x>0)$. If $\zeta(w+1)$ is free of zeros in the region (2), then under the Tauberian condition (1) one has the estimate (3). This result improves an earlier result of the author on Lambert series (Bull. Amer. Math. Soc. Abstract 59-6-627). (Received February 26, 1954.)

496. M. Z. Krzywoblocki: *On the generalization of Iglisch's existence theorem to parametric differential equation.*

Iglisch considered the existence for a third order ordinary differential equation of the boundary layer problem, subject to two point boundary conditions. Superposition of additional conditions upon the boundary value problem in question furnished the existence proof of the flow in the boundary layer. The author in the past generalized some Whyburn's existence theorems of ordinary differential systems to differential systems with a parameter, thus furnishing a much wider class of possible solutions of the boundary layer problem. On this base, as well as using Rellich's criteria for continuation of a solution of a differential system, the author generalized Iglisch's proof of existence to a parametric third order ordinary differential equation subject to two point boundary conditions. (Received February 22, 1954.)

497*t*. W. S. Loud: *A nonexceptional element of Wiener space.*

Let C be the space of continuous functions on $0\leqq t\leqq 1$ which vanish at 0. Cameron and Martin (Bull. Amer. Math. Soc. vol. 53 (1947) pp. 130–137) define $\int_0^1|dx(t)|^2$, which they show exists and equals 1/2 for almost all points of C (Wiener measure). The quantity $\int_0^1|dx(t)|^2$ is equal to zero for any element of C which is of bounded variation, and for many totally nondifferentiable functions of C as well. An explicit example is given of an element of C for which $\int_0^1 |dx(t)|^2 = 1/2$. The construction is a modification of the author's (Proc. Amer. Math. Soc. vol. 2 (1951) pp. 358–360) with $A = 1/2$, $\alpha = 1/2$. (Received March 10, 1953.)

498. E. B. McLeod, Jr.: *The solution of a free-boundary problem by conformal mapping.*

By considering either a special case of the physical problem of finding the two-dimensional, irrotational flow about a bubble having a significant surface tension, or an equivalent variational problem in the complex domain, we obtain a condition on the free boundary which states that the square of the absolute value of the derivative of the velocity potential is proportional to the curvature of the boundary. Examination of the conformal mapping properties of the function whose derivative is equal to the square of the derivative of the velocity potential leads to a first order differential equation in $Z(\zeta)$, which maps a point ζ in the exterior of the unit circle into a point Z in the exterior of the free boundary. This differential equation has the explicit solution $Z = \zeta - 2/3\zeta - 1/27\zeta^3$. (Received March 3, 1954.)

499. E. J. McShane: *A dominated convergence theorem.*

In a previous publication Daniell's method of defining an integral has been generalized so as to apply to a wide class of elementary mappings (integrals) of a subset E of a lattice F into a partially ordered set G. This elementary map is extended to a larger subset of F so as to attain useful convergence properties. The program of defining the extended mapping by use of order properties alone was incomplete in two respects; G was assumed to be a group, and in its partial ordering was assumed "normal," a concept involving real functions on G and stronger than the Hausdorff property. Here it is shown that the hypothesis that G is a group can be replaced by a weaker hypothesis involving order alone, and that normality of G can be replaced by a hypothesis weaker than assuming that G is a Hausdorff space in its lattice topology. With these hypotheses, the integral can be defined and the generalization of Lebesgue's dominated-convergence theorem established. (Received March 10, 1954.)

500. Immanuel Marx: *On the structure of recurrence relations.* II.

Consider differential recurrence relations $p(x)A_n(x)dy_n/dx + B_n(x)y_n = y_{n-1}$, $p(x)F_n(x)dy_{n-1}/dx + G_n(x)y_{n-1} = y_n$ between contiguous solutions of a differential equation $(d/dx)(pdy/dx) + q(x, \lambda)y = 0$, With $y_m^{(1)}$, $y_m^{(2)}$ denoting, for each m, a fundamental system of solutions corresponding to the characteristic value λ_m, a pair of relations is called *general* if it is valid both for $y_n = y_n^{(1)}$, $y_{n-1} = y_{n-1}^{(1)}$ and for $y_n = y_n^{(2)}$, $y_{n-1} = y_{n-1}^{(2)}$. A pair of relations is called *particular* if it is valid for a pair y_n, y_{n-1} of particular solutions, but not valid for any pair of solutions linearly independent of y_n, y_{n-1}. It is proved that the solutions of every differential equation of the type considered have a pair of general recurrence relations with uniquely determined coefficients; that every set of contiguous particular solutions y_n, y_{n-1} has a family of pairs of particular recurrence relations depending on two arbitrary functions of x; and that all relations are included under these two cases. Explicit expressions for the coefficients are given. Similarly obtainable are differential recurrence relations between solutions corresponding to any two values λ', λ'' of the parameter, or indeed (suggestion of G. Y. Rainich) first-order differential relations between the solutions of any two second-order linear differential equations. Relations among three contiguous solutions, etc., are trivially obtainable from the differential relations. (Received February 11, 1954.)

501*t*. E. P. Merkes: *Binary complex functions.*

A binary function $w=f(z)=u(x, y)+jv(x, y)$, defined on a domain D of the binary complex $z=x+jy$ plane where x, y are real variables and $j^2=aj+b$, is binary polygenic on D if the real functions u, v are of class C' on D. In this paper the derivative of one binary polygenic function with respect to a second is obtained from which the generalized binary Cauchy-Riemann equations are derived. It is noted that when the discriminant of the quadratic in j is non-negative, binary monogenity does not imply binary analyticity. In the non-monogenic case analogues of the three characteristic properties of Kasner for the derivative of one polygenic function with respect to another are secured. Also studied are general linear fractional transformations in the binary variables z and $\bar{z}$ of which special sub-groups are analogues of the Moebius and the Laguerre transformations. Moreover, the analogues of the Cauchy integral theorem, of Cauchy's integral formulas, of Morera's theorem, and of the Pompieu areolar derivative are derived. (Received March 11, 1954.)

502. E. P. Merkes: *On a generalization of polygenic functions.*

A complex function $f(z)$ of a complex variable z which is continuous on an open set G is said to be quasi-polygenic if there exists a finite constant $M\geqq 0$ such that $|J(f, I)| \leqq 2M|I|$, where I is any interval in G and where $J(f, I)$ is the line integral of f along the boundary of I. The constant M is called a radius of f on G. The class of quasi-polygenic functions with radius M on G includes the class of polygenic functions with radii of the derivative circles for z on G not exceeding M. Theorem: A continuous function $f(z)$ on an open set G is quasi-polygenic with radius M if and only if, for all z in G except at most for a denumerable number, lim sup $|J(f, Q)|/|Q|<\infty$ as diam $Q\rightarrow 0$, and for almost all z of G, lim inf $|J(f, Q)|/|Q|\leqq 2M$ as diam $Q\rightarrow 0$, where Q is any square in G containing z. An analogue of the Looman-Menchoff theorem and other theorems are obtained. (Received February 15, 1954.)

503*t*. Josephine Mitchell: *Potential theory in the geometry of matrices.* Preliminary report.

Let D be the domain consisting of the set of points for which the quadratic form $I-Z\bar{Z}'$ is positive definite, where Z is an n by n matrix of complex numbers, $\bar{Z}'$ its conjugate transpose, and I the identity matrix; let B be that part of the boundary for which $Z\bar{Z}'=I$. The Laplace equation corresponding to the invariant metric $[\sigma((I-ZZ')^{-1}dZ(I-\bar{Z}'Z)^{-1}d\bar{Z}')]^{1/2}$ ($\sigma(A)$ being the trace of the matrix A) was calculated and a fundamental solution of the equation obtained. The Green and Neumann's functions with respect to the set $D\cup B$ were constructed and their properties found. The Poisson formula for the real part of an analytic function was obtained easily from the Cauchy formula for analytic functions which is already known [Bochner, Ann. of Math. vol. 45 (1944)]. The relations among the Poisson, Bergman, and Szegö kernel functions and the Green and Neumann's functions were found to be exact generalizations of the case $n=1$. Finally a boundary value problem of Dirichlet type was solved for the set $D\cup B$. (Received March 9, 1954.)

504*t*. S. B. Myers: *Algebras of differentiable functions.*

Let M, N be compact differentiable manifolds of class C^r $(1\leqq r<\infty)$, let $C^r(M)$

and $C^r(N)$ be the algebras of differentiable functions of class C^r on M and N respectively. The following theorems are proved. (1) If $C^r(M)$ is isomorphic to $C^r(N)$, there is a nonsingular differentiable homeomorphism of class C^r of M onto N. (2) If M and N are provided with Riemannian metrics of class C^{r-1}, and if $C^r(M)$ is made into a normed algebra (a Banach algebra if $r=1$) by $\|f\| = \max_{x\in M} |f(x)| + \max_{\theta \in \overline{M}} |\partial f/\partial \theta|$, where $\partial f/\partial\theta$ is the directional derivative of f in the unit direction θ and $\overline{M}$ is the compact unit tangent bundle to M, and if $C^r(N)$ is similarly normed, then if $C^r(M)$ is equivalent to $C^r(N)$ there is an isometry of class C^r of M onto N. (Received February 17, 1954.)

505*t*. Leopoldo Nachbin: *Closed algebras of analytic functions of one variable with an isolated singularity.*

Let P be the vector field of all convergent power series $f(z) = \sum_{-\infty}^{+\infty} a_n z^n$ regular or with a pole in $z=0$. For each $r>0$ and integer N, let P_{rN} be the vector subspace of P of all $\sum_N^{+\infty} a_n z^n$ converging for $\{z;\ 0<|z|<r\}$, with the topology of uniform convergence on compact subsets of this set. Endow P with the inductive limit topology as $N\to-\infty$ and $r\to 0$. Consider also on P the natural valuation given by $v(f)=N$ if $a_N\neq 0$. Theorem 1. Every subalgebra of P closed in the inductive limit topology is also closed in the valuation topology and vice versa. The closed subalgebras satisfy the ascending chain condition. Let next E be the algebra of all $f(z) = \sum_{-\infty}^{+\infty} a_n z^n$ converging around 0, essential singularities being allowed. For each $r>0$, let E_r be the subalgebra of all $f\in E$ converging in $\{z;\ 0<|z|<r\}$, with the topology of uniform convergence on compact subsets of this set. Endow E with the inductive limit topology as $r\to 0$. The residue functional $r(f) = (1/2\pi i)\int f(z)dz$, $f\in E$ (integral around 0) is a trace on E in the sense that, if to every $g\in E$ one associates the linear continuous functional $\tilde{g}$ on E given by $\tilde{g}(f) = r(fg)$, $f\in E$, the mapping $g\to\tilde{g}$ is a topological linear isomorphism of E onto its dual E^* endowed with the strong topology. Call $\tilde{g}$ truncated if g is regular or has a pole at 0. Theorem 2: Every subalgebra of E closed in the inductive limit topology is also closed in the weak topology defined by the truncated functionals and vice versa. (Received March 10, 1954.)

506*t*. Zeev Nehari: *On the coefficients of R-univalent functions.*

An analytic function $w=f(z)$ is said to be R-univalent in a region B if it maps B onto a domain embedded in a given closed Riemann surface R. If S_R denotes the class of functions $f(z) = z + a_1 z^{-1} + a_2 z^{-2} + \cdots$ which are regular and R-univalent in $1<|z|<\infty$, it is shown that the region of variability of the coefficient a_1 within the class S_R is contained in a disk of radius 1. This result is used to obtain estimates for a_1 in a number of special cases. (Received April 30, 1954.)

507. Howard Osborn: *On conservation laws.* Preliminary report.

A system of n quasi-linear homogeneous partial differential equations in two independent variables and n unknowns, the coefficients depending only on the unknowns, can sometimes be represented as a system of *conservation laws*, i.e., n or more two-vectors depending only on the unknowns are solenoidal in the space of the independent variables. For $n=1$, 2 one can always represent the given system by ∞^n conservation laws, i.e., n functions of one variable may be arbitrarily prescribed. For $n>2$ the determination of those systems with such a representation leads to an *overdetermined* system of $C_{n,2}$ second order linear homogeneous partial differential equations in n variables and one unknown. With certain restrictions it is shown that when a set of integrability conditions is satisfied, one can solve the latter system with Cauchy data given on n characteristic curves. This leads to a representation of the original system by ∞^n conservation laws. As an application one can display many systems which *cannot* be derived from a variational principle. (Received March 11, 1954.)

508t. L. E. Payne and H. F. Weinberger: *Bounds for harmonic and biharmonic functions.* II.

In a previous abstract (Bull. Amer. Math. Soc. Abstract 60-3-352), the authors gave bounds for a harmonic function at a point with respect to which the boundary B is star-shaped. The authors here give bounds at any point assuming only that B is star-shaped with respect to a single point 0. More precisely, if r is the distance from 0, it is required that $h=r\partial r/\partial n$ is positive on B. If the Green's function in $N\geqq 3$ dimensions is written as $G(P, Q) = -(\omega_N r_{PQ}^{N-2})^{-1}+g(P, Q)$, it is proved that (1) $(N-2)g(P, P)+2r(P)g_r(P, P) = \int_B h(\partial G/\partial n)^2 d\sigma$. From this together with the Green's function representation of the harmonic function u and Schwarz's inequality follows the inequality (2) $u^2(P) \leqq [(N-2)g(P, P)+2r(P)g_r(P, P)]\int_B h^{-1}u^2 d\sigma$. Putting $u(Q) = (N-2)g(P, Q)+2r(P)g_r(P, Q)$ results in the inequality (3) $(N-2)g(P, P)+2r(P)g_r(P, P) \leqq \int_B h^{-1}[(N-2)\omega_N^{-1}(r(Q)^2-r(P)^2)r_{PQ}^{-N}]^2 d\sigma_Q$ which, together with (2), gives bounds for u in terms of a mean square integral of its boundary values and the geometry of B. In two dimensions the analogue of (1) is (4) $(2\pi)^{-1}+2r(P)g_r(P, P) = \oint_B h(\partial G/\partial n)^2 ds$ and inequalities corresponding to (2) and (3) are easily obtained from this. A similar result also holds for biharmonic functions. Let the clamped plate Green's function in two dimensions be $\Gamma(P, Q) = (8\pi)^{-1}r_{PQ}^2 \ln r_{PQ}+\gamma(P, Q)$. Then the analogue of (1) is $\gamma(P, P)-r(P)\gamma_r(P, P) = 2^{-1}\oint h(\Delta\Gamma)^2 ds$. (Received March 4, 1954.)

509. Pasquale Porcelli: *Concerning integrals.* IIa.

The author establishes relationships between the Lebesgue-Stieltjes integral $L\{f, \phi\}_a^b$, f and ϕ functions on $[a, b]$, and the mean integrals $\mathcal{f}_a^b f d\phi$ studied recently by R. E. Lane and the author (Bull. Amer. Math. Soc. Abstracts 59-5-538 and 59-6-638). The author starts with $L\{f, \phi\}_a^b = \sum f(x_i)[\phi(x_i+)-\phi(x_i-)]$, where f is bounded on $[a, b]$ and ϕ is a nondecreasing step function on $[a, b]$, and then by use of a theorem reported earlier (Abstract 59-6-638) arrives at an integral $L\{h, g\}_a^b$ which exists when (i) one of h and g is of bounded variation and the other has discontinuities of the first kind only, and (ii) g is of bounded variation and h is bounded and g-measurable, in this case $L\{h, g\}_a^b$ is the Lebesgue-Stieltjes integral. We show in case (i) that $L\{h, g\}_a^b = h(b)g(b)-h(a)g(a)-L\{g, h\}_a^b$ if, and only if, $L\{h, g\}_a^b = \mathcal{f}_a^b h dg$ where the latter integral is a mean integral. (Received March 5, 1954.)

510t. Pasquale Porcelli: *Concerning integrals.* IIb.

In considering the integration by parts formula for the Lebesgue-Stieltjes integral, the author uses the solution to the following operator problem: Let $Q(a, b)$ denote the collection of all real-valued functions on $[a, b]$ such that $f\in Q(a, b)$ if, and only if, f is the uniform limit of a sequence $\{u_n\}_{n=1}^{\infty}$ of normalized step functions on $[a, b]$ and u_n is continuous at each of a and b for $n=1, 2, \cdots$; let L be a linear, homogeneous, and bounded operator on $Q(a, b)$, then there exists one, and only one, function γ of bounded variation on $[a, b]$ such that $\gamma(a)=0$ and $L(f) = \mathcal{f}_a^b f d\gamma$ for each f in $Q(a, b)$. Theorem: If ϕ is of bounded variation on $[a, b]$, then there exists one, and only one, function γ of bounded variation on $[a, b]$ such that $\phi(a)=\gamma(a)$ and $L\{f, \phi\}_a^b = \mathcal{f}_a^b f d\gamma$

for each f in $Q(a, b)$; moreover, $\gamma(x)=(1/2)\{\phi(x+)+\phi(x-)\}$ if $a<x<b$. Corollary: If $f\in Q(a, b)$, ϕ a normalized function of bounded variation on $[a, b]$, then $L\{f, \phi\}_a^b$ $=f(b)\phi(b)-f(a)\phi(a)-L\{\phi, f\}_a^b$. (Received March 5, 1954.)

511. W. T. Reid: *A note on the Hamburger and Stieltjes moment problems.*

For $\{\mu_n\}$ $(n=0, 1, \cdots)$, a sequence of real numbers satisfying with a monotone decreasing sequence of positive numbers $\{a_n\}$ the condition $(*)$ $\mu_0\geqq a_0$, $\mu_{2n}\geqq a_n$ $+(a_{n-1}-a_n)^{-1}\sum_{j=0}^{n-1}\mu_{n+j}^2$ $(n=1, 2, \cdots)$, it is shown that the Hamburger moment problem $\mu_n=\int_{-\infty}^{\infty}t^n d\alpha(t)$ $(n=0, 1, \cdots)$ has a nondecreasing solution $\alpha(t)$ with infinitely many points of increase; moreover, if $0<\lim a_n$ then the problem is indeterminate. If in addition to condition $(*)$ there is a monotone decreasing sequence of positive numbers $\{b_n\}$ such that $\mu_1\geqq b_0$, $\mu_{2n+1}\geqq b_n+(b_{n-1}-b_n)^{-1}\sum_{j=0}^{n-1}\mu_{n+1+j}^2$ $(n=1, 2, \cdots)$, then the Stieltjes moment problem $\mu_n=\int_0^{\infty}t^n d\alpha(t)$ $(n=0, 1, \cdots)$ has a nondecreasing solution with infinitely many points of increase, and the problem is indeterminate if $0<\lim a_n$, $0<\lim b_n$. In particular, this latter result contains as a very special instance a result proved by R. P. Boas in 1939 (see D. V. Widder, *The Laplace transform*, pp. 140–142). (Received March 10, 1954.)

512. L. V. Robinson: *Solutions of some irreducible linear partial differential equations of the second order.*

It is shown that, when but two independent variables are involved, all linear partial differential equations of the second order reduce to $[(\lambda+q)(\Lambda+Q)+N]u=V$, where $\lambda=\partial/\partial x+p\partial/\partial y$, $\Lambda=\partial/\partial x+P\partial/\partial y$, the p, P, q, Q, N, and V being functions of x and y. It is then shown how some of these equations can be solved. (Received March 9, 1954.)

513. R. J. Silverman: *Semigroups with the Hahn-Banach extension property.*

An abstract semigroup $\overline{G}$ has the *Hahn-Banach extension property* (HBEP), if and only if for every collection (Y, G, X, p, f, V) satisfying the following properties: (a) Y is a linear space, (b) G is a representation of $\overline{G}$ on Y, (c) X is a subspace of Y which is invariant under G, (d) V is a partially ordered linear space with the property that every subset of V with an upper bound has a least upper bound, (e) p is a positive homogeneous subadditive function from Y to V such that $p(gy)\leqq p(y)$ for all y in Y and g in G, (f) f is a linear function from X to V such that $f(gx)=f(x)$ and $f(x)\leqq p(x)$ for all x in X and g in G, there exists a linear function F defined on all of Y to V such that $F(gy)=F(y)$, $F(y)\leqq p(y)$, $F(x)=f(x)$ for all g in G, y in Y and x in X. The following results are obtained: $\overline{G}$ has the HBEP if $\overline{G}$ satisfies one of the listed conditions. **1.** $\overline{G}$ is a commutative semigroup. 2. $\overline{G}$ is a solvable semigroup. 3. $\overline{G}$ is a finite group. **4.** $\overline{G}$ is a semigroup which contains a collection of subsemigroups directed by set inclusion and whose union is all of $\overline{G}$ and such that every member of the collection has the HBEP. 5. $\overline{G}$ is a homomorphic image of a group with the HBEP. 6. $\overline{G}$ is an extension of a group with the HBEP by a group with the HBEP. 7. $\overline{G}$ is a subgroup of a group with the HBEP. (Received February 25, 1954.)

514*t*. C. J. Titus: *Linear vector spaces of elliptic mappings.* Preliminary report.

Let F be the family of mappings into the plane which are continuously differentiable on a plane domain D for which the Jacobian of any mapping in F is non-negative and zero only if rank zero. All real linear vector spaces V of mappings in F are characterized. These spaces lead to a concept of elliptic spaces of 1st and 2nd kind so-called since the Bers' theory of pseudo-analytic functions of 1st and 2nd kind is characterized therein. Various other theories of pseudo-analytic functions as related to systems of partial differential equations are also characterized therein. It is noted that all continuously differentiable mappings of bounded eccentricity, thus all quasi-conformal mappings, are in F. The relationship between the elliptic spaces, quasi-conformal mappings, and 1st order elliptic systems of partial differential equations is discussed. (Received March 11, 1954.)

515. J. L. Ullman: *On Tchebycheff polynomials.*

Let C be a bounded, closed set of positive capacity, with complement D, $T_n(z)$ be the Tchebycheff polynomial of degree n associated with C, $u_n(E)$ the total multiplicity of zeros of $T_n(z)$ in the set E, $U(z)$ the generalized conductor potential of C, and $m(E)$ the set function corresponding to the equilibrium distribution of charge on C. Theorem I. If E is a closed set bounded from C, $u_n(E)=o(n)$. Theorem II. If K is a Jordan curve consisting of points of D, and $I(K)$ its interior, then $\lim u_n(I(K))/n =m(I(K))$. Theorem III. If D is bounded by a finite number of analytic Jordan curves, and E is a closed set bounded from C, then $u_n(E)<M$, a constant independent of n. Theorem IV. If E is a closed set bounded from C, then $\lim \int_E \left| \, |T_n(z)|^{1/n} - \exp (U(z)) \right| dA^i = 0$. Theorem V. If D is bounded by a finite number of analytic Jordan curves, then $\lim |T_n(z)|^{1/n} = \exp (U(z))$, except for a set of measure zero. (Received March 18, 1954.)

516. F. M. Wright: *Some results relative to indeterminate Stieltjes moment sequences.*

The author has stated in earlier papers that an S-fraction transformation theorem previously presented but as yet unpublished can be used to obtain rather easily and extend somewhat the results proved by H. S. Wall (Trans. Amer. Math. Soc. vol. 31 (1929) pp. 91–116) relative to the backward extension of a given indeterminate Stieltjes moment sequence; in this paper it is first shown exactly how this can be accomplished. The author then uses the S-fraction transformation theorem mentioned above to obtain a necessary condition for a determinate Stieltjes moment sequence $\{\lambda_n\}$ such that the S-fraction expansion of the formal power series $\sum_{n=0}^{\infty}\lambda_n w^{n+1}$ is nonterminating to be the first backward extension of an indeterminate Stieltjes moment sequence $\{\mu_n\}$. (Received March 11, 1954.)

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517. A. A. Blank: *The non-euclidean geometry of binocular visual space.*

According to Luneburg (*Mathematical analysis of binocular vision*) the binocular sensory space is Lobachevskian. This conclusion follows in part from his inference upon experimental evidence that the space possesses a complete six-parameter group of rigid transformations. The conclusion is valid since the space appears to be Desarguesian, every triple of points being spanned by a convex two-dimensional subspace. However, the phenomena which led Luneburg to assume free movability in visual space are better explained by a replacement of his transformation equations for the mapping of physical space onto visual sensory space. An individual's binocular space is completely characterized once this mapping is known. The determination of the mapping of physical space onto visual sensory space is reduced to the specification of visual radial distance as a function of convergence disparity. (Received March 6, 1954.)

518*t*. H. D. Block: *A remark on integral invariants.*

Let the $2n$ variables $q_1, q_2, \cdots, q_n, p_1, p_2, \cdots, p_n$ be related to the $2n$ variables $Q_1, Q_2, \cdots, Q_n, P_1, P_2, \cdots, P_n$ by a canonical transformation. Let σ be the unit square: $0 \leqq u \leqq 1$, $0 \leqq v \leqq 1$, and let $q_i = f_i(u, v)$, $p_i = g_i(u, v)$ $(i = 1, 2, \cdots, n)$, where f_i and g_i have continuous derivatives on σ. This induces the relationships $Q_i = F_i(u, v)$, $P_i = G_i(u, v)$. Let $s_i = \bigcup_{(u,v) \in \sigma}(f_i(u, v), g_i(u, v))$ and $S_i = \bigcup_{(u,v) \in \sigma}(F_i(u, v), G_i(u, v))$, i.e. the maps of σ on the (q_i, p_i) and (Q_i, P_i) planes respectively. It is widely believed that, under the conditions stated, $\sum_{i=1}^{n} \iint_{s_i} dq_i dp_i = \sum_{i=1}^{n} \iint_{S_i} dQ_i dP_i$. It is the purpose of this paper: (1) to show by a simple example that this is not true, and (2) to discuss the sources of the mistake and point out correct formulations. The details will appear in the Quarterly of Applied Mathematics. (Received March 8, 1954.)

519*t*. H. D. Block: *On the minimality of the variational principles of classical particle mechanics.*

This paper deals principally with the variational principles which are associated with the names of Hamilton, Jacobi, Maupertuis, and Hilbert. Each principle involves a functional in the form of an integral. The requirement of the vanishing of the first variation of the functional provides the differential equations characterizing the motion. For this reason these principles are called variational or stationary principles. If, in addition, the actual motion minimizes the functional, then the principle is called a minimum principle. We show that some of the classical variational principles are minimum principles under certain conditions and only stationary principles under other conditions; others are always minimum principles, while still others are always only stationary principles. Some facts of this type are well known, but it is hoped that the treatment presented here not only provides more complete results, but also is more elementary in method, simpler in development, and yields results in more convenient form. (Received March 8, 1954.)

520. P. C. Hammer (p) and Jack Hollingsworth: *Numerical treatment of differential equations.* I.

There is proposed here a general class of methods of integrating ordinary differential equations numerically. These methods extend from the concept of differential equivalence introduced by Hammer. The class includes many methods such as those of Adams, Milne, and Moulton but does not directly include the Runge-Kutta type methods. In particular this class of methods includes all those which approximate the solution function and/or its derivatives by means of selecting from classes of functions, depending on a finite number of parameters to each class, particular functions for a designated range of independent variable. The selections may be made on the basis of explicit or implicit numerical determination of a finite number of parameters in any manner. Specific new methods resulting from this general attack show promise for difficult problems. Test calculations are now being carried out on a variety of methods at the University of Wisconsin Numerical Analysis Laboratory. (Received March 10, 1954.)

521. P. C. Hammer and Jack Hollingsworth (p): *Numerical treatment of differential equations.* II.

Given a system of differential equations whose behavior one wishes to study numerically, the question of what constitutes a good method is significant. The answer cannot be given categorically, but must be modified for example by the equipment and personnel available. The class of methods proposed in I is quite general and can often be made to yield a method tailored to fit the problem. Various specializations to meet certain situations are discussed. Let an nth order differential equation with suitable continuity in a closed region be given. Then there exists a family of functions of $n+2$ parameters from which arcs of any member (or from several members) can be joined to form a curve which uniformly approximates a solution. Moreover a sequence of such approximate solutions can be constructed, a subsequence of which converges uniformly to a solution. If a Lipschitz condition is met, this solution is unique. (Received March 10, 1954.)

522. W. C. Hoffman: *Scattering of electromagnetic waves from a random surface.*

Suppose that $z(x, y)$ is real and continuous in the mean over a finite region D, with mean value zero and covariance function $r(u, x; v, y)$. As such $z(x, y)$ defines a random surface. Then by Karhunen's theorem on the representation of a stochastic process, $z(x, y)$ has an expansion in the eigenfunctions of the covariance function. Employing this expression in the Stratton-Chu formulation of the electromagnetic field then leads to an approximate expression for the radiation scattered from the random surface, from which mean and covariance of the scattered field may be determined. (Received March 10, 1954.)

523. R. S. Ledley: *Digitalization, systematization, and formulation of the theory and methods of the propositional calculus.*

The procedures of the propositional calculus previously used almost exclusively in the set theoretical proofs of pure mathematics have recently become of fundamental importance in applied mathematics as well. In many scientific, social, and industrial fields, complex logical non-numerical problems arise more frequently than do problems of a numerical nature. "Logic machines" capable of handling such problems must be based on systematized and digitalized manipulations for carrying out computations of the propositional calculus. Consequently, the need arises for a systematic method of formulating, analyzing, and solving propositional functions and equations. The author has obtained methods for: (1) expressing the propositional calculus uniquely in a manner amenable to easy mechanization; (2) systematically generating all true and false implications of given propositions; (3) systematically generating all possible changes of variables (i.e. substitution of new variables) that can be made on a propositional calculus while preserving the logical meaning of the calculus (these substitutions being subject to arbitrary restrictions); and (4) the systematic generation of all solutions to any number of simultaneous propositional equations in any number of unknowns. The solution to this latter problem imples the ability to perform manipulations with propositional variables in a straightforward manner analogous to manipulations of ordinary algebraic equations. (Received March 9, 1954.)

524. W. S. Loud: *Existence and uniqueness of a periodic solution for Duffing's equation with damping and a forcing term.*

The equation considered is (1) $\ddot{x}+c\dot{x}+x+\beta x^3=\cos \omega t$. Analogue computer studies suggest that for certain values of the parameters c, β, and ω, (1) has more than one periodic solution of period $2\pi/\omega$, with the stable ones not odd-harmonic. Using results of Levinson (Journal of Mathematics and Physics vol. 22 (1943)) and Cartwright (Annals of Mathematics Studies, no. 20) it is shown that for c positive, and β non-negative and sufficiently small, this can not occur, and that (1) has a unique, odd-harmonic, solution of period $2\pi/\omega$, to which all solutions converge. An explicit, but not best possible, range of allowable values of β is given. (Received March 10, 1954.)

525. Y. L. Luke: *Numerical integration near a logarithmic singularity.*

Purpose of paper is to present coefficients to facilitate computation of $\int_0^{rh} \log x f(x)dx$. Integration by parts yields the integrals $\int_0^{rh} f(x)dx$ and $C_{nr}=\int_0^{rh} x^{-1}\int_0^x f(u)du$. The former can be integrated by several well known techniques. Let f be represented by a polynomial through the $(n+1)$ points $x=mh$, $m=0(1)n$. Paper gives exact coefficients of the Lagrangian type for the evaluation of C_{nr}; $n=1(1)10$, $r=1(1)(n+1)$. The remainder is studied after the manner of W. E. Milne (*The remainder in linear methods of approximation*, Journal of Research, National Bureau of Standards, vol. 43 (1949) pp. 501–511). Write $R_{nr}=\int_0^{rh} f^{(n+1)}(S)G_{nr}(S)dS$. Mean value theorem is applicable if $G(S)$ is one signed (that is, definite) in $(0, c)$ where c is the larger of n and r. $G_{nr}(S)$ is definite for all reported values of r and n such that $r\leqq n$. If $r=n+1$, $G_{n,n+1}(S)$ is definite (indefinite) if n is even (odd). In the indefinite cases, the function has one zero in $(0, n+1)$ open. Let ζh be the vanishing point. Also tabulated are $\int_0^c G_{nr}(S)dS$, ζh, and $\int_0^{\zeta h} G_{n,n+1}(S)dS$ for all r and n. (Received March 8, 1954.)

526. Hillel Portisky and R. P. Jerrard (p): *Expression of a wave function over a half-space in terms of its values over the boundary plane.*

A solution of the wave equation in the half-space $z>0$ is expressed in terms of its values $u=f(x,y)$ over the plane $z=0$ as follows: $u|_{P_0}=(1/2\pi)(\int [f]d\Omega-(1/c)[\partial f/\partial t]Rd\omega)$, where the integration is carried out over the plane $z=0$. Here R is the distance between the field point P_0 and a point on the plane of integration, $d\Omega$ is the element of solid angle subtended by the plane element $dx'dy'$ at P_0, and brackets indicate "retarded" values; in other word $[f]$ denotes the value of f not at the time t but at the time $t'=t-R/C$ such that a spherical wavelet starting from the point in question at the time t', with a radius expanding at the rate c, will reach the point P_0 at the time t. Two proofs of this relation are outlined. The first is based on Fourier time resolution; the second utilizes the proof used by Kirchhoff in his well known expression of the solution of the non-homogeneous wave equation in terms of retarded potentials of the sources. Several examples are given by way of illustration. (Received March 11, 1954.)

Geometry

527t. John DeCicco: *An inverse theory of conservative fields of force.*

The extremals of the variation problem: $\int_1^2 g(x, y, \tau, \mathfrak{T}, t)ds=$minimum possess an integral of the form: $v^2/2=\mathfrak{T}(x,y, \tau, t, E)$, where $E=$constant, if and only if $\bar{g}=g(x, y, \tau, \mathfrak{T}(x, y, \tau, t, E), t)$ is independent of the time t. The rectangular components ϕ and ψ of a field of force such that a constrained motion is possible along every extremal of this variation problem depend on $\mathfrak{T}$, $\bar{g}$, and an arbitrary resistance R. If these extremals form a system S_k, then ϕ and ψ are determined in terms of $\mathfrak{T}$, $\bar{g}$, and k. These results are applied to the case when $g=v^{1+k}H$ and $H=H(x, y, \tau, \mathfrak{T},$

t). In particular, when $H=1$, ϕ and ψ are determined in terms of $\mathfrak{T}$ and k. This field of force is independent of k if and only if the energy equation is of the form $v^2/2 = \mathfrak{T}(x, y, E)$. Similar results are obtained for velocity systems S_∞. For these generalized conservative fields of force, the Lagrangian and Hamiltonian equations of a system S_k, including $k=\infty$, are obtained. (Received March 5, 1954.)

528. John DeCicco: *Some theorems in the integral theory of polygenic functions.*

For a clock congruence whose central and phase point vectors are the polygenic functions $f(z)$ and $f(z)+g(z)$, the vector $\lambda = f_{\bar{z}} - g_z$ has the phase point and center of the derivative clock congruences of $f(z)$ and $f(z)+g(z)$ as its initial and terminal points. If $I(\Gamma) = \int_\Gamma f(z)dz + g(z)d\bar{z}$, then $|I(\Gamma)| \leqq 2RA$, where $R = \text{l.u.b.}\ |\lambda|$ for z within or on Γ, and A is the area enclosed by Γ. The generalized areolar derivative is $\lim I(\Gamma)/A = 2i\lambda$. For $f(z)$ of class $2q$, $(1/2\pi i)\int_\Gamma f(z)dz/(z-z_0) = \sum_{k=0}^{q-1} (r^{2k}/2^2 \cdot 4^2 \cdots (2k)^2) \nabla^{2k} f(z_0) + R$, where $|R| \leqq (r^{2q}/2^2 \cdot 4^2 \cdots (2q)^2)\ \text{l.u.b.}_{|z-z_0| \leqq r} |\nabla^{2q} f(z)|$, and Γ is $|z-z_0| = r$. If $f(z)$ is a nonconstant real q-harmonic function in $|z-z_0| \leqq r$, and if $\nabla^2 f(z_0) = 0, \cdots, \nabla^{2q-2} f(z_0) = 0$, then z_0 is not a maximum or minumum of $f(z)$. If $f(z)$ is real, nonconstant, and of class $2q$ in $|z-z_0| \leqq r$, and if $\nabla^2 f(z_0) = 0, \cdots, \nabla^{2m-2} f(z_0) = 0$, $\nabla^{2m} f(z_0) \neq 0$, where $1 \leqq m < q$, then in $|z-z_0| < S \leqq r$, $f(z)$ can not have a minimum or maximum at z_0 according as $\nabla^{2m} f(z_0)$ is <0 or >0. Finally the integrals $(1/2\pi i) \cdot \int_\Gamma f(z)dz/(z-z_0)^{m+1}(\bar{z}-\bar{z}_0)^n$ are studied in detail. (Received March 11, 1954.)

529t. John DeCicco: *Some theorems in the mapping theory of surfaces.*

In a cartogram Γ between two surfaces S and $\bar{S}$, the metric scale $\sigma = d\bar{s}/ds$ and the projective scale $\tau = d\bar{\phi}/d\phi$ are studied. There always exist the principal metric scales σ_1 and σ_2, and the principal projective scales τ_1 and τ_2 for a cartogram Γ. If Γ is not conformal, σ_1 and σ_2 are assumed along the unique orthogonal net Ω which is preserved by Γ, and if Γ is not quasi-projective, τ_1 and τ_2 are assumed along the unique conjugate net Π which is preserved by Γ. The ratio of surface elements of S and $\bar{S}$ is equal to $\sigma_1\sigma_2$, and the ratio of the Gaussian curvatures of S and $\bar{S}$ is equal to $(\tau_1\tau_2/\sigma_1\sigma_2)^2$. The totality of all cartograms which carry a given orthogonal net Ω or a given conjugate net Π of a surface S into an orthogonal net or a conjugate net of any surface $\bar{S}$ forms a group $G(\Omega)$ or a group $G(\Pi)$. Characterizations of the spherical representation of a surface S are obtained. Finally are studied cartograms for which the lines of curvature on S correspond to those on $\bar{S}$. (Received March 4, 1954.)

530t. Edward Kasner and John DeCicco: *Physical systems of curves in general fields of force.*

Physical systems S_k of curves of a field of force in the plane, where the force depends not only on the position of the point but also on the velocity and the time, are studied. A system S_k consists of ∞^4 trajectories or else, in the directional case, it consists of ∞^3 trajectories. In a general field of force, there are ∞^3 lines of force unless the field of force is directional together with a tangential resistance, in which case there are only ∞^1. The rest trajectory of a system S_k at a point P is initially tangent to a line of force at P, and the ratio of the initial curvatures of the rest trajectory of S_k and the line of force at P is $(1-\lambda)/((3+k)/(1+k)-\lambda)$, where λ is a certain angular rate at P. Generalizations of this formula are obtained for higher order contact. This generalizes a well known theorem of Kasner. Fields of force which retain some of the properties of positional fields of force are also studied. (Received March 5, 1954.)

STATISTICS AND PROBABILITY

531. Miriam C. Ayer, H. D. Brunk (p), G. M. Ewing, W. T. Reid, and Edward Silverman: *An empirical distribution function for sampling with incomplete information.*

For $i=1, 2, \cdots, n$, let N_i independent trials be made of an event with probability p_i, and suppose that the probabilities p_i are known to satisfy the inequalities $p_1 \geqq p_2 \geqq \cdots \geqq p_n$. Particular examples of this situation are found in bio-assay (cf. C. H. Goulden, *Methods of statistical analysis*, New York, Wiley, 1952) and in the proximity fuze problem discussed by M. Friedman (E. Eishenhart, M. W. Hastay, W. A. Wallis (Editors), *Techniques of statistical analysis*, New York, McGraw-Hill, 1947, Chap. 11). The maximum likelihood estimators, $\bar{p}_i$, of the probabilities p_i are determined and their consistency is proved. Let a_i denote the number of successes in the ith trial, and set $A(r, s) = \sum_{\nu=r}^{s} a_\nu / \sum_{\nu=r}^{s} N_\nu$. One finds that $\bar{p}_i = \min_{1 \leqq r \leqq i} \max_{i \leqq s \leqq n} A(r, s) = \max_{i \leqq s \leqq n} \min_{1 \leqq r \leqq i} A(r, s) = \min_{1 \leqq r \leqq i} \max_{r \leqq s \leqq n} A(r, s) = \max_{i \leqq s \leqq n} \min_{1 \leqq r \leqq s} A(r, s)$. The first of these expressions is useful in proving the consistency of the estimators, but other, simpler, means are available for the rapid calculation of the $\bar{p}_i$. (Received March 10, 1954.)

532t. G. E. Baxter: *Stationary "space" state of the Brownian motion of continuous one-dimensional media.*

Let a one-dimensional mechanical system have kinetic energy $2^{-1}\int_a^b r(x)(\partial\xi/\partial t)^2 dx$ and potential energy $(\alpha/2)\ \xi^2(a) + 2^{-1}\int_a^b \{p(x)(\partial\xi/\partial x)^2 + q(x)\xi^2\} dx + (\beta/2)\xi^2(b)$ where $\alpha, \beta \geqq 0$, $r(x) > 0$ and continuous, $q(x) \geqq 0$ and continuous, $p(x) > 0$ and continuously differentiable. Under the assumption that this system is the limit of n particles, it is shown that the stationary "space" state of its Brownian motion is described by a Gaussian stochastic process $\{\xi_x, x \in [a, b]\}$ with $E\{\xi_x\}$ zero and $E\{\xi_x\xi_y\}$ a multiple of the Green's function of the system. It is also shown that the exponent of the n-variate density function of $\xi_{x_1}, \xi_{x_2}, \cdots, \xi_{x_n}$ $(a < x_1 < x_2 < \cdots < x_n < b)$ for this process is equal to a multiple of $\sum_{j=1}^{n+1} \min \{\int_{x_{j-1}}^{x_j} \{p(x)(\partial\xi/\partial x)^2 + q(x)\xi^2\} dx + (\alpha/2)\xi^2(a)\delta_{1j} + (\beta/2)\xi^2(b)\delta_{n+1,j}\}$ where the minimization of the ith subintegral is subject to the conditions $\xi(x_j) = \xi_j$, $\xi(x_{j-1}) = \xi_{j-1}$. Finally, there is considered the effect of additional terms in the potential energy which correspond to constant forces acting on the system. The only change is to make the mean function $u(x) = E\{\xi_x\}$ the rest position of the system. (Received March 15, 1954.)

533t. J. H. Curtiss: *Sampling from a lot which is itself a probability sample.*

First it is proved that virtually any form of probability sampling *without* replacements from a lot which is itself a random sample, yields a random sample from the process. The sample and the uninspected remainder (u.r.) of the lot will be distributed independently. It is shown that in spite of this independence it is possible to draw inferences from sample to u.r. Various elementary inequalities linking parameter of the u.r. to those of the lot and of the sample are derived. For the normal case and attributes case, various confidence intervals for parameter of the u.r. are also derived. Then conditional inferences into over-all lot quality are considered. To develop this point of view, various conditional distributions of sample statistics relative to hypotheses on the corresponding lot statistics are derived for the normal, binomial, Poisson cases. (E.g. the conditional distribution of the sample variance for fixed lot variance is in the normal case a beta distribution.) Other results include general formulas for the unconditional distribution of the sample mean in sampling *with* replacements, and for conditional mean values in sampling without replacements. (Received March 15, 1954.)

534t. Ernest Ikenberry: *The convergence of Type* A *Gram-Charlier series.*

It is shown that the partial sums $f_R(x)=\alpha(t)\sum_{r=0}^{R}c_rH_r(t)$, where $t=h(x-X)$, X and $1/h^2$ are the mean and variance of a sample in which the relative frequency of occurrence of x_j is $f(x_j)$, $\alpha(t)=(1/(2\pi)^{1/2})\exp.(-t^2/2)$, $d(t)\cdot H_r(t)=(-D)^r\alpha(t)$, and the coefficients c_r are evaluated from $c_r=(1/r!)\sum_j f(x_j)H_r(x_j)$, converges to zero as R approaches infinity, except for $x=x_j$, at which points there is divergence. The sum function as R approaches infinity may conveniently be written in the form $f(x)=\sum_j f(x_j)[\alpha(t)/\alpha(t_j)]^{1/2}\cdot\delta(x-x_j)$ where $t_j=h(x_j-X)$ and $\delta(x-x_j)$ is the Dirac delta function. This result adds to the list of disadvantages of Type A Gram-Charlier series given by Kendall (*The advanced theory of statistics*, London, Griffin, 2d ed., rev., 1945, vol. 1, pp. 151–154). (Received March 8, 1954.)

535t. J. H. B. Kemperman: *Isotropic random flight in k-dimensional space.*

Consider a sequence X_i $(i=1, 2, \cdots)$ of independent k-dimensional random vectors $(k\geqq 1)$, each having the same isotropic distribution. Let $F(r)=\Pr(|X_i|\leqq r)$ and suppose that $F(0)<1$. Let $Z_j=X_1+X_2+\cdots+X_j$ and let n be the random variable defined by $|Z_j|<R$ $(j=1,\cdots,n-1)$ and $|Z_n|\geqq R$, where R is a positive constant. Finally, let $p_N=\Pr(n=N)$ $(N=1, 2, \cdots)$. Neglecting the excess $|Z_n|-R$ over the boundary (causing only small errors if $E(|X_i|^2)\ll R$), the following generating function is derived by means of Wald's fundamental identity. (*) $\sum_{N=1}^{\infty}p_Nw^{-N}=g(R^2t)^{-1}$ for $0\leqq t\leqq\epsilon$, ϵ being a sufficiently small positive quantity. Here, $w=\int_0^\infty g(r^2t)dG(r)$ and $g(t)=\Gamma(k/2)\sum_{h=0}^{\infty}(-t)^h/\Gamma(h+k/2)\Gamma(h+1)$. Provided that $E(|X_i|^{2h})<\infty$ one may differentiate (*) h times term by term. Differentating only once gives $E(n)=R^2/E(|X_i|^2)$. Differentiating twice gives $E(n^2)$ which, however, depends on the dimension k. (Received March 11, 1954.)

536. H. G. Landau: *Distribution of solution times for random communication in a task-oriented group.*

A task-oriented group consists of n individuals, $i=1, 2, \cdots, n$, each having initially one piece of information which must be transmitted to all the others to complete the task. At every sending time, $t=1, 2, 3, \cdots$, each individual sends all the information received previously to one of his possible recipients chosen at random. The communication net, specifying the possible paths for messages, is given by the matrix $S=(s_{ij})$, where $s_{ii}=1$, and for $i\neq j$, $s_{ij}=1$ if i can send to j, $s_{ij}=0$ otherwise. The information state at any time is given by a matrix $C_\alpha=(C_{\alpha,ij})$ where $C_{\alpha,ii}=1$, and for $i\neq j$, $C_{\alpha,ij}=1$ if j has received the information which i had initially, $C_{\alpha,ij}=0$ otherwise. The information state after each sending time is given by a Boolean matrix product. The solution time is the time when C_α first becomes (1), all elements equal unity. The C_α can be considered as the states of a finite, stationary Markov chain, which is particularly simple because the C_α can be ordered so that the transition probability matrix has only zeros below the principal diagonal. The generating function for the distribution of solution times can then be calculated very simply from the transition probability matrix. The transition probabilities themselves can be obtained by a straightforward calculation. Some special cases are considered. (Received February 22, 1954.)

TOPOLOGY

537. Maurice Auslander: *Compound group extensions*. IV.

Let $\psi: K \rightarrow G$ be a homomorphism of the group K into the group G with kernel X. By a continuation of this homomorphism we mean a pair (E, ϕ) where E is a group containing K as a subgroup and $\phi: E \rightarrow G$ is a homomorphism onto satisfying the conditions (1) $\phi^{-1}(1) = X$. (2) $\phi | K = \psi$. It is shown that every continuation of $\psi: K \rightarrow G$ induces a homomorphism $\theta: G \rightarrow A(X)/I(X)$ satisfying certain conditions. We define (E_1, ϕ_1) to be isomorphic to (E_2, ϕ_2) if there is an isomorphism $w: E_1 \approx E_2$ such that (1) $w | K = \text{identity}$, (2) $\phi_2 w = \phi_1$. It is then shown that the relative cohomology group $H^2(G, \psi(K); Z_X)$, where Z_X is the center of X, operates simply transitively on the isomorphism classes of continuations of $\psi: K \rightarrow G$ which induce the same homomorphism $\theta: G \rightarrow A(X)/I(X)$. An obstruction to the existence of continuations is defined to be a certain element of $H^3(G, \psi(K), Z_X)$. Finally, the results of J. H. C. Whitehead on operator extensions (Quart. J. Math. Oxford Ser. (2) vol. 1 (1950)) are obtained as an application of above results. (Received March 8, 1954.)

538t. Maurice Auslander: *Relative cohomology groups of groups*. Preliminary report.

Let G be a group operating on an abelian group A and N a subgroup of G. Let $r: C^q(G, A) \rightarrow C^q(N, A)$ be the restriction map of the group of q-dimensional cochains on G onto the q-dimensional cochains on H. Define $C^q(G, N; A) = r^{-1}(0)$ for $q > 0$ and $C^0(G, N; A) = 0$. We designate by $H^q(G, N; A)$ the cohomology groups of the cochain complex $\{C^q(G, N; A), \delta\}$, where δ is induced by the coboundary operator of $C^q(G, A)$. The following results are obtained: (1) there is an exact sequence $\cdots \rightarrow H^q(G, N; A) \rightarrow H^q(G, A) \rightarrow H^q(N, A) \rightarrow H^{q+1}(G, N; A) \rightarrow \cdots$. (2) For all $q > 0$, $H^q(G, N; A) \approx H^q(G, C)$ for a well determined C. (3) If N is a proper normal subgroup such that for all A, $H^q(G, N; A) = 0$, then $H^i(G, N; A) = H^i(G, A) = H^i(N, A) = \{0\}$ for all A and all $i \geqq q$. (4) If G contains an element of finite order and N is a proper normal subgroup, then the restriction map $H^q(G, A) \rightarrow H^q(N, A)$ can not be an isomorphism for two successive values of q and all A. (Received March 8, 1954.)

539t. R. H. Bing: *Monotone upper semicontinuous decompositions of E^3*.

Suppose G is a monotone upper semicontinous decomposition of E^3 such that G has only a countable number of nondegenerate elements and the complement of each is topologically equivalent to the complement of a point. It is shown that the decomposition space G is topologically equivalent to E^3 if either (a) each element of G is starlike, (b) each nondegenerate element of G is a tame arc, or (c) the sum of the nondegenerate elements of G is a G_δ set. An example is given of an upper semicontinuous decomposition H of E^3 such that each nondegenerate element of H is a tame arc but where it is not known whether or not the decomposition space H is topologically equivalent to E^3. However, it is shown that such a decomposition space H is topologically equivalent to E^3 if each element of the decomposition H is a subset of a vertical line. (Received March 11, 1954.)

540t. R. H. Bing: *Some monotone decompositions of a cube.*

Suppose G is a monotone upper semicontinuous decomposition of a cube such that each nondegenerate element of G is a subset of the boundary of the cube. The decomposition space G is shown to be topologically equivalent to the closure of the complement of a cactoid in S^3. The topological character of the decomposition space is determined by using the characterization of S^3 given in *A characterization of 3-space by partitionings*, Trans. Amer. Math. Soc. vol. 70 (1951) pp. 15–27. (Received March 11, 1954.)

541. H. J. Curtis: *A metrisation problem concerning lattices.* Preliminary report.

Let L be a semi-modular, atomistic lattice with the following properties: the point space, P, of L is a metric space; every element of L, considered as a point set, is closed in the topology of P; the g.l.b. of distances from a given point to points of any nonzero element of L is continuous in any set of independent points determining the element. Wilcox (Duke Math. J. vol. 8 (1941) pp. 273–285) extended the topology of P to L in such a way that L is a Hausdorff space. In this paper a sufficient condition that L be metrisable is established. It is that L satisfy the following three conditions: P is separable; no element of dimension greater than one, considered as a point set, has arbitrarily small diameter; a generalization of Wilcox's axiom pertaining to the g.l.b. of distances from a point to points of a nonzero element of L. This result is achieved by the use of the theory of compact spaces. Non-isomorphic examples of lattices which satisfy the sufficient condition are the real euclidean and projective spaces. (Received March 22, 1954.)

542. O. G. Harrold, H. C. Griffith (p), and E. E. Posey: *A characterization of tame curves in 3-space.*

Let J be a 1-manifold in 3-space. If J lies in the combinatorial boundary of a disk (closed, topological 2-cell) K which is locally polyhedral at each point of $K-J$, say J has *property* Q. If for each point x of J there is a disk K_x such that (i) K_x is locally polyhedral at points of K_x-J, (ii) $K_x\cap J$ is an arc, and (iii) $K_x\cap J$ is the closure of a neighborhood of x relative to J, say J has property Q at x. There are examples of arcs and curves having property Q that are not tamely imbedded. By combining property Q with a property P given earlier (Bull. Amer. Math. Soc. Abstract 59-4-434) the following results are obtained. If a homeomorphism h on 3-space maps J into a plane, J has properties P and Q. If J is tame, J has property P and the local form of property Q at each point. If J has property P and the local form of property Q, J is tame. If J has property P and Q, J is tame and unknotted. (Received March 8, 1954.)

543. G. R. Livesay: *A separation theorem.*

Let A be a compact subset of E_n, B closed subset not intersecting A. Then for any open set $U\supset A$, there exist a finite number of compact manifolds, disjoint from one another, and from $A\cup B$, such that the union of these manifolds is contained in U, and separates A from B in E_n. Further, if A is compact and connected, and B is closed and contained in a single component of E_n-A, then there is a single compact manifold satisfying the above conditions. The manifolds are triangulable. (Received March 8, 1954.)

544. R. J. Nunke: *Two theorems on the cohomology groups of a chain complex.* Preliminary report.

For abelian groups A and C let Ext (A, C) be the group of abelian group extensions of C by A. If Hom $(A, Z) = 0 =$ Ext (A, Z), where Z is the group of integers, then A is trivial. This result is used to prove the following two theorems: I. If K is a chain complex composed of free abelian groups and if, for every integer $q > n$, $H^q(K; Z) = 0$, then for all $q > n$, $H_q(K; Z) = 0$. II. Let K and L be chain complexes composed of free abelian groups and let $f: K \to L$ be a chain transformation. In order that f be a chain equivalence, it is necessary and sufficient that the induced homomorphism $f^*: H^q(L; Z) \to H^q(K; Z)$ be an isomorphism for all dimensions q. (Received March 10, 1954.)

545. D. E. Sanderson: *On isotopy of homeomorphisms in E_3 and S_3.* Preliminary report.

The principal results obtained in this paper are (i) given $\epsilon > 0$, every homeomorphism of Euclidean 3-space, E_3, onto itself is ϵ-isotopic to a piecewise linear homeomorphism, and (ii) the space of homeomorphisms of the 3-sphere, S_3, onto itself is uniformly locally connected. Both results are obtained by applications of Moise's piecewise linear approximation theorem for homeomorphisms on 3-manifolds (Ann. of Math. vol. 56 (1952) p. 97), Alexander's deformation theorem for an n-cell (Proc. Nat. Acad. Sci. U.S.A. vol. 9 (1923) pp. 406–407) and a theorem on fitting together homeomorphisms previously announced by the author (Bull. Amer. Math. Soc. Abstract 59-6-717). By a slight modification of the proof of (i) it can be shown that the space of homeomorphisms of E_3 onto itself under the compact-open topology defined by Fox (Bull. Amer. Math. Soc. vol. 51 (1945) p. 429 ff.) is locally connected. (Received March 11, 1954.)

546*t*. C. T. Yang: *A generalization of Borsuk-Ulam's theorem.*

Let R^{n+1} be the euclidean $(n+1)$-space and 0 its origin. Let X be a compact subset of $R^{n+1} - \{0\}$ symmetric with respect to 0. It is proved that the following are equivalent: (i) X separates 0 and ∞. (ii) Every mapping of X into the euclidean n-space maps some pair of symmetric points into a single point. (iii) X has a symmetric n-cycle (mod 2), with respect to which the order of 0 is not zero. That (i) implies (ii) is a generalization of a well known theorem of Borsuk-Ulam (Borsuk, Fund. Math. (1933)). (Received March 9, 1954.)

547. C. T. Yang: *Mappings from spheres to euclidean spaces.* Preliminary report.

In this paper it is proved that for any positive integers m, n there exists a positive integer k such that every mapping of a k-sphere into the euclidean m-space maps the end points of some n mutually orthogonal diameters into a single point. Denote by $k(m, n)$ the smallest of such integers k. In addition to the known results that $k(m, 1) = m$ (Borsuk, Fund. Math. (1933)) and $k(1, n) = n$ (Yang, Bull. Amer. Math. Soc. Abstract 59-6-720), the author has proved (i) $k(m, n) \leqq mn + n - 1$, (ii) $k(m, 2) \leqq 2m$ and (iii) $k(2, 2) = 4$. These results are actually established in a more general form as in an early work of the author (Bull. Amer. Math. Soc. Abstract 60-1-144). (Received March 9, 1954.)

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