

Here  $J_r$  and  $Y_r$  are Bessel functions of first and second kinds, respectively;  $K_r(x)$  is the modified Bessel function of the third kind, and  $H_r(x)$  is Struve's function. The second part of the volume contains integrals of higher transcendental functions, most of which could not properly have come into the foregoing tables of transforms. They are classified under the following titles: orthogonal polynomials, gamma and related functions, Legendre functions, Bessel functions, and hypergeometric functions. As in Volume I there is an appendix for notations and definitions and an index thereto.

It is the fate of all tables to be incomplete, and in spite of the ambitious scope of the present set most users will probably spot omissions. For example, the reviewer would have welcomed a chapter on the Weierstrass (or Gauss) transform. The omission is not serious since this transform can easily be related to the Mellin or Fourier transform. The appearance of this second volume confirms the reviewer's earlier opinion that these tables will ultimately be among the indispensable tools of many analysts and applied mathematicians.

D. V. WIDDER

*Geometrie der Zahlen.* By O. H. Keller. Enzyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen, Vol. I<sub>2</sub>, No. 11, Part III. 2d ed. Leipzig, Teubner, 1954. 84 pp. 8.80 DM.

The title *Geometrie der Zahlen* was introduced by Minkowski over half a century ago. Its subject matter has naturally expanded greatly, particularly in the last twenty years, and the author commendably presents a more modern account of the subject. The aim of the *Enzyklopädie* is "to find a middle road between the . . . historical presentation and the . . . systematic presentation." The reviewer's impression is that the work tends to be more systematic than historical, but the few isolated remarks that follow can in no way detract from the debt owed to the author for his pioneering compilation.

One major pre-Minkowskian development is the study of the minima of quadratic forms in  $n$  variables, elegantly interpreted as the densest lattice packing of equal spheres in  $n$  dimensions. The discovery, essentially, that the symmetric tetrahedral packing (with  $n+1$  mutually tangent spheres) is no longer densest when  $n=4$  is one of the earliest indications that  $n$ -dimensional "Euclidean geometry" would depend on  $n$  with "number-theoretic" irregularity. Strangely enough, the footnote reference to this result (footnote 184a) does not refer to its discovery (by Korkine and Zolotareff in 1872) but

rather to a series of papers starting in 1948. The footnotes catch up in the long run, if the reader uses his powers of deduction. But more difficult is the deduction that the aforementioned result led to the development of extremal methods and polyhedron packings.

The Minkowskian synthesis (i.e., the packing of space with general convex bodies), certainly receives due attention, although a sense of continuity with earlier work, particularly with that of Hermite, is not made apparent. Likewise the contact between the geometry of numbers and algebraic number theory seems lost in the mass of details again. The reviewer refers to the contact occurring when the algebraist had to work with inequalities not only to find bounds on norms but even to prove the discriminant theorem, which implies the following: "Every irreducible monic polynomial of degree at least two and with integral coefficients contains a repeated (polynomial) factor modulo some prime." Conversely in the last twenty years (owing largely to the influence of Mordell, Davenport, and Mahler), the intense desire for unimprovable bounds in algebraic number theory caused the replacement of the convex body by the more general star body. One contrast in presentation that the reader will immediately note is that while the older methods (Minkowskian and earlier) are often presented with enough detail (diagrams, tables, etc.) to excite the non-specialist, the methods developed in the last twenty years are given in little detail, with emphasis instead on lists of definitions, theorems, and conjectures, and with a valuable bibliography drawn up for 1951. Incidentally, a more modern bibliographical format (alphabetically arranged, and containing the titles of the papers) probably would be more useful.

The chapter titles are as follows: A. Convex bodies. B. Star bodies. C. Linear forms. D. Minima of homogeneous forms. E. Inhomogeneous forms. F. Quadratic definite forms. G. Continued fractions. H. Algebraic numbers. I. Partitions and lattice point figures. The reviewer feels that the inclusion of the last chapter is hardly justified by the historical continuity presented, but that otherwise the titles suggest that the geometry of numbers still would not be beyond Minkowski's recognition.

HARVEY COHN

*Economic activity analysis.* Ed. by O. Morgenstern. New York, Wiley; London, Chapman and Hall, 1954. 18+554 pp. \$6.75.

This is a collection of essays by members of the Princeton University Economics Research Project who were studying the mathematical structure of American type economies with support from the