contained. But neither is ultimately physically self-contained, and this is particularly the case for quantum mechanics. Such a basic phenomenon as the dual particle-wave aspects of light and matter, treated in the first chapter, cannot be well understood without the introduction of systems with an infinite number of degrees of freedom. And it is generally believed that the nature of the interaction between light and matter can be comprehended successfully only in a relativistic theory, although as yet there is no wholly satisfactory mathematical treatment of the matter. In other terms, quantum mechanics is logically more indivisible than classical mechanics—which manifests itself in the circumstance that more of the physics is in the mathematics. But a book of reasonable size will probably never be able to go into all such matters with anything like the exceptional thoroughness and clarity with which this book illuminates the fundamentals of "classical" quantum mechanics, i.e. the part of quantum mechanics thought by most informed persons to be in fairly definitive form.

I. E. SEGAL

Methods of algebraic geometry. Vol. III. *Birational geometry.* By W. V. D. Hodge and D. Pedoe. Cambridge University Press, 1954. 10+336 pp. $7.50.

The book we are reviewing is the third volume of a series devoted to the methods of algebraic geometry. Since a common spirit animates these books, the present review would be incomplete without a glance at the three volumes.

The first part of Volume I (reviewed in this Bulletin vol. 55 (1949) pp. 315–316) contains various algebraic preliminaries, ranging from linear algebra, matrices and determinants to field-theory and the study of polynomials along classical lines. Its second part presents an account of the "linear geometry" in projective spaces, with a full analytic treatment of Grassmann coordinates, collineations and correlations, and with a long side-trip into synthetic projective geometry.

With the second volume (reviewed in this Bulletin vol. 58 (1952) pp. 678–679) begins algebraic geometry proper. Irreducibility, generic points, dimension and associated forms are discussed. A chapter on algebraic correspondences introduces a theory of intersection multiplicities which, without being as generally applicable as more sophisticated sister-theories, is however sufficient for most purposes. As an illustration we find two remarkably exhaustive chapters on quadrics and Grassmann varieties.

In the third volume the emphasis shifts from projective to affine space, from a study in the large to a local theory. After an introductory chapter on ideal theory, we find the "arithmetic" theory of varieties, i.e. the correspondence between ideals and varieties, the study of simple points, and that of normal varieties. More powerful algebraic tools, i.e. valuations, are then brought into play, and the book reaches its climax with a proof of the local uniformization theorem and of the reduction of singularities for surfaces.

At a time when abstract algebraic geometry is becoming more and more popular among mathematicians, many of us are confronted with the problem of how to learn (or teach!) it. Some good books are now available, and, for the time being, no two of them have the same purpose. Walker's *Algebraic curves* presents a carefully expounded special and significant theory. A sketchy and suggestive treatment of many important topics may be found in Lefschetz's *Algebraic geometry*. In A. Weil's *Foundations* we find a systematic and logical book, centered on intersection theory, and which some people prefer to read backwards. A book assuming from the start an exhaustive knowledge of commutative algebra is still missing, but perhaps not for long.

With Hodge and Pedoe's *Methods* we now see a quite different type of book. In its three volumes we find a leisurely description of an impressively extensive part of algebraic geometry, and of the algebraic methods which are used nowadays. Motivations are given. Examples of significant and useful varieties are numerous. All the algebra which is needed is given, and, what is more, these books tell how to translate geometry into algebra, and conversely. The powerful and useful method of associated forms (called "Cayley forms" by the authors) is given a leading part in Volume II. The authors do not hesitate to repeat the same thing twice in different words, thus easing the reading for a beginner: e.g., prime ideals and factor rings are introduced in Chapter X in a special case, and in full generality in Chapter XV. None of these features impairs the value of these books for the working geometer, and all of them enhance their value for the student.

The only things which may be regretted by the working geometer (and, mostly, for the sake of its own references!) are the restriction to characteristic 0, and the use of an intersection theory which is not the most general one. The authors give, in the introductions, their reasons for having chosen such a course: broadly speaking the inclusion of characteristic $p \neq 0$ would have been inconsistent with the spirit of these books, which only intend to put classical algebraic geometry on a sound basis.

We now come to a more detailed account of Volume III, the subtitle of which is *Birational geometry*. It is written for readers who have mastered the classical methods of algebraic geometry (e.g. who have studied the first two volumes), and who wish to get acquainted with the methods and the results of the modern algebraic school (read Oscar Zariski) on local problems.

The introductory chapter (Chap. XV) on ideal theory has been written "with the needs of geometry in mind," and the reviewer is convinced that the authors have successfully reached their goal. Besides classical topics (like factor rings, noetherian rings and modules, primary representation, quasi-equality, integral dependence), we find many others which algebraic geometry has recently introduced into algebra: extension and contraction of ideals, quotient rings, detailed study of integral closures.

Chapter XVI is devoted to the arithmetic theory of varieties. Since the preceding volumes dealt with projective varieties, we first encounter the passage from projective to affine space, a feature rarely found in books, and thus most welcome. Then comes the correspondence between ideals and varieties, based upon Hilbert's zero theorem (proved in Volume I by using elimination theory), and upon the primary representation of ideals. Quotient rings of simple points are studied, and uniformizing parameters introduced. A careful study of the behavior of quotient rings by ground-field extension leads to Zariski's characterization of simple subvarieties. Affine normal varieties (i.e. affine varieties whose coordinate ring is integrally closed) are introduced, and the correspondence between an affine variety and a normal model is studied. Projective varieties having an integrally closed homogeneous coordinate ring are called "projectively normal," a name which seems excellent to the reviewer; useful features of this section are the detailed study of the integral closure of a graded integral domain $K[x_0, \cdots, x_n]$, and the characterization of projectively normal varieties by the completeness of their systems of plane sections. At many points in this chapter the connection with associated forms is made.

Chapter XVII presents for the first time in book form the kind of valuation theory that is needed by algebraic geometers. Besides classical topics, like linearly ordered group and the extension of a valuation to an algebraic extension, it contains the study of the rank, and that of the compounded valuations. In the case of a function-field, the dimension of a valuation is introduced, and its relations with rank and rational rank are carefully studied. Some helpful words are written about the connection between valuations and branches. Then the authors define the center of a valuation on a projective model, and give many properties relating valuations and their centers. The behavior of valuations under ground-field extension is studied. The reviewer was, however, slightly disappointed not to find any clear-cut theorem giving the existence of valuations with given center (a very comprehensive result of Zariski's is quoted, but without proof); the difficulty lies perhaps in the fact that, instead of the powerful theorem about the extension of a specialization to a place, the authors proved only that any integrally closed domain is an intersection of valuation rings.

In the last chapter (Chapter XVIII) valuation theory is applied to geometric problems. Corresponding subvarieties in a birational correspondence are defined in a geometric way, and characterized by means of valuation theory. Transforms and total transforms of subvarieties are discussed, and a weak form of Zariski's "main theorem" is proved. The authors derive from a more general result the fact that, if P is a normal point and if the total transform of P under a rational mapping F consists of a finite number of points, then F is regular at P. After an introduction devoted to monoïdal transformations, comes the climax of this book, i.e. the proof of the local uniformization theorem. This proof follows, broadly speaking, Zariski's method, but a valiant effort has been made toward greater intelligibility. The existence of finite resolving systems for a function-field F is then proved without topologizing the Riemann surface of F. And, as in Zariski's "simplified proof," the method of replacing, in a resolving system, two varieties by a single one, leads to the reduction of singularities for surfaces.

A one-page bibliographical note tells more about the history of the subject than could many pages in the usual historical style, and the authors should not apologize about that in their introduction. Here as in the remainder of their books, the authors should be commended for having given the facts, many useful facts, in a straightforward way. They should also be commended for having successfully steered a course which is equally remote from bashfulness about using algebra and from oversophistication in its use. In writing their books they have rendered an invaluable service to the mathematical community.

P. Samuel

The printing of mathematics. Aids for authors and editors and rules for compositors and readers at the University Press, Oxford. By T. W. Chaundy, P. R. Barrett, and Charles Batey. Oxford University Press, 1954. 10+105 pp. $2.40.

Mathematics in type. Richmond, Va., The William Byrd Press. 12+58 pp. $3.00; $1.50 to staff members of educational institutions.

Printing is a necessary evil: there is substantial agreement among mathematicians that an alleged piece of mathematics has no standing until it has appeared in print for all interested people to read. There