RESEARCH PROBLEMS

16. Richard Bellman: *Number theory.*

Let $f(t)$ be the number of solutions of $y^3+2=tx^2$ in integers x and y. It is known that $f(t)$ is finite for each t. What bound can one obtain for $f(t)$, and what asymptotic relation, if any, holds for

$$\sum_{t=1}^{T} f(t) \quad \text{as} \quad T \to \infty?$$

(Received January 10, 1955.)

17. Richard Bellman: *Analysis.*

Let

$$x = \sum_{n=1}^{\infty} t^n/n^s, \qquad y = \sum_{n=1}^{\infty} t^n/n^{s+1}, \qquad |t| < 1.$$

Eliminating t we have $y = \sum_{n=1}^{\infty} c_n x^n$. What is the radius of convergence of this series as a function of s for $-\infty < s < \infty$? (Received January 10, 1955.)

18. Ernest Michael: *Continuity of multiplicative linear functionals.*

Let A be a closed subalgebra of the cartesian product (product topology) of countably many commutative, complex Banach algebras. Is every complex-valued multiplicative linear functional on A continuous? (Reference: E. Michael, Memoirs of the American Mathematical Society, no. 11, 1952, section 12.) (Received January 18, 1955.)

19. E. T. Parker: *A tensor equation.*

Let $[g_{ab}]$ be a symmetric $n \times n$ matrix of real analytic functions of n real variables, with nonzero determinant over a neighborhood. Let g_{ab} be the covariant metric tensor of a Riemannian space. Find a necessary and sufficient condition on g_{ab} in order that there exist a scalar whose second covariant derivative is g_{ab} over a neighborhood. One might also find conditions for the existence of such a scalar over the space. It may be possible to obtain a result under a hypothesis weaker than that the functions of g_{ab} be analytic. The question might be of interest in non-Riemannian spaces. (Received February 2, 1955.)

20. Casper Goffman and G. M. Petersen: *Consistent matrix summability methods.*

Regular matric methods A and B are called consistent if every bounded sequence summed by both methods is summed to the same value. Show, for every set S of regular methods which are mutually consistent, that there is a set $T \subset S$ of regular methods which are mutually consistent and such that every bounded sequence is summed by a method in T. In particular, consider the case where S has two elements. (Received February 25, 1955.)

Problems Discussed by the XIth General Assembly of the International Radio Scientific Union, and Called to the Attention of Mathematicians

(Communicated to the International Mathematical Union by Professor B. van der Pol, and submitted to the Bulletin by Professor E. Bompiani, Secretary of the International Mathematical Union. Received by the Bulletin, February 7, 1955.)

1. The propagation of radio waves through a medium (i.e. troposphere, ionosphere), in which turbulence occurs causing a statistical spatial distribution of the dielectric constant, which in turn produces a scattering (irregular reflection and diffraction) of the radio waves. (Booker, Carrol, Megaw and others.)

2. The relation—if it exists—between the "frequency bandwidth" of a linear electrical circuit and its "time constants" was investigated and discussed. This problem is analogous to the mathematical problem of the geometrical relation between a given function of a real variable and the form of its Fourier cosine and sine transforms.

3. Mean spectrum of a series of impulses which in principle are identical and periodic, but are actually spaced and deformed at random. This problem has been treated by R. Fortet, L'Onde Electrique vol. 34 (1954) p. 683.

If further generalized it can be stated in the following form:

4. *Spectrum of a "random" signal compared with that of a simple signal emitted by the same system.* It is convenient, for determining the spectrum of a given emitter, to apply either a single elementary signal $E(t)$ or a periodic sequence of signals $E(t)$. But the spectrum emitted under actual operating conditions, corresponding to a sum

$$\sum_{i=1}^{n} E(t - t_i),$$

of elementary signals shifted in time, where t_i are random, does not necessarily bear a simple relation to the spectrum of a single signal or that of a periodical signal.

In all cases of amplitude modulation or pulse modulation a simple solution to the problem of comparing the two types of spectra has been effected easily. In fact, in those cases there corresponds to the sum

$$\sum_{i=1}^{n} E(t - t_i)$$

applied to the input of the system a sum

$$\sum_{i=1}^{n} S(t - t_i)$$

representing the output signal since the system effects only a linear transformation on the signal.

But the problem has yet to be resolved for the case when the transformation is not linear. In particular, in frequency modulation, to a signal of the form

$$\sum_{i=1}^{n} E(t - t_i)$$

there corresponds an output signal which can be put in complex terms in the form of a product

$$\prod_{i=1}^{n} S(t - t_i)$$

and the relationship between the various spectra does not seem to be very simple for those cases when the index of modulation is not very small.