of the problem of linear relationship, alias the Riemann problem or (as the author would prefer) the Hilbert problem, the problem being to find a sectionally holomorphic function $F(z)$ with a line of discontinuity L, the boundary values of which from the left and from the right satisfy the condition

$$F^+(t) = G(t)F^-(t) + f(t) \text{ on } L$$

(except at the ends), where $G(t)$ and $f(t)$ are functions given on L and $G(t) \neq 0$ everywhere on L.

Part VII becomes a little more three-dimensional, dealing with extension, torsion and bending of homogeneous and compound bars.

This is a book to be recommended in the highest terms to every serious student of the mathematical theory of elasticity. And to engineers also, for though they may find some of the work too purely mathematical for their taste, they will be rewarded by solutions of definite problems completely worked out. The reviewer was tickled by the enlivenment imparted to the solution of the torsion problem for a circular cylinder reinforced by an eccentric bar; one part of the solution is due to the cylinder itself and the other part to the "indignation" aroused by the presence of the reinforcement!

J. L. SYNGE

Vorlesungen über Differential- und Integralrechnung. Vol. III. *Integralrechnung auf dem Gebiete mehrerer Variablen*. By A. Ostrowski. Basel, Birkhäuser, 1954. 475 pp., 36 figs. 78 Swiss fr.; paper bound 73.85 Swiss fr.

This is the third and concluding volume of Professor Ostrowski's comprehensive text on calculus. For the reviews of volumes I and II, both by the present writer, the reader is referred to Bull. Amer. Math. Soc. vol. 52 (1946) pp. 798–799 and vol. 58 (1952) pp. 513–515. In this third volume there are seven chapters, all of them dealing to some extent with integration. There is an index for volumes II and III, a glossary of symbols for these two volumes, and a short list of corrections for volumes I and II.

Chapter I deals with the technique of integration. The topics are: complex numbers, partial fractions and integration of rational functions, integration of algebraic and transcendental functions, and the transcendence of e. The discussion of partial fractions is complete, with proofs.

Chapter II, on the definition of multiple integrals, is actually one third devoted to "the general case" of integrals of functions of one variable. The author first discusses sets of zero Jordan content, hereafter called *null sets*. He then defines the double integral of f over R, where R is a closed rectangle with sides parallel to the axes and f is continuous in R except on a null set, and is bounded. Such a function is called integrable, and the existence of its integral is proved. The same method is used for single and triple integrals. The author explains in the preface his reasons for this choice of a basic definition of integration for this particular book. Integrals over other bounded regions are defined by enclosing the region G in a rectangle R and extending the function by zero values in $R-G$. The boundary of G is required to be a null set. There follows the theory of evaluating multiple integrals by iterated integrals.

Chapters III and IV are concerned with line integrals, surface integrals, the divergence theorem, Stokes's theorem, exact differentials, change of variable in multiple integrals, and applications of multiple integrals to the finding of volumes, centers of mass, and so on. The treatment is careful and thorough. There is, for example, a proof that any plane region whose boundary consists of a finite number of smooth arcs can be cut up into a finite number of regions of "canonical" form relative to the axes (i.e. regions for which the intersection of the interior of the region with a line parallel to an axis is at most a single segment). It is shown that the divergence and curl, as expressed in rectangular coordinates, are invariants relative to rotations of axes.

Improper integrals are considered in Chapter V (functions of one variable) and Chapter VI (functions of several variables). Here also we find the discussion of important special integrals, a good deal about the gamma function, and consideration of the double limit situations (e.g. change of order of integration in iterated improper integrals) so crucial in many calculations. Chapter VI concludes with a demonstration of the integral inequality of Pauli, which can be used to express the uncertainty principle of quantum mechanics. The treatment of improper multiple integrals was very interesting to the reviewer, for this is a subject on which most books at this level are very sketchy. If G is any open set in the plane, the integral of f over G is defined in such a way that, if it exists, then the integral of $|f|$ over G also exists. Here, as so often elsewhere throughout these volumes of Ostrowski, there is much profit for the teacher of calculus and analysis.

The final chapter deals with Fourier series and integrals. The convergence theory is disposed of neatly and briskly (for functions differentiable from right and left at a point, when defined there by the limit from the corresponding side). Among the further topics are:

Parseval's formula, Poissons's summation formula, and the Rayleigh-Plancherel formula. The exercises contain many important results, including Fejér's theorem on summation of Fourier series by arithmetic means. Contrary to the standard practice, the author defines complex-valued functions f and g to be orthogonal if the integral of fg (instead of $f\bar{g}$) is 0. The discussion of Bessel's inequality and Parseval's formula, however, is solely for real-valued functions.

As in the first two volumes, in the third also we find a large and valuable collection of exercises. They occupy more than one fifth of the space in the book. Teachers of analysis will find a perusal of them rewarding.

The book as a whole is an admirable exposition of the fundamentals of calculus in a thoroughgoing way at a reasonably elementary level. The author does not hesitate to take time and space for unusually thorough discussion of matters which are often glossed over in elementary texts and ignored in more advanced works.

It is too bad that the book is so expensive. The cost of the three volumes is 184.10 Swiss francs, or about $43.00 at the current rate of exchange. This is formidable to the point of putting the book completely out of reach of most European students. As I was told in one German university town, a student can live for a month on less than it takes to buy the three volumes of this book.

ANGUS E. TAYLOR

The elements of probability theory and some of its applications. By Harald Cramér. New York, Wiley, 1955. 281 pp. $7.00.

This book may properly be called the junior students' *Mathematical methods of statistics*, which has won the author wide recognition. It fills a need for an introductory text for a class whose main interest is statistics. More than half (roughly from Chapter 8 on) of the material belongs to conventional statistics rather than conventional probability; in fact the latter part of the book is a small compendium on sampling and testing. Statisticians will, however, find this part rather old-fashioned: the Neyman-Pearson theory is barely touched upon while Wald's sequential analysis and decision theory are only mentioned. It would seem that the elements of such modern theories have interesting probability content and are no less amenable to an elementary discussion than some of the topics chosen here. As a probability text for the general mathematics student the book will be found somewhat lacking in attractions, although quite adequate and very respectable. It goes as far as the Bernoulli and De Moivre theorems and a statement of the central limit theorem and some of