BOOK REVIEWS

Limit distributions for sums of independent random variables. By B. V. Gnedenko and A. N. Kolmogorov. Trans. and annotated by K. L. Chung. Cambridge, Addison-Wesley, 1954. 264+9 pp. $7.50.

This book is a compact and beautifully presented account of the subject matter of its title. Some of the material is either new or previously available only in the Russian language. The authors have strived with good success to make the development complete and definitive and at the same time direct and simple. The main principle guiding them is to attain an exhaustive treatment of results within their framework at the expense of a broader coverage of subjects (such as error terms in the non-equidistributed cases, multi-dimensional limit theorems, generalizations to dependent random variables, etc.) for which complete results await future research. The authors certainly have much that is pertinent and interesting to say about such matters but have refrained from disrupting the unity and conciseness of their exposition by doing so.

Perhaps the most striking feature of the book lies in the incisive simplicity of its organization and exposition. The material is very cleanly and economically motivated—there is no attempt to overwrite or "sell" the subject, just as there is no apology when the authors heuristically discuss here and there connections between some of the theorems and the theory of additive processes, etc. Indeed there may be a risk here, as with any work which is extremely elegant, that the uninitiate, while able to carry through the proofs, may not appreciate nor even understand the theorems.

The following is a brief outline. The book is divided into three parts: I Introduction, II General limit theorems, III Identically distributed summands. Each of these parts consists of three chapters and there are two appendices, one by the translator and one by J. L. Doob. In a short preface sketching the development and history of the subject the authors set forth their main problem—viz., that of delineating the class of all limit laws for sequences of sums of independent and individually negligible random variables and the refinements of these results when the basic random variables are specialized in various ways.

The first two chapters give a compact account of measure and probability theory, convergence of measures and distributions and the basic theorems on Fourier transforms. Here the authors adopt a very inclusive axiomatic framework rather than the usual exposition of probability measure in n-dimensional Euclidean space as would suffice here where the theorems are concerned exclusively with convergence in distribution (and where the notions of conditional probabilities and expectations do not arise). This treatment, evidently a precursor of a forthcoming book by Kolmogorov on measure theory, is simplified in various ways, e.g. by the introduction of "perfect" measures, and some of the results here and in the succeeding chapters are stated in terms of more general (signed) measures. In the first appendix J. L. Doob has elaborated on this measure theoretic setting.

The introduction closes with the third chapter on infinitely divisible distributions which are defined as those which can be factored into the convolution of an arbitrary number of identical components. The canonical representation for these distributions is obtained quite easily from this definition, and this representation, together with theorems on the convergence of sequences of infinitely divisible distributions, is the cornerstone of all the subsequent development.

In chapter 4, which opens part II, the basic problem of establishing necessary and sufficient conditions for a sequence of sums of independent and individually negligible random variables to converge in distribution and showing that when the conditions are met the limiting distribution is necessarily infinitely divisible, is solved. From these results there flow, in a natural way, the various forms for the central limit theorems (uniform negligibility of the summands, truncated moments and other variants of the Lindeberg condition, etc.) convergence to the Poisson law, and the general limit laws for cumulative sums.

The last part deals with sums of identically distributed random variables. A stable law is defined as one which is invariant, within a linear transformation, under compositions with itself. After determining the canonical form of the stable laws, the necessary and sufficient conditions that a random variable belong to the domain of attraction of one of them are deduced, and the domains of partial attraction of the infinitely divisible laws are discussed. The penultimate chapter deals with improvements of convergence to the normal distribution: asymptotic expansions, the Berry-Esseen theorems, convergence of densities and the extremal nature of the lattice variables. The final chapter treats lattice valued identically distributed random variables in the domain of attraction of the stable laws and the corresponding convergence of the discrete mass points.

The authors have in certain major respects altered and inverted the historical development of the topics. For example, the various forms of the central limit theorem are viewed from a subordinate, and in the reviewer's opinion, proper perspective. There are numerous simple examples and counter examples which neatly point up the text.

One obtains the impression on reading the book that the creation and execution of the subject is almost exclusively Russian with an occasional interloper here and there. There are only one or two major results which are unequivocally attributed to outsiders. And while it is perhaps pointless here to press these matters it seems equally profitless for everyone concerned to obtain the authors' view that Schwartz' inequality, Hermite polynomials, etc. ought to bear Russian names.

Though there is a slight nonuniformity in the printing, due to the photo reproduction of the displayed formulas, the main effect is quite pleasing. There are surprisingly few misprints and only a very few rough spots in the analysis, in the latter sections. (The definition of the symbol $A\backslash B$ in the footnote on page 16 should read "the set of points in A, but not in B.")

In closing a word of commendation on the translation. This is much more than a pedestrian transliteration of the Russian text, and the translator has really made a critical analysis, correcting errors, emending the text in numerous places, bridging lacunae, etc. In appendix II he has disproved and analyzed an erroneous theorem of the original. In addition he seems to have caught the flavor of the authors' trenchant style, and the book is pleasant to skim through for a surface taste of the topics. It is, in short, eminently readable.

It seems clear that this book will serve as an authoritative model of clarity, simplicity and definitiveness for some time to come.

DONALD A. DARLING

Methods of theoretical physics. By P. M. Morse and H. Feshbach. New York, McGraw-Hill, 1953. Part I, 22+998+40 pp.; Part II, 18+979 pp. $30.00 a set.

The present two-volume book is a gigantic compendium of methods of mathematical physics. It is truly staggering in scope and one cannot but admire the authors for accomplishing a task of this magnitude. The two most notable features are: 1. an excellent account of the Wiener-Hopf technique with many important applications; 2. a systematic use of the Green's function technique in dealing with differential equations of physics.

The arrangement of material follows a "handbook" pattern, i.e. the methods and techniques are not necessarily arranged according to logical interconnections but rather according to their specific use