

THE APRIL MEETING IN CHICAGO

The five hundred twenty-third meeting of the American Mathematical Society was held at the University of Chicago on Thursday, Friday and Saturday, April 12, 13 and 14. There were a total of 360 registrations, including 317 members of the Society.

By decision of the Council of the Society, there was a Symposium on *Calculus of variations and its applications*. The Symposium was supported by contract with the cosponsoring organization, the Office of Ordnance Research. Sessions of the Symposium were held on Thursday and Friday morning and on Thursday afternoon.

In the first session on Thursday morning, Professor Eric Reissner of the Massachusetts Institute of Technology spoke on *Variational methods in linear theory of elasticity*; Professor D. C. Drucker of Brown University on *Variational principles in the mathematical theory of plasticity*; and Professor J. B. Keller, New York University, on *Variational methods in wave propagation*. The Chairman of the session was Professor L. M. Graves of the University of Chicago.

The Chairman for the session on Thursday at 2:00 P.M. was Professor C. A. Truesdell of Indiana University. The three addresses were entitled *Upper and lower bounds for eigenvalues*, *Stationary principles for forced vibrations in elasticity and electromagnetism*, and *Applications of variational methods in the theory of conformal mapping*. The speakers were, respectively, Professor J. B. Diaz, University of Maryland; Professor J. L. Synge, Dublin Institute for Advanced Studies; and Professor M. M. Schiffer, Stanford University.

At the session on Friday at 9:30 A.M., Dr. R. E. Bellman of the RAND Corporation, Professor Subrahmanyan Chandrasekhar, University of Chicago, and Professor E. H. Rothe of the University of Michigan were the speakers. The topics were, respectively, *Dynamic programming and its application to variational problems in mathematical economics*, *Variational principles in stability problems in hydrodynamics and hydromagnetics*, and *Some applications of functional analysis to the calculus of variations*. The Chairman of the session was Professor J. J. Gergen of Duke University.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor R. C. Buck of the University of Wisconsin addressed the Society on the topic *Linear transformations on function spaces*. Professor Buck's lecture was given on Friday afternoon with Professor T. H. Hildebrand presiding.

Sessions for the presentation of contributed papers were held on Friday afternoon, Saturday morning and afternoon. Presiding officers were Professors M. L. Curtis, M. R. Hestenes, D. H. Brunk, M. F. Smiley, and Drs. F. P. Peterson and E. H. Brown, Jr.

The Society takes this opportunity to thank the ladies of the Department of Mathematics for entertaining at tea on Thursday and Friday afternoons.

Abstracts of the papers presented follow. The number of a paper presented by title is followed by "t". In the case of joint authorship, the name of the person presenting the paper is followed by (p).

ALGEBRA AND THEORY OF NUMBERS

396. A. A. Albert: *On partially stable algebras.*

Consider a commutative simple power-associative algebra A of degree two over an algebraically closed field F of characteristic $p > 5$. Then $A = C + L$ where $C = A_u(1) + A_u(0)$ and $L = A_u(1/2)$, where u is a nontrivial idempotent of A . We call A partially stable if there is an idempotent u such that $CL \subseteq L$. In this paper we shall determine the structure of all such algebras. We shall show that $C = B + B(u - v)$ where B is either a simple algebra of degree one or is associative. It is not known whether or not there exist simple algebras of degree one which are not one-dimensional but, if they do exist, they will yield algebras $A = B + Bz + Bw$ where $(bw)(cw) = bc$, $b(cw) = (bc)w$, $(Bz)(Bw) = 0$ for every b and c of B where $B + Bz$ is the direct product of B and the associative algebra $F[z]$. When B is associative we have $(Bz)L = 0$ and the multiplicative formulas for the algebra A are essentially those of an earlier paper (Trans. Amer. Math. Soc. vol. 74 (1953) pp. 336-343). (Received February 10, 1956.)

397t. R. M. Baer: *Closure operators on partially ordered sets.*

A closure operator ϕ is an idempotent order-homomorphism which carries a partially ordered set X into itself and which satisfies $x \leq \phi(x)$, all $x \in X$. A nonempty set F in X is called a *partial ordinal* in X if $(x) \cap F$ is a nonempty set having a first element, for every dual principal ideal (x) in X . The set C_x of all ϕ on X is partially ordered in a natural way. Then (*) *A nonempty subset F of X is the fixed-point set of some closure operator ϕ on X if and only if F is a partial ordinal in X .* Using (*), the relation between lattice properties of X and lattice properties of C_x is studied and certain results of Ward [Ann. of Math. vol. 43 (1942) pp. 191-196] and Dwinger [Neder. Akad. Wetensch. vol. A58 (1955) pp. 36-40] are generalized. Further, it is shown that *if X satisfies the finite chain condition, then C_x is a lattice.* (Received April 13, 1956.)

398. S. K. Berberian: *Reduction of the projection geometry of a finite AW^* -algebra.*

Let A be a finite AW^* -algebra. The continuous geometry of projections of A can be reduced into a complete system of irreducible continuous geometries in the following way. Take any maximal ideal M , and let I be the ideal generated (algebraically) by the projections of M . Then the projections of A/I form an irreducible continuous geometry. If C is the regular ring attached to A , and J is the ideal of C generated by the projections of M , then C/J is the regular ring attached to A/I . (Received February 6, 1956.)

399. Richard Block: *New simple Lie algebras of order $p^n - 2$.*

Let F be a field of characteristic p and G an n -dimensional vector space ($n > 1$) over the prime field which is a direct sum of subspaces G_0, \dots, G_m , with $m = m(G) > 1$ in case $p = 2$ and $G_0 = 0$. Let δ_i be a nonzero element of G_i for $i = 1, \dots, m$. Let f_i be a nondegenerate skew-symmetric bilinear function on (G_i, G_i) to F such that for $i > 0$ there are linear functions g_i, h_i , on G_i to F with $g_i(\delta_i) = 0$ and $f_i(\alpha_i, \gamma_i) = g_i(\alpha_i)h_i(\gamma_i) - g_i(\gamma_i)h_i(\alpha_i)$, and in case $G = G_1$, with (δ_1) the kernel of g_1 . Let $L(G, \delta, f)$ be the algebra with basis $\{v(\alpha) \mid \alpha \in G, \alpha \neq 0, -\delta_1 - \dots - \delta_m\}$ and multiplication $v(\alpha)v(\gamma) = \sum_{i=0}^m f_i(\alpha_i, \gamma_i)v(\alpha + \gamma - \delta_i)$, $v(0) = 0$. Then L is a simple Lie algebra of dimension $p^n - 2$ (in case $G = G_0$, of dimension $p^n - 1$), not of type A unless $p = 3$ and $n = 2$. These algebras generalize the algebras L_δ, L_0 , and V_m of Albert and Frank [Rend. Sem. Mat. Torino vol. 14 (1954-1955) pp. 117-139]. A nondegenerate invariant form is determined by setting $\iota\{v(\alpha), v(\gamma)\} = 1$ if $\alpha + \gamma = -\delta_1 - \dots - \delta_m$, $= 0$ otherwise. $L(G, \delta, f)$ is restricted if and only if $G_0 = 0$ and G_i is 2-dimensional for $i > 0$. The derivations are determined and show that $L(G, \delta, f), L(G', \delta', f')$ are isomorphic only if $m(G) = m(G')$. Examples are given showing that an algebra may have Cartan subalgebras of distinct dimensions. (Received February 27, 1956.)

400. S. J. Bryant and J. L. Zemmer (p): *A theorem on topological ring extensions.*

A ring is called a *semi-topological ring* if its additive group is a topological group. A subring of a semi-topological ring is called a *topological subring* if it is a closed subset and its multiplication is continuous in the induced topology. The following theorem and several corollaries are proved: If a compact semi-topological ring A contains an open semi-simple topological subring then A is a topological ring. (Received February 17, 1956.)

401t. John DeCicco: *Quadratic extensions of a field.*

A quadratic extension of a field R is a commutative ring Γ with unit. This ring Γ admits an involutorial automorphism for which the self-conjugate elements form a field isomorphic to R . Characterizations within isomorphisms are obtained for such rings Γ . As applications, the systems of complex, dual complex, and hyperbolic complex numbers are characterized. Similarly quadratic extensions of modular number systems and other known fields are studied. (Received February 9, 1956.)

402. D. E. Edmondson: *Modular lattices.*

This is an algebraic study of the arithmetical properties of modular lattices and its application to the study of the congruence relations on a modular lattice. The principal tool is a fundamental characterization of the reducible and irreducible ideals of the lattice. This property characterizes modularity and is used to show that any congruence relation on a modular lattice can be generated by a collection of irreducible ideals. Finally it is shown that the congruence relations on the lattice of ideals of a modular lattice can be a Boolean algebra if and only if the lattice is finite dimensional. (Received March 1, 1956.)

403. C. C. Faith: *Extensions of normal bases and completely basic fields.*

Let K/F be normal. $u \in K$ is *basic* in K/F , if u generates a normal basis for K/F . The existence of a normal basis of K/F which is the extension of a normal basis of

K/M , for any inter-field M , is settled by the construction of $u \in K$ which is basic in K over every inter-field, if F is infinite. Any such u is a *completely basic element* of K/F . The class \mathfrak{C} of all *completely basic extensions*, i.e., all normal extensions for which every basic u is completely basic, is shown to properly include the Kummer extensions. Let $\mathfrak{B}_e/\mathfrak{F}$ be cyclic of deg p^e , p a prime with $g = g(\mathfrak{F}, p) \geq 1$, where g is the largest integer for which $x^{p^g} - 1$ factors linearly in \mathfrak{F} (if no such g exists, set $g = \infty$). Result: If $g(\mathfrak{F}, p) \geq e - 1$, then $\mathfrak{B}_e/\mathfrak{F} \in \mathfrak{C}$. Otherwise, $\mathfrak{B}_e/\mathfrak{F} \in \mathfrak{C}$ if and only if $g(\mathfrak{B}_e, p) = g(\mathfrak{F}, p)$. Let P be the root field over \mathfrak{F} of $x^{p^{e-1}} - 1 = 0$, contained in a field $\supseteq \mathfrak{B}_e$. If p is odd with $e - 1 > g(\mathfrak{F}, p) \geq 1$, then $\mathfrak{B}_e/\mathfrak{F} \in \mathfrak{C}$ if and only if $\mathfrak{B}_e \wedge P = \mathfrak{F}$. Other results: if $\mathfrak{B}/\mathfrak{F}$ is cyclic of deg n with generating automorphism S , then $u \in \mathfrak{B}$ is basic in $\mathfrak{B}/\mathfrak{F}$ if and only if $\sum_{i=0}^{n-1} u^{S^i} x^i = 0$ has no root ζ obeying $\zeta^n = 1$; Kummer fields K/F have a basis $\{\theta_i\}$ such that $u = \sum_{i=1}^n \alpha_i \theta_i$, $\alpha_i \in F$, is basic in K/F , and hence completely basic, if and only if each $\alpha_i \neq 0$; if $N/F \in \mathfrak{C}$, and if $N = N_1 X \cdots X N_t$ over F , then each $N_i/F \in \mathfrak{C}$. (Received February 29, 1956.)

404. L. E. Fuller: *Congruence of matrices over a principal ideal ring modulo p^k .*

In a previous paper (*A canonical set for matrices over a principal ideal ring modulo m* , Canadian Journal of Mathematics vol. 7 (1955) pp. 54-59) a definition of the degree of an element in the system was made. By a slight modification of the concept of degree, the Hermite form for a field under row equivalence becomes a special case of that paper. A diagonal form under general equivalence is also shown to include that for the field. Congruent equivalence is considered for both symmetric and skew symmetric matrices. The diagonal forms obtained are similar to those for the case of a field. The major difference is the appearance of the prime of the modulus to various powers. By making a generalized definition of quadratic residue, a further simplification in the symmetric case becomes possible for some systems. The diagonal elements can be reduced to the form $\pm p^t$ where $t \leq k$. Again the form for a real symmetric matrix is shown to be a special case. (Received February 21, 1956.)

405. Franklin Haimo: *Normal automorphisms of a class of holomorphs.*

The group of normal automorphisms of a group G is the centralizer of the group of inner automorphisms in the group of automorphisms of G . Consider the relative holomorph of G over a group B of automorphisms of G where B includes the group of inner automorphisms and is included in the centralizer of the group of normal automorphisms. It is shown that the group of normal automorphisms of this holomorph is isomorphic to a splitting extension of the (additive) group of homomorphisms of B into the group F of fixed points of G under the mappings from B by the group of automorphisms of G , the members of which induce the identity on G/F . (Received February 28, 1956.)

406. D. R. Hughes: *Regular collineation groups.*

A λ -plane π with parameters v, k, λ (i.e., a (v, k, λ) configuration, or a symmetric balanced incomplete block design) is termed *regular of degree m* if π possesses a collineation group G of order m , no non-identity element of which fixes any point or line of π . Then $t = v/m$ is an integer. If π is a λ -plane with parameters v, k, λ , regular of degree m , then there is a square matrix A of order t , consisting entirely of non-negative

integral entries, such that $AA^T = A^T A = B$, where B has $(k-\lambda) + \lambda m$ on the main diagonal and λm elsewhere. Thus, if t is odd, it follows that the equation $x^2 = (k-\lambda)y^2 + (-1)^s \lambda m z^2$, where $s = (t-1)/2$, has a nontrivial solution in integers. It is thus possible to show that for many choices of v, k, λ , any λ -plane cannot be regular of degree greater than one (nontrivial results for $\lambda=1$ are included). Since any λ -plane is regular of degree one, the above results (with $m=1$) apply to any λ -plane, and yield the well-known incidence matrix equations. (Received February 20, 1956.)

407. Bernard Jacobson: *Sums of distinct divisors of algebraic integers.*

In a recent paper B. M. Stewart discussed sums of positive distinct divisors of rational integers [Amer. J. Math. vol. 76 (1954) pp. 779-785]. In this article these results are generalized. Let $\alpha(M)$ be the number of positive integers n which can be written in the form $n = \sum d$, where the d are distinct positive or negative divisors of M . The author has proved that $\alpha(M) = \sigma(M)$ if and only if n is of the form $n = 2^b 3^c \prod_{i=1}^k p_i^{a_i}$ where b and c are not both zero, $3 < p_1 < p_2 < \dots < p_k$, $p_i \leq 2\sigma(2^b 3^c) + 1$ and $p_{i+1} \leq 2\sigma(2^b 3^c \prod_{j=1}^i p_j^{a_j}) + 1$ for $j=1, 2, \dots, k-1$. The function $\alpha(M)/\sigma(M)$ is everywhere dense on the interval 0 to 1. In the quadratic fields $x+y(2^{1/2})$ and $x+y(5^{1/2})$ every integer in the field can be written as a finite sum of distinct units, the algorithm used depending upon the representation of each integer of the field as a lattice point in the plane. In any real quadratic field there exist infinitely many integers n , having the property that every integer in the field can be written as a finite sum of distinct divisors of n . Explicitly if $a+b(m^{1/2})$ is the unit of smallest absolute value for which $a>0$ and $b>0$, then any integer $2^{t+1}(m^{1/2})$ where $2^t > a$ satisfies this condition. Analogous results have been found for imaginary quadratic fields. (Received February 27, 1956.)

408. E. D. Nering: *An integral basis theorem for algebraic function fields.*

Let K be an algebraic function field with field of constants k . Let x be an element in K transcendental over k so that K is finite algebraic over $F = k(x)$. Let $\mathfrak{o} = k[x]$ be the ring of polynomials with coefficients in k , and let $\mathfrak{D} \subset K$ be the ring of elements in K integral over \mathfrak{o} . Let h be a discrete non-archimedean valuation of k extended to a valuation of F by the Gaussian definition. Let H_i ($i=1, \dots, s$) be all the extensions of h to valuations of K . Let F^* denote the ring of elements in F integral with respect to h , and let K^* be the ring of elements in K integral over F^* . For any set \mathfrak{A} in K , let $\mathfrak{A}^* = \mathfrak{A} \cap K^*$. If all the extensions of H on F to H_i on K have radical degree 1, then for every ideal \mathfrak{A} of \mathfrak{D} , there exists a basis of K over F which is an integral basis of \mathfrak{A}^* over \mathfrak{o}^* . (Received February 27, 1956.)

409t. C. A. Nicol: *On the number of solutions of certain linear congruences.*

Let $F_n(z, x) = \prod_{j=1}^{n-1} (z - x^j)$. If this product is expanded as a polynomial in x , the coefficients are polynomials in z , whose coefficients have combinatorial properties related to partitions. The combinatorial properties of these polynomials in z are shown to be related to the number of solutions of $x_1 + \dots + x_n \equiv t \pmod{n}$. Then the number of solutions of these linear congruences is expressed in terms of the Von Sterneck number [A. Von Sterneck, *Arithmetical function and restricted partitions with respect to a modulus*, C. A. Nicol and H. S. Vandiver, Proc. Nat. Acad. Sci. U.S.A. vol. 40 (1954)

pp. 825–35]. Some of these results may be found in *Niedere Zahlentheorie*, P. Bachmann, Part 2 (Leipzig, Teubner, 1909). (Received March 1, 1956.)

410. A. M. Yaqub: *On the identities of certain algebras.*

Let $\mathcal{A} = (A, \times, \circ, \dots)$ be a binary algebra with primitive operations \times, \circ, \dots . An A -function $f(\zeta, \eta, \dots)$ is a function from A, A, \dots to A . An \mathcal{A} -function is an A -function which is a primitive composition of some indeterminate-symbols ζ, \dots of A and a (possibly empty) set of constants ($=$ fixed $\in A$). A *strict* \mathcal{A} -function is an \mathcal{A} -function which involves no constants. An \mathcal{A} -identity $f(\zeta, \dots) = g(\zeta, \dots)$ is an identity between the \mathcal{A} -functions f, g . When both f and g are *strict*, the identity is called *strict* \mathcal{A} -identity. \mathcal{A} is *strictly complete* if A is finite and if each A -function may be expressed as some *strict* \mathcal{A} -function (see A. L. Foster, *Math. Zeit.* vol. 58 (1953) pp. 306–336). The following theorem is proved: A strictly complete algebra with more than one element has a finite basis, i.e., a finite set of identities from which *all* the strict identities of the algebra are logical consequences. The proof uses the concept of free algebra. Finally, an example by Lyndon (*Proc. Amer. Math. Soc.* vol. 25 (1954) pp. 8–9) is treated in regard to the above finite basis theorem. (Received February 23, 1956.)

ANALYSIS

411t. R. W. Bass: *Global continuation of periodic solutions of ordinary differential equations.*

When Poincaré's method of continuation is formulated abstractly, it appears that his classical hypothesis (concerning the nonexistence of nontrivial solutions of the variational equations of the known solution) merely asserts that the Leray-Schauder index does not vanish. Similarly, in the important but difficult degenerate cases, a result of the author supplies the LS index as the Kronecker index of the bifurcation equations. These two results permit establishment of the existence of periodic solutions by *non-local* perturbation processes, which was Poincaré's original goal. Moreover, the known results become special cases of a single theory. The integral equation technique of D. C. Lewis, Jr. (based on that of L. Lichtenstein and E. Hölder) is brought to completion in a form which can be shown "best possible of its type." In illustration, a direct perturbation proof is applied to van der Pol's equation. It is often convenient, e.g. for autonomous oscillations, to employ instead the method of undetermined Fourier coefficients (following G. W. Hill, A. Wintner, C. L. Siegel). In this approach, several new techniques are developed. There are applications to non-linear mechanics. (Received February 29, 1956.)

412t. R. W. Bass: *On solution of singular functional equations by the Leray-Schauder theory.*

For each fixed $\lambda \in [0, \Delta]$ let $F(x; \lambda)$ be a completely continuous map of a Banach space into itself. Let F be uniformly continuous in λ for x in any fixed ball, and suppose that x_0^k ($k=1, \dots, n$) satisfy (E): $x - F(x; \lambda) = 0$ at $\lambda=0$. Suppose that the Fréchet differentials $L^k = F_x(x_0^k; 0)$ exist and are completely continuous linear operators. The "regular" case occurs when $(I - L^k)x = 0$ implies $x = 0$ ($k=1, \dots, n$); it is then usually possible to compute the Leray-Schauder total index $i(F)$. If $i(F) \neq 0$, and if a uniform *a priori* bound for the solutions of (E) is known, then (E) has a solution for every $\lambda \in I_\Delta$. In the singular case, finite dimensional bifurcation equations (V): $g(x, c; \lambda) = 0$ occur; this case has been treated purely *locally* (i.e., for values of λ

sufficiently near to zero) by Liapounoff, Schmidt, Iglisch, Lichtenstein, Friedrichs, Cronin, Bartle, and Graves. However it is frequently possible (by inspection, or Newton's polygon) to replace (V) by modified equations (V'): $g'(x, c; \lambda) = 0$ in such a way that $i(F) = KI(g')$, where KI , the Kronecker Index of g' , is just the sum of the signs of the Jacobians of the initial solutions of (V'). In this way (E) can be solved for all $\lambda \in I_A$ even in the singular case. (Received February 29, 1956.)

413. G. U. Brauer: *Tauberian theorems for general integral transforms.*

Let $\phi(t)$ denote a real-valued function, integrable on each finite interval $[0, T]$, and let $K(x, t)$ be defined for nonnegative values of x and t , and be in L^1 on $0 \leq t < \infty$, for each non-negative value of x . The K -transform of ϕ is the function $\int_0^\infty K(x, t)\phi(t)dt$. The integral is assumed to converge for $x > 0$. If $\phi(t) = o(1/t)$, $t \rightarrow \infty$, and the kernel K satisfies the conditions (i) $|K(x, t) - K(0, t)| \leq Lxt$, where L is a universal constant, (ii) $\int_0^\infty |K(x, t)|dt = O(1/x)$, $x \rightarrow 0+$, then the existence of $\lim_{x \rightarrow 0+} \int_0^\infty K(x, t)\phi(t)dt$ ensures the convergence of the integral $\int_0^\infty K(0, t)\phi(t)dt$. If the condition on $\phi(t)$ is replaced by $\int_0^\infty t[\phi(t)]^2 dt < \infty$ and condition (ii) on K is replaced by $\int_0^\infty [K(x, t)]^2 dt = O(1/x)$ as $x \rightarrow 0+$, then the same conclusion holds. These results are analogues of well-known theorems on Taylor series. If x is a complex variable and the kernel is analytic in x , then an analogue of Riesz' theorem can be obtained. (Received March 2, 1956.)

414. T. A. Elkins: *Orthogonal harmonic functions in space.*

In two dimensions it is easily shown by the use of the complex variable that given any nonconstant harmonic function, another nonconstant harmonic function can be found such that the product of the two is harmonic. It is shown by presenting a counter example that this result does not extend to three dimensions. A study is made of some classes of harmonic functions in space for which such a nonconstant harmonic multiplier does exist. This is the case for all harmonic functions depending on only one or two variables. Finally, solutions are given to the following two problems: (1) determining whether or not for a given harmonic function of three variables there exists a nonconstant harmonic function such that the product of the two is harmonic; (2) determining all such functions in case one does exist. When a harmonic multiplier v exists for a given harmonic function u , the two functions are easily shown to be orthogonal in the sense that $u_x v_x + u_y v_y + u_z v_z = 0$; which explains the name given to them in the title. (Received February 27, 1956.)

415. Jacob Feldman and J. M. G. Fell (p): *Separable representations of rings of operators.*

Let A be a ring of operators in a Hilbert space H . A *-representation of A (that is, a *-homomorphism of A into the bounded operators on some Hilbert space K) is separable if both H and K are separable. Now any *-representation L is norm-continuous; if in addition it is σ -weakly continuous, then the kernel of L is a direct summand of A , and the range of L is a ring of operators in K . It is shown that a separable *-representation L of A is always σ -weakly continuous if one of the following conditions holds: (1) A is purely infinite, i.e., contains no finite direct summand, (2) A is a factor of Type II₁, (3) A is finite of Type II and the range of L is contained in a finite ring of operators in the range space K . An immediate corollary of this result is

the nonexistence of separable irreducible *-representations of A when A is purely infinite or a factor of Type II₁. It remains an open question whether, for an arbitrary Type II finite algebra A , all separable *-representations of A are σ -weakly continuous. (Received February 27, 1956.)

416t. Casper Goffman: *Sector properties.*

In his work (*Exceptional sets*, Fund. Math. vol. 32, pp. 3–32) on sets derived from interval properties, a simple observation made by H. Blumberg was that the set of points of anti-symmetry of an arbitrary interval property is countable. He then obtained analogues of this fact, in the plane, for segment properties, region properties, etc. The purpose here is to obtain a result regarding circular sectors in the plane of which these facts are special cases. It is to be remarked that while an interval property is a subdivision of the set of all intervals into two disjoint sets, those which have the property, and those which do not have the property, the present result pertains to a subdivision of the set of sectors into three disjoint sets, A , B , and C where, for two mutually piercing sectors, if one belongs to A the other does not belong to B . A set S , for example, leads to such a subdivision if a sector is in set A if it is a subset of S ; in set B , if it is a subset of CS ; and in set C , otherwise. Certain converse results are also obtained. (Received February 10, 1956.)

417. R. R. Goldberg (p) and R. S. Varga: *Moebius inversion of Fourier transforms.*

Let $F(t) = \int_0^\infty \phi(u) \cos tu \, du$. Let $G_N(t) = (1/t)[F(0)/2 + \sum_{k=1}^N (-1)^k F(k\pi/t)]$. If (1) $\int_0^\infty |\phi(t)| \, dt < \infty$ and (2) $\phi(t)$ is of bounded variation on every finite interval, then $G(t) \equiv \lim_{N \rightarrow \infty} G_N(t) = \sum_{k=1}^\infty \phi[(2k-1)t]$ almost everywhere ($0 < t < \infty$). Let $\{\mu_n\}_{n=1}^\infty$ be the Moebius numbers defined as $\mu(1) = 1$, $\mu(n) = (-1)^s$ if n is the product of s distinct primes, $\mu(n) = 0$ if n is divisible by a square. Then, using the well-known property $\sum_{d|m} \mu(d) = \delta_{1,m}$, one can show that $\sum_{n=1}^\infty \mu_{2n-1} G[(2n-1)t] = \phi(t)$ almost everywhere ($0 < t < \infty$), provided that, in addition to conditions (1) and (2) above, the condition (3) $\int_1^\infty |\phi(t)| \log t \, dt < \infty$ holds also. The sine transform may be inverted by a similar procedure. (Received February 20, 1956.)

418. J. H. B. Kemperman: *A functional equation.*

Let A be an open connected subset of the Euclidean space E_m and let Ω be the class of all functions on A equal to a polynomial. Then Ω satisfies the following property Q. Let a_i be real constants, $a_i \neq a_0 = 0$ ($i = 1, \dots, k$). Let B be a set of positive measure in E_m and let $f_i(t)$ be defined on $A + a_i B$ ($i = 0, \dots, k$) such that, for $x \in B$, $\sum_0^k f_i(a_i x + t) \in \Omega$. If, moreover, $f_0(t)$ is bounded on a set $C \subset A$ of positive measure then $f_0(t) \in \Omega$. This yields an easy proof of a result due to V. P. Skitovic (Izv. Akad. Nauk vol. 18 (1954)) concerning the normal distribution. Property Q is shown to hold for many other classes Ω . As a corollary to one of the auxiliary theorems the author obtains (cf. H. Steinhaus, Fund. Math. vol. 1 (1920)): let B_1 and B_2 be subsets of E_m , each of positive measure. Then, after deleting proper subsets of measure 0 from B_1 and B_2 , the set of differences $B_1 - B_2$ becomes an open subset of E_m . (Received February 28, 1956.)

419. G. L. Krabbe: *Rings and spectra of factor-sequence operators on L^p .*

Suppose $1 \leq p < \infty$ throughout, and let \mathcal{Q}_p denote the set of all factor-sequences of

type (L^p, L^p) (see Hille, *Functional analysis and semi-groups*, pp. 343-344). Any a in \mathfrak{A}_p defines a transformation T_a belonging to the set \mathfrak{E}_p of all bounded linear mappings of L^p into itself. *Lemma*: If $a \in \mathfrak{A}_p$, then the point-spectrum of T_a is the range of a . If \mathfrak{R} is a ring, denote by \mathfrak{R}^* the set of all regular elements of \mathfrak{R} . Recall that $\mathfrak{A}_1 \subset \mathfrak{A}_p \subset \mathfrak{M}$, where \mathfrak{M} is the ring of all bounded sequences. *Theorems*: (I). Suppose that \mathfrak{R} is a subring of \mathfrak{A}_p with $\mathfrak{R} \cap \mathfrak{M}^* \subset \mathfrak{R}^*$; if $a \in \mathfrak{R}$, then the spectrum of T_a is the closure $E(a)$ of the range of a . (II). Suppose a Banach algebra \mathfrak{R} satisfies the conditions of (I). If the topology of \mathfrak{R} is finer than the product-topology, then the mapping $a \rightarrow T_a$ of \mathfrak{R} into \mathfrak{E}_p is a continuous isomorphism; moreover, if $a = \{a_n\}_n \in \mathfrak{R}$ and if f is an analytic function on $E(a)$, then the sequence $\{f(a_n)\}_n$ is in \mathfrak{R} . This extends a result of Hille [17.2.3 in loc. cit.] concerning the case $\mathfrak{R} = \mathfrak{A}_1$, $p=1$. When $p=2$, then $\mathfrak{R} = \mathfrak{A}_2$ verifies (II). For $p>1$, a ring $\mathfrak{R} = \mathfrak{R}_p$ studied by Marcinkiewicz [Studia Math. vol. 8 (1939) pp. 78-91] satisfies the conditions of (I), and the set of all sequences of bounded variation form a ring $\mathfrak{R} \subset \mathfrak{R}_p$ satisfying the conditions of (I) and (II). (Received February 23, 1956.)

420. M. Z. Krzywoblocki: *On the generalized Bergman's method of linear integral operators.*

Assume that in the fundamental equation underlying the final form to which Bergman's method is applied, there appears an arbitrary given function of independent variables. Then the author shows that under these assumptions the Bergman method of the linear integral operator can be applied directly to that generalized equation. The proof of existence follows directly; it is shown what kind of changes one has to introduce in the tabular representation of Bergman's functions in order to adjust them to the new assumption. (Received February 17, 1956.)

421. R. G. Kuller: *Representations of locally convex vector lattices.*

The Banach lattice has been defined as a lattice ordered Banach space in which the norm and the ordering are related by the axiom (A) If $|x| \leq |y|$ then $\|x\| \leq \|y\|$. In this paper the locally convex vector lattice is defined as a locally convex Hausdorff linear space, which is lattice ordered, and in which there exists a fundamental system of neighborhoods $[V_\alpha]$ of 0, such that the corresponding semi-norms satisfy the inequality (a). If $|x| \leq |y|$, then $p_\alpha(x) \leq p_\alpha(y)$. A complete locally convex vector lattice is the projective limit of Banach lattices. If, with (a), one postulates (m) If $x \wedge y \geq 0$, then $p_\alpha(x \cup y) = \text{Max}[p_\alpha(x), p_\alpha(y)]$, for all α , one obtains the locally m -convex vector lattice, a natural generalization of the abstract M space defined by Kakutani. If, with (a), one postulates (1) If $x \wedge y \geq 0$, then $p_\alpha(x+y) = p_\alpha(x) + p_\alpha(y)$ for all α , one obtains the locally 1-convex vector lattice, the generalization of the abstract L space of Kakutani. Representation theorems for these two types of topological vector lattices are given. (See the Ann. of Math. (2) vol. 42 (1941) for Kakutani's papers.) (Received February 22, 1956.)

422t. Karel deLeeuw: *Homogeneous Banach algebras. II. Classification of uniformly closed algebras.*

Let G be a compact abelian topological group. A homogeneous Banach algebra on G is a separating, translation invariant Banach algebra of complex valued continuous functions on G that contains 1 and is generated by the characters of G that it contains and also whose topology is stronger than pointwise convergence. *Theorem*: There is a natural 1-1 correspondence between the class of homogeneous algebras A on G that are uniformly closed on their maximal ideal space and the class of all pairs (S, p)

where S is a subsemigroup with unit of G^* that generates G^* and p is a real valued function on S that satisfies 1. $p(s) \geq 1$ for all s in S ; 2. $p(e) = 1$; 3. $p(s_1 s_2) \leq p(s_1) p(s_2)$; 4. $p(s^2) = p(s)^2$. The correspondence is set up by taking S to be the subset of characters in G^* that occur in A and defining p by $p(s) = \|s\|$, where $\|\cdot\|$ is the sup norm. The only real difficulty arises in showing that no two distinct algebras can give rise to the same p . This is accomplished by the use of a result in the theory of several complex variables that relates polynomial and monomial convexity. (See Bull. Amer. Math. Soc. Abstract 62-1-142.) Those algebras having $p \equiv 1$ have been studied extensively by Arens and Singer in *Generalized analytic functions*, Trans. Amer. Math. Soc. vol. 81 (1956) pp. 379-393. (Received February 27, 1956.)

423. Karel deLeeuw: *Silov boundary and Cauchy integral formula for circular subsets of the space of n complex variables.*

For any point z in C^n , z_i will denote its i th coordinate. Let X be a bounded subset of C^n that contains, for each x in X , every y that satisfies $|y_i| = |x_i|$. Let A be the Banach algebra obtained as the uniform closure of polynomials on X . *Theorem:* The maximal ideal space of A is identified with $Y = \{y: |m(y)| \leq \text{Sup}_{x \in X} |m(x)|, \text{ all monomials } m\}$ by associating to each y in Y the multiplicative linear functional $f \rightarrow f(y)$. A point y in Y is called a *multiplicative extreme point* if $(y_1^2, \dots, y_n^2) = (u_1 v_1, \dots, u_n v_n)$ for points u and v in Y implies $|u_i| = |v_i|$. *Theorem:* The Silov boundary of A , that is, the smallest closed subset of Y on which every f in A attains its maximum modulus, is the closure of the multiplicative extreme points in Y . Let $\{u\}$ be a collection of multiplicative extreme points in Y and $\{M_u\}$ a partition of the set of monomials such that each monomial in M_u attains its maximum modulus at u . These can always be found. Let T_u be $\{(u_1 t_1, \dots, u_n t_n): |t_i| = 1\}$ and μ_u the measure on T_u induced by Haar measure on $\{t: |t_i| = 1\}$. Let $K_u(x, y) = \sum_{m \in M_u} m(x) m^{-1}(y)$. *Theorem:* For any f in A , in particular for any f analytic in a neighborhood of Y , and any x in the interior of Y , $f(x) = \sum_u \int_{T_u} f(y) K_u(x, y) d\mu_u(y)$. (Received February 27, 1956.)

424t. Karel deLeeuw: *Subsemigroups of compact groups are equidistributed.*

Let G be a compact topological group with normalized Haar measure μ . Let S be a subsemigroup that generates a dense subgroup of G and $C(S)$ be the space of bounded complex valued functions on S with the uniform topology. *Theorem:* If f in $C(S)$ is the restriction of a continuous function h on G , then the unique constant function in the convex closure of the translates of f on S has the value $\int_G h d\mu$. In most special cases the equidistribution can be exhibited more explicitly. Let $\{U_m\}_{m=1}^\infty$ be a collection of finite subsets of S that satisfies $N(s U_m \Delta U_m) / N(U_m) \rightarrow 0$ for every s in S , where $N(X)$ is the number of elements of X and Δ is symmetric difference. *Theorem:* For f and h as above, $\sum_{s \in U_m} f(sx) / N(U_m) \rightarrow \int_G h d\mu$ uniformly in x . As an application, if α and β are irrational, the semigroup that consists of all $m\alpha + n\beta$, for all lattice points (m, n) in a sector of the plane, is equidistributed mod 1. Also if γ and δ are positive real with γ/δ irrational, the semigroup of $m\gamma + n\delta$, for m and n as before, is equidistributed in the almost-periodic compactification of the reals. (Received February 27, 1956.)

425. Lee Lorch: *The Gibbs phenomenon for Borel means.*

The following theorem is proved: Let $B_x(t)$ denote the x th Borel exponential or

integral mean of the Fourier series $\sum_1^{\infty} n^{-1} \sin nt$ and let T be given, $0 \leq T \leq \infty$. Then $B_x(x) \rightarrow \int_0^T v^{-1} \sin v \, dv$ as $x \rightarrow \infty$, where $x \rightarrow 0+$ and $xt_x \rightarrow T$. Thus, the Gibbs ratio is the same for Borel means as for convergence, and is achieved for the same value, π , of the parameter T . For the exponential means the derivation is similar to that of the Lebesgue constants for the corresponding case (Duke Math. J. vol. 11 (1944) pp. 459-467). For integral means the result then follows from a standard identity connecting the two Borel methods [cf. Hardy, *Divergent series*, 1949, p. 182, formula (8.5.5)]. (Received February 21, 1956.)

426. T. D. Oxley, Jr. (p) and H. K. Hughes: *On a generalized factorial series.*

It was shown (T. Fort, *Infinite series*, Oxford, 1930) that the series $\sum_{n=1}^{\infty} a_n A_n^{(k)}(z)$, where $A_n^{(k)}(z) = k\Gamma(nk)\Gamma(z+k)/[(z+k-1)\Gamma(z+nk)\Gamma(k)]$, converges in a half-plane $R(z) > \lambda$ (absolutely for $R(z) > \mu$) and has other function theoretic properties common to the simple factorial and Dirichlet series which it includes. (1) Let $\Omega_k(z) = \sum_{n=1}^{\infty} a_n A_n^{(k)}(z)$, $k \geq 1$ and m a positive number $> \mu$. Then $\Omega_k(z) = \int_0^1 t^{k-1} \phi(t) dt$, where $\phi(t) = 1/2\pi i \int_{m-i\infty}^{m+i\infty} t^{-z} \Omega_k(z) dz = \sum_{n=1}^{\infty} a_n k \Gamma(nk) t^{k-1} (1-t)^{(n-1)k} / [\Gamma(k) \Gamma\{(n-1)k+1\}]$, for $t \in [0, 1]$. (2) If $\phi(t)$ is analytic in a region including the interval $0 \leq t \leq 1$, then $\Omega_k(z)$ is analytically continued to the entire z -plane (except at the points where $z-1$ is an integer) by $\Omega_k(z) = 1/[e^{2\pi i(z-1)} - 1] \int_L t^{z-1} \phi(t) dt$, where L is a contour proceeding from 1 along the upper side of the positive real t -axis, circling the origin, and returning to 1 along the lower side. (3) If k is complex, $R(k) > 0$, $|\arg k| \leq \pi/2 - \epsilon$, and there exists $g(w)$ for which $(-1)^n g(n) = a_n$, $g(w)$ is single-valued, analytic for $R(w) \geq 1/2$, $|g(x+iy)| \leq Kx^p e^{p|y|(\pi-\epsilon)}$, p a positive number, then for all z (except when $z+k=1, 0, -1, \dots$) $\Omega_k(z) = -k\Gamma(z+k)/[2(z+k-1)\Gamma(k)] \int_{-\infty}^{\infty} g(\frac{1}{2}+iy)\Gamma(\frac{1}{2}k+iky)/[\Gamma(z+\frac{1}{2}k+iky) \cosh \pi y] dy$. Results (2) and (3) reduce to those of H. K. Hughes (Amer. J. Math. vol. 53 (1931) pp. 757-780) for the factorial series. (Received February 27, 1956.)

427. W. T. Reid: *A note on adjoint linear differential operators.*

With the aid of the fundamental lemma of the calculus of variations the following result is established for linear differential operators $L(y) = \sum_{\mu=0}^n p_{\mu}(x)y^{(\mu)}$ ($n \geq 1$), with coefficients in the space \mathfrak{L} of complex-valued Lebesgue integrable functions on $ab: a \leq x \leq b$. If \mathfrak{C}_n is the class of functions with continuous derivatives of the first n orders on ab , $\mathfrak{C}_n^0 = \{y; y \in \mathfrak{C}_n: y^{(\alpha)}(a) = 0 = y^{(\alpha)}(b), \alpha = 0, \dots, n-1\}$, $\mathfrak{A}_n = \{y; y \in \mathfrak{C}_{n-1}: y^{(m-1)} \text{ absolutely continuous on } ab\}$, and \mathfrak{D}^* is the set of $z(x)$ with $z \in \mathfrak{L}$, $\mathfrak{z} p_{\mu} \in \mathfrak{L}$, ($\mu = 0, \dots, n$), for which there exists a corresponding $f_z \in \mathfrak{L}$ such that $\int_a^b [zL(y) - \bar{f}_z y] dx = 0$ for arbitrary $y \in \mathfrak{C}_n^0$, then in case $x^\lambda/\lambda! \in \mathfrak{D}^*$ ($\lambda = 0, \dots, n$), we have: (i) there exist functions $\pi_0(x) \in \mathfrak{L}$, $\pi_j(x) \in \mathfrak{A}_{kj}$ ($k_j = [(j+1)/2]$; $j = 1, \dots, n$), such that $L(y) = \sum_{j=0}^n \Lambda_j(x; \pi_j)$, where $\Lambda_0(y; \pi) = \pi(x)y$, $\Lambda_{2r}(y; \pi) = [\pi(x)y^{(r)}]^{(r)}$, and $\Lambda_{2r-1}(y; \pi) = \{[\pi(x)y^{(r-1)}]^{(r)} + [\pi(x)y^{(r)}]^{(r-1)}\}/2$, ($r = 1, 2, \dots$); (ii) $\mathfrak{A} \subset \mathfrak{D}^*$, and $f_z = \sum_{j=0}^n \Lambda_j(z; (-1)^j \bar{\pi}_j)$ for $z \in \mathfrak{A}_n$. Various results related to the above are established; in particular, there are presented relations between the above result and certain theorems of H. L. Hamburger (Proc. London Math. Soc. (3) vol. 3 (1953) pp. 446-463) for Sturm-Liouville differential operators. (Received February 28, 1956.)

428t. E. M. Stein: *Generalization of a theorem of Paley and Wiener.*

The author generalizes a well-known result of Paley and Wiener concerning functions of exponential type to the case of n dimensions. One such generalization has been given by Plancherel and Pólya (Comment. Math. Helv. vol. 9, pp. 224-248) but

the following is of different nature. Let Ω be a compact, convex, and symmetric domain in Euclidean n -space. Let Ω^* denote its polar domain, $\psi(x)$ the characteristic function of Ω^* , and $\psi_t(x) = t^{-n}\psi(x/t)$. Finally, let $H_t(x) = \int f(x+y)\psi_t(y)dy$, where it is assumed that $f(x) \in L_2(E_n)$, and that $f(x)$ is bounded. It is asserted: The Fourier transform of $f(x)$ vanishes outside Ω if and only if $H_t(x)$ is analytic and of exponential type ≤ 1 in t , for each x . It is an easy matter to verify that the case $n=1$ implies the classical result of Paley and Wiener. (Received February 27, 1956.)

429*t.* E. M. Stein: *Inequalities of Bernstein and Kolmogoroff.*

A general method is given for passing from theorems involving the L_∞ norm to their analogues for the L_p norms. Among applications are the following. Let $f(x)$ be analytic and of exponential type σ . Suppose that $f(x) \in L_p(-\infty, +\infty)$, $1 \leq p < \infty$. Then the following is well known: $\|f'(x)\|_p \leq \sigma \|f(x)\|_p$. It is shown that $\|f'(x)\|_p = \sigma \|f(x)\|_p$ implies that $f(x) = 0$. The analogous statement for the sup norm is well known. Another application is the following: Let $f(x)$ be a function whose derivatives up to order n are continuous and lie in $L_p(-\infty, +\infty)$. It is shown that $\|f^{(k)}(x)\|_p^n \leq C_{k,n} \|f(x)\|_p^{n-k} \cdot \|f^{(n)}(x)\|_p^k$. Here, $0 < k < n$, and $C_{k,n}$ are the same constants which are valid for the limiting case $p = \infty$. The case $p = \infty$ is due to Kolmogoroff (see Translation of the American Mathematical Society, no. 4). (Received February 27, 1956.)

430*t.* E. M. Stein: *Mean convergence below the critical index.*

Let $f(x) \in L_p(E_n)$ ($E_n =$ Euclidean n -space, $n \geq 2$), and let $S_R^\delta(f)$ denote the Bochner-Riesz spherical means of order δ of the "partial integrals" of the Fourier expansion of $f(x)$. The index $\delta_0 = (n-1)/2$ is known as the "critical" index. It is shown that if $1 < p < 2$, and $f \in L_p$, then $S_R^\delta(f)$ converges to f in L_p norm as $R \rightarrow \infty$, whenever $\delta > (2p^{-1} - 1)\delta_0$. A similar statement holds if $2 < p < \infty$. In particular, if $1 < p < \infty$, then there is convergence of $S_R^\delta(f)$ to f in L_p norm for some index δ below the critical index. An analogous theorem holds for multiple Fourier series. (Received February 27, 1956.)

431. F. M. Wright: *Sufficient conditions for certain C-fraction expansions.*

Let $\{k_j\}$ and $\{p_j\}$ ($j=0, 1, 2, \dots$), be given infinite sequences of real integers^s such that $k_0=1$, $p_0=-1$, and $k_j \geq 0$, $p_j \geq 0$ for $j=1, 2, 3, \dots$. Let $\{\alpha_n\}$ ($n=1, 2, 3, \dots$) be the sequence of positive integers defined by the following rules: (i) $\alpha_1=1$; (ii) $\alpha_n=1$ for $n=2 \cdot \sum_{j=0}^h (k_j+p_j) + i$ ($i=2, 3, \dots, 2k_{h+1}+1$; $h=0, 1, 2, \dots$); (iii) $\alpha_n=2$ for $n=2 \cdot \sum_{j=0}^h (k_j+p_j) + 2k_{h+1} + i$ ($i=2, 3, \dots, 2p_{h+1}+1$, $h=0, 1, 2, \dots$). Certain assumptions made concerning the sequences $\{k_j\}$ and $\{p_j\}$ assure that the sequence $\{\alpha_n\}$ is infinite. The author considers here C -fractions of the form $K_{(n)}[c_n w^{\alpha_n}/1]$ which are either nonterminating or terminating with $[2 \cdot \sum_{j=0}^{m+1} (k_j+p_j) + 1]$ partial quotients, where m is any given non-negative integer. Let $C(w)$ denote this fraction or this fraction with a partial quotient $cw/1$ attached according as this C -fraction is nonterminating or terminating. In a previous paper, the author developed and discussed certain results which hold in case a given formal power series is the corresponding power series of $C(w)$. This paper is essentially devoted to showing that certain assumptions concerning a given formal power series $\sum_{(n)} \mu_n w^{n+1}$ are sufficient for this power series to have $C(w)$ as its C -fraction expansion. (Received February 27, 1956.)

APPLIED MATHEMATICS

432t. R. W. Bass: *Equivalent linearization and the existence of periodic solutions of ordinary differential equations.*

A recent heuristic criterion, the "Nyquist diagram for nonlinear servomechanisms," is currently used by engineers as an indication of whether or not a given nonlinear differential equation has periodic solutions. The method, reminiscent of that of Kryloff-Bogoliuboff, has been developed in particular cases by involved "frequency response" arguments. Here the method is clarified and systematized by noticing that it results from time-averaging what the author calls the "instantaneous characteristic polynomial" to obtain the characteristic polynomial for a corresponding "linearized" equation. This formulation renders trivial the extension to difference-differential equations and many other cases not hitherto treated. Also a companion heuristic criterion for determining stability of the periodic solutions is presented. This criterion, based on certain new identities, extends Bulgakov's idea of time-averaging the variational equations. It is shown indirectly (by comparison with the explicit solutions) that both criteria are valid for a certain three parameter family of discontinuous difference-differential equations. By combining the method of undetermined Fourier coefficients with the Leray-Schauder theory, it is also proved directly that in a range of typical cases the first criterion is legitimate. (Received February 29, 1956.)

433. Bernard Friedman: *Virtual eigenvalues.* Preliminary report.

The resolvent $R(\lambda)$ of a class of self-adjoint ordinary differential operators over the interval $(0, \infty)$ may be extended to an analytic function of λ in a two-sheeted Riemann surface. The poles of the resolvent in the "physical" sheet are eigenvalues of the operator. For a class of operators the resolvent has poles in the second non-physical sheet. These are "virtual" eigenvalues and the corresponding solutions of the differential equations are "virtual" eigenfunctions. Even though these solutions are exponentially large for large x , they may be used to give an expansion of the solution of a partial differential equation. Applications to the Breit-Wigner nuclear level resonance formula, the Stark effect in Quantum Mechanics and the problem of α -particle decay are considered. (Received March 1, 1956.)

434. P. C. Hammer and W. H. Peirce (p): *Numerical integration over planar regions.* Preliminary report.

Hammer and Wymore have previously [Bull. Amer. Math. Soc. Abstract 61-4-318] reported on methods of obtaining numerical integration formulas exact for polynomials over certain symmetric regions. In this paper are presented explicit formulas for the circle, square, regular octagon, hexagon, and the arbitrary circular annulus. Using a confluence process, formulas for numerical integration over boundaries of some of these regions are established. From these, certain integration formulas for surfaces are established. Formulas for asymmetrical regions using mappings are established. (Received February 27, 1956.)

435. G. E. Hay: *On axially symmetric torsion.*

The problem of axially symmetric torsion reduces to the determination of a function F satisfying a certain second order partial differential equation, F being constant on the surface of the body and on the axis of revolution of the body. Cylindrical coordinates or spherical coordinates are usually employed. In the present paper,

the equation satisfied by F is expressed in terms of general curvilinear coordinates. Certain nonorthogonal coordinates systems are then introduced; these lead to the solution of special problems. (Received February 29, 1956.)

436. Bayard Rankin: *An algebra for minimizing machine performance time within a class of algebraically equivalent programs.*

A program for a computing machine is a finite sequence, p , of real parameters together with a finite sequence, i , of instructions. Let PXI be a collection of pairs (p, i) . The real functions $T(p, i)$ and $O(p, i)$ defined on PXI correspond respectively to the performance time and output for (p, i) . The computing machine M is defined as the triplet (PXI, T, O) . (p, i) and (p', i') are called algebraically equivalent (a.e.) if in the absence of round-off and overflow errors $O(p, i) = O(p', i')$. It is desired to find (\bar{p}, \bar{i}) within a class of a.e. programs such that $T(\bar{p}, \bar{i})$ is minimum. In this paper the author constructs an algebra of instructions for an exemplary three address machine. He uses the symbols A : add, S : subtract, M : multiply, D : divide, R : modify previous instruction, C : make a two-valued choice, $()^n$: perform bracketed sequence n times. By convention, an instruction on the left of another is performed first in time. We investigate laws with which to generate a.e. programs from known ones. Example, distributive law for addition: $(AR)^2M \equiv (MRAR)^2$. By letting instructions depend on the output of other programs the nonlinear dynamic structure of programs is expressed as simultaneous equations. Partial sponsorship by the Office of Naval Research. (Received February 28, 1956.)

GEOMETRY

437. Eugenio Calabi: *Examples of 6-dimensional almost-complex and complex manifolds and their properties.*

It is known that any orientable, differentiable hypersurface imbedded in Euclidean 7-space R^7 , considering the latter as the set of purely imaginary Cayley numbers, is *ipso facto* an almost-complex manifold. The integrability condition for the induced almost-complex structure to be derivable from a complex analytic one is expressed in terms of the second fundamental form of the hypersurface and of the invariants of the Cayley algebra, with the following results. (I) Every compact, orientable hypersurface in R^7 has a domain in which the only holomorphic functions in any subdomain are constants. (II) The only hypersurfaces for which the induced almost complex structure is integrable form a special class of minimal hypersurfaces, including the direct products of minimal surfaces in a distinguished $R^3 \subset R^7$ with the orthogonal R^4 . Following this one constructs a compact, complex manifold, differentially homeomorphic to the direct product of a closed surface of odd genus ≥ 3 with a 4-torus, which, contrary to the "trivial" model, admits no Kähler metric, and has 1st Chern class equal to zero. (Received January 25, 1956.)

438*t*. John DeCicco: *Velocity systems upon a surface.*

For a given field of force F on a surface S , there is defined a velocity family corresponding to a given constant speed $v_0 > 0$. If v_0 varies, the resulting set is a velocity system S_v . The Lagrangian and Hamiltonian equations of an S_v are derived. A family of ∞^2 curves on the surface S is a velocity family if and only if the locus of centers of geodesic curvature constructed at each point P of S of the ∞^1 curves of the family that pass through P is a straight line, not passing through P . As applications, natural

and isogonal families on the surface S are each characterized. Finally the transformation theory of these families is studied. (Received February 9, 1956.)

439. P. C. Hammer: *Overlapping areas of convex sets*. Preliminary report.

Let B and C be two planar convex sets of positive area. Define the point functional $f(x) = \text{Area}[B+x \cap C]$. Then f is quasi-concave, that is, $f(x_p) \geq \min[f(x_0), f(x_1)]$ where x_p is on the line segment x_0x_1 . This result is generalized. Properties of quasi-concave functionals are developed. The infimum of a class of quasi-concave functionals is a quasi-concave functional. A monotonic function of a quasi-concave functional is a quasi-concave functional. A necessary and sufficient condition that a functional be quasi-concave is that its infimum on every compact set be the same as its infimum on the class of extreme points of the set. (Received February 27, 1956.)

440*t*. Shoshichi Kobayashi: *On holomorphic affine connections*.

The existence of a holomorphic affine connection on a compact complex manifold imposes a very strong restriction on the space. In fact, if there exists a holomorphic affine connection on a compact complex manifold M which admits a Kaehlerian metric, then all the Chern classes of M vanish. In particular, if M is compact homogeneous Kaehlerian, there exists a holomorphic affine connection on M if and only if M is a complex torus. It can be also proved that the homogeneous holonomy group of a holomorphic affine connection is a complex Lie subgroup of the complex general linear group. More generally, the holonomy group of a holomorphic connection in a holomorphic principal fibre bundle with complex Lie structure group G is a complex Lie subgroup of G . It should be remarked (due to Serre) that there exists always a holomorphic connection in every holomorphic principal fibre bundle whose base space is a Stein manifold. (Received February 15, 1956.)

441*t*. Shoshichi Kobayashi: *On isometries of pseudo-Kaehlerian spaces*.

Every isometry of an irreducible pseudo-Kaehlerian space M of real dimension $2n$ preserves the almost complex structure or gives the conjugate structure except in the case where n is even, say $2m$, and the homogeneous holonomy group is contained in $Sp(m)$. This improves the previous result of the author (Bull. Amer. Math. Soc. Abstract 62-4-534) as well as that of Lichnerowicz (C. R. Acad. Sci. Paris vol. 239, p. 1344) and Schouten-Yano (Indagationes Math. vol. 17, p. 565). The proof is extremely simple. If $J: T(M) \rightarrow T(M)$ defines the almost complex structure and if f is an isometry of M , then $\delta f^{-1} \circ J \circ \delta f$ gives also an almost complex structure which commutes with every element of the holonomy group. The theorem follows immediately from the fact that the algebra of all linear transformations which commute with every element of an irreducible group of linear transformations of a real vector space is isomorphic to the field of real numbers, complex numbers or quaternions. (Received February 15, 1956.)

442. Albert Nijenhuis: *Derivations on differential forms*.

The graded ring \mathfrak{R} of real-valued differential forms (scalar forms) on an n -dimensional manifold \mathfrak{M} admits derivations D of every degree r , $-1 \leq r \leq n$, where D satisfies $D(\mathfrak{R}_p) \subset \mathfrak{R}_{p+r}$, $D(\phi + \psi) = D\phi + D\psi$, $D(\phi_p \wedge \psi_q) = D\phi_p \wedge \psi_q + (-1)^p \phi_p \wedge D\psi_q$. If $D|_{\mathfrak{R}_0} = 0$ (D of the first kind), D is determined by a tangent vector-valued dif-

ferential form (vector form) $L_{r+1} \cdot D\phi_p = \phi_p \perp L_{r+1}$, with $\phi \perp L(u_1, \dots, u_{p+r}) = (1/(p-1)!(r+1)!) \sum_{\alpha \in \mathcal{E}_{p+r}} (-1)^\alpha (L(u_{\alpha_1}, \dots, u_{\alpha_{r+1}}, u_{\alpha_{r+2}}, \dots, u_{\alpha_{p+r}})$. If $Dd = (-1)^r Dd$ (D of the second kind) there is a vector r -form L such that $D\phi = [L, \phi] = d\phi \perp L - (-1)^r d(\phi \perp L)$. Every derivation is a sum of a derivation of the first, and one of the second kind. The derivations (of the first/second kind) form a graded module \mathfrak{D} ($\mathfrak{D}^1/\mathfrak{D}^2$), where \mathfrak{D}^1 and \mathfrak{D}^2 are subalgebras. The vector form $[L, M]$ associated with the commutator of two derivations of the second kind determined by vector forms L and M is a differential concomitant whose existence was already known, but not its function. The identities between $[L, \phi]$ and $[L, M]$, $\phi \perp L$, etc., which express the Lie algebra nature of \mathfrak{D} appear to be extremely useful in the theory of almost complex structures j , and also in almost-product structures. The operations defined there arise naturally in our scheme; e.g. $d''\phi = [I - ij, \phi]/2$, $T = [j, j]/2$, $d''d''\phi = -[T, \phi]$. (Received February 28, 1956.)

443. W. L. Stamey: *On generalized euclidean and non-euclidean spaces.*

A semimetric space is generalized {euclidean, r -hyperbolic}, i.e. has each of its n -dimensional subspaces congruent with {euclidean, r -hyperbolic} n -space, provided it is complete, convex, externally convex and has the weak {euclidean, r -hyperbolic} four-point property. (See L. M. Blumenthal, *Theory and applications of distance geometry*, Oxford, at the Clarendon Press, 1953.) Similar definitions and results with respect to a diameterized class of spaces instead of externally convex spaces hold for generalized r -spherical and r -elliptic space. (See Blumenthal, loc. cit. and J. D. Hankins' University of Missouri doctoral dissertation, 1954.) Recently Blumenthal (*An extension of a theorem of Jordan and von Neumann*, Pacific Journal of Mathematics vol. 5 (1955)) has shown that for euclidean spaces a "feeble euclidean four-point property" can replace the weak euclidean four-point property. In the present note the generalized euclidean, r -hyperbolic, r -spherical, r -elliptic spaces are characterized among the complete, convex, semimetric spaces having segments which can be uniquely locally extended. It is shown that the appropriate feeble four-point property characterizes each of these generalized spaces among the above class of spaces. (Received February 28, 1956.)

LOGIC AND FOUNDATIONS

444t. M. L. Keedy: *On a relation which orders a set with a first element.* Preliminary report.

An element a in an abstract relation algebra, as developed in (Chin and Tarski, *Distributive and modular laws in the arithmetic of relation algebras*, University of California Publications in Mathematics, Vol. 1, No. 9, pp. 341-384), is defined to be an "order element" if it satisfies: $a; a = a$, $a \cdot \bar{a} = 1'$, and $a + \bar{a} = 1$. If, in addition, $a + 0 \neq 0$, then a is called a "first-point" order element. Such an element in a proper relation algebra over a set U is a reflexive linear order relation, and it orders the set U such that one member of U precedes all other members of U . In case the relation a orders a proper subset of U in a similar manner its arithmetic characterization is identical to the above, but is expressed in the subalgebra consisting of all relations included in $[(a + \bar{a}); 1] \cdot [1; (a + \bar{a})]$. (Received March 20, 1956.)

445t. A. R. Schweitzer: *An outline of potential theory.*

This paper consists of an introduction and two parts. The introduction has three

sections A, B, C outlining a transition from geometry to mathematical physics. A. Geometry and dynamics. B. Dynamics and potential theory. C. Potential theory and mathematical physics. Part I has title: Chapters in potential theory. Chapter I. Potential theory and dynamics; Gravitation. Newton, Lagrange. Chapter II. Potential theory and analysis; harmonic functions. LaPlace. Chapter III. Potential theory and mathematical physics; Magnetism and electricity. Green. Part II has title: Potential theory and related subjects. For Part II the author refers to O. D. Kellogg, *Foundations of potential theory*, Berlin, 1929, pp. VIII-IX, pp. 94-97. On pp. 94-97 Kellogg comments on the Heine-Borel theorem with application to uniform convergence of series. In concluding his paper the author mentions lectures on potential theory delivered by David Hilbert in Göttingen, 1901-1902. (Received February 27, 1956.)

446t. A. R. Schweitzer: *On the foundations of mathematical analysis.*

The author holds that uniformity, including uniformity of convergence of series, is the most important concept in recent mathematical analysis. Reference is made to articles and treatises by W. F. Osgood including *Introduction to infinite series*, 3d ed., Cambridge, 1928; *A geometrical method for the treatment of uniform convergence and certain double limits*, Bull. Amer. Math. Soc., November, 1896, pp. 59-86. Reference is also made to E. J. Townsend, *Der Doppel Limes*, dissertation, Göttingen; G. D. Birkhoff, *Collected mathematical papers*, vol. III, p. 153. An extension of uniform convergence to "relatively uniform convergence" is due to E. H. Moore who has used his extension in the construction of a form of general analysis (New Haven, 1906). (Received February 27, 1956.)

STATISTICS AND PROBABILITY

447. G. E. Baxter (p) and M. D. Donsker: *On the distribution of the supremum functional for processes with stationary independent increments.*

Let $\{x(t), 0 \leq t < \infty\}$ be a separable stochastic process with stationary independent increments whose sample functions vanish at the origin. Such a process is characterized by $E\{\exp(i\xi x(T))\} = \exp(T\psi(\xi))$, $0 \leq T < \infty$, where $\exp(\psi(\xi))$ is the Lévy-Khintchine representation of the characteristic function of an infinitely divisible distribution. For $f(u, \lambda) = \int_0^\infty \int_0^\infty \exp(-uT - \lambda\alpha) d_\alpha \sigma(\alpha, T) dT$ with $\sigma(\alpha, T) = P\{\sup_{0 \leq t \leq T} x(t) < \alpha\}$, it is shown that for all positive u and λ ; (1) if $\psi(\xi)$ is real-valued, $uf(u, \lambda) = \exp\{1/2\pi \int_u^\infty \int_{-\infty}^\infty \lambda \psi(\xi) / [(\lambda^2 + \xi^2)s(s - \psi(\xi))] d\xi ds\}$; and (2) if $\psi(\xi)$ is complex and such that for some positive δ , $\int_{-\delta}^\delta |\psi(\xi)/\xi| d\xi < \infty$, $uf(u, \lambda) = \exp\{1/2\pi \int_u^\infty \int_{-\infty}^\infty \lambda \psi(\xi) / [\xi(\xi - i\lambda)s(s - \psi(\xi))] d\xi ds\}$. These results are used to calculate $\sigma(\alpha, T)$ in various examples. (Received February 23, 1956.)

448. S. C. Moy: *Conditional expectations of Banach space valued random variables and their properties.*

Let $(\Omega, \mathfrak{F}, \mu)$ be a probability space and \mathfrak{X} be a Banach space. A random variable X is a function on Ω to \mathfrak{X} which is strongly \mathfrak{F} -measurable. It is proved: If X is a random variable which is Bochner integrable and \mathfrak{F}' is a σ -subalgebra of \mathfrak{F} , there is a unique random variable Y which is strongly \mathfrak{F}' -measurable and Bochner integrable and $\int_E Y d\mu = \int_E X d\mu$ for every $E \in \mathfrak{F}'$. Y is defined to be the conditional expectation of X relative to \mathfrak{F}' and is designated by $E[X|\mathfrak{F}']$. The following properties of conditional expectations are established. 1. $\|E[X|\mathfrak{F}']\| \leq E[\|X\||\mathfrak{F}']$. 2. Let $B_p(\mathfrak{X})$ be the Banach

space of all functions X from Ω to \mathfrak{K} which are strongly \mathfrak{F} -measurable and $\int \|X\|^p d\mu < \infty$. The mapping $T: TX = E[X|\mathfrak{F}']$ is a linear, norm decreasing, idempotent transformation of $B_p(\mathfrak{K})$ into itself. 3. By ϕ is a complex valued function on Ω which is \mathfrak{F}' -measurable and $\int |\phi|^q d\mu < \infty$ where $1/p + 1/q = 1$ then $E[X\phi|\mathfrak{F}'] = \phi E[X|\mathfrak{F}']$ for every $X \in B_p(\mathfrak{K})$. (Received February 27, 1956.)

449. J. M. Shapiro: *Some properties of infinitely divisible distributions*. Preliminary report.

The principal result is as follows: Let X be a random variable with an infinitely divisible distribution. Then either X has a unitary distribution or $P(|X| > A) > 0$ for any constant A . The proof uses some of the results of the basic limit theorems for sums of independent infinitesimal random variables. Note that this theorem gives a large class of distributions which are not infinitely divisible. More precise results as to when $P(X > A) > 0$ and $P(X < -A) > 0$ where $A > 0$ are also given. (Received February 27, 1956.)

TOPOLOGY

450. F. W. Anderson (p) and R. L. Blair: *Ideals in lattices of continuous functions*. I.

Let $L = L(X)$ be the lattice of all real-valued continuous functions on a completely regular space X , and let L_+ be the sublattice of all $f \geq 0$. An ideal I of L_+ is an R -ideal if $f \in I$ implies $f \ll g$ for some $g \in I$ and $h \in I$ whenever $h \leq f$ (cf. T. Shirota, Osaka J. Math. vol. 4 (1952)). An ideal of L_+ is a maximal R -ideal iff it consists of all $f \in L_+$ vanishing on an X -neighborhood of some unique $p \in \beta X$. Hence, $L(X)$ characterizes βX . Also, a class of "closed" R -ideals serves to characterize the lattice of regular closed subsets of X . If P is a prime ideal of L , it is "associated" with some unique point $p \in \beta X$ (cf. Kaplansky, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 617-623), and the set of all prime ideals of L associated with p is denoted by $\mathcal{O}(p)$. The space νX is then characterizable as the subspace of all $p \in \beta X$ for which $\mathcal{O}(p)$ has a countable cofinal subset. An investigation is made of a class of ideals important in the study of automorphisms of L , namely, those $\mathcal{O} \in \mathcal{P}(p)$, called *nodes*, which are comparable with every $Q \in \mathcal{O}(p)$. (Received February 28, 1956.)

451t. F. W. Anderson and R. L. Blair: *Ideals in lattices of continuous functions*. II.

A point $p \in X$ is an N -point if the chain of nodes in $\mathcal{O}(p)$ (see previous abstract for terminology) is separable in the interval topology. If p is an N -point and $f, g \in L$, then $f(p) < g(p)$ iff $f \in P$, $g \notin P$ for some node in $\mathcal{O}(p)$. A topological characterization of N -points is given, and it is shown then that $p \in X$ is an N -point if (a) the first axiom of countability holds at p , (b) p has a connected neighborhood, or (c) p is a P -point (Gillman and Henriksen, Trans. Amer. Math. Soc. vol. 77 (1954)). In fact, p is a P -point iff $\mathcal{O}(p)$ is a chain separable in the interval topology. A point $p \in X$ is an M -point if it is either an N -point in X or a P -point in some regular closed subset of X . Again, it is possible to determine $f(p) < g(p)$ algebraically at M -points. Finally, the following statements are equivalent: (i) X consists entirely of M -points; (ii) every automorphism of L_+ sends strictly positive functions onto strictly positive functions; (iii) every automorphism of L is continuous in the m -topology. (Cf., also, Kaplansky, Amer. J. Math. vol. 70 (1948) pp. 626-634). (Received February 28, 1956.)

452. R. H. Bing: *A peculiar decomposition of E^3 .*

An example is given of an upper semicontinuous decomposition G of E^3 such that G has only a countable number of nondegenerate elements, each of these elements has a complement topologically like the complement of a point, but the decomposition space associated with G is topologically different from E^3 . Each element of G lies in a plane. The nondegenerate elements of G are topologically alike—in fact there is such a G where they are all pseudo-arcs. It is not known if there is such a G each of whose elements is a continuous curve. (Received February 28, 1956.)

453t. C. F. Briggs: *Locally normed commutative *-algebras.*

Call a linear algebra A , with identity, a locally normed algebra if it is a topological group under addition, and if the component of $O(B)$ is a Banach subalgebra, topologically open and closed in A . A is a *-algebra if there is a *-operation on A with the properties: (1) $(x^*)^* = x$, (2) $(ax)^* = \bar{a}x^*$, (3) $(xy)^* = y^*x^*$, (4) $(x+y)^* = x^*+y^*$, (5) B is a Banach *-subalgebra. A is B -related if xx^*e has an inverse in B for every x in A , and A is regularly semi-simple if the intersection of the closed maximal left ideals is 0 and the union is the set of all singular elements. The set of all closed maximal ideals of A can be embedded topologically in the set of all maximal ideals of B if A is commutative, locally normed, *, and B -related, where the set of closed maximal ideals is topologized by embedding in the conjugate space of B as a Banach space. If M is the set of closed maximal ideals of A so topologized, then A is isomorphic and "isometric" to a subalgebra of $O(M)$ under the uniform topology. (Received February 27, 1956.)

454. E. H. Brown, Jr.: *Computability of homotopy groups.*

M. M. Postnikov associates with each arcwise connected space X a sequence of algebraically defined complexes $P_1(X), P_2(X), \dots$ which he calls the natural system of X (Doklady Akad. Nauk SSSR. vol. 76, pp. 356–362.) The writer shows that if X is a finite simply connected simplicial complex, then a complex $P_{n,q}(X)$, $q > n > 0$, homologically equivalent to $P_n(X)^q$ (q -skeleton) can be finitely constructed. $P_{n,q}(X)$ is constructed by induction on n and in the process of going from $P_{n-1,q}(X)$ to $P_{n,q}(X)$ $\pi_n(x)$ is computed. This process gives an explicit finite procedure for calculating $\pi_n(X)$ but it must be emphasized that the techniques involved are much too complicated to be considered practical. (Received February 23, 1956.)

455. E. R. Fadell: *On extending the range of the covering homotopy theorem.*

The covering homotopy theorem (CHT) is said to hold for a space Y with respect to a triple (X, B, π) , where $\pi: X \rightarrow B$ is a given map, if for any homotopy $H: Y \times I \rightarrow B$ and map $g: Y \times \{0\} \rightarrow X$ such that $\pi \circ g = H$ on $Y \times \{0\}$, there exists a homotopy $G: Y \times I \rightarrow X$ which extends g such that $\pi \circ G = H$ on $Y \times I$. An open cover $\{U_\alpha\}$ of a space Y is said to be normal if there exists, for each α , real-valued maps f_α on Y which are positive on U_α and 0 elsewhere. In this note the following theorem is proved. If $\{U_\alpha\}$ is a locally finite, normal, open cover of Y and if, furthermore, the CHT holds for each \bar{U}_α (closure of U_α) with respect to (X, B, π) , then the CHT holds for Y with respect to (X, B, π) . The following is an immediate corollary. The CHT for cells implies the CHT for metric spaces which are locally polyhedral, e.g., manifolds. If one desires to condition B , one can show using the above theorem that if B is a metric ANR, the CHT for cells implies the CHT for separable metric ANR's which are locally compact and finite dimensional. (Received February 27, 1956.)

456. Jesús Gil de Lamadrid: *Higher derivatives of mappings of topological vector spaces*. Preliminary report.

Let E and F be two such spaces. Denote $F^{\mathfrak{Z}}$ by $\mathfrak{F}_1(E, F)$ and define $\mathfrak{F}_n(E, F)$ by induction as $\mathfrak{F}_1[E, \mathfrak{F}_{n-1}(E, F)]$ for any n . Let \mathfrak{Z} be a family of subsets of E . The \mathfrak{Z} -topology of $\mathfrak{F}_n(E, F)$, which for $n=1$ reduces to the topology of uniform convergence over the sets of \mathfrak{Z} , is defined by an induction process. If $(E)^n$ denotes the cartesian product of E taken n times, and \mathfrak{Z}^n the family of products $A_1 \times A_2 \times \cdots \times A_n$, $A_i \in \mathfrak{Z}$, it is shown that there exists a homeomorphic isomorphism M of $\mathfrak{F}_n(E, F)$ onto $\mathfrak{F}_1[(E)^n, F]$, the latter space with the \mathfrak{Z}^n -topology. If τ is the \mathfrak{Z} -topology of $\mathfrak{F}_1(E, F)$, the \mathfrak{Z} -derivative $f' \in \mathfrak{F}_2(E, F)$ of $f \in \mathfrak{F}_1(E, F)$, defined by the author in an earlier paper (Bull. Amer. Math. Soc. vol. 61 (1955) p. 315), is called the \mathfrak{Z} -derivative of f . By induction one can define the n th \mathfrak{Z} -derivative $f^{(n)} \in \mathfrak{F}_{n+1}(E, F)$. In the literature the n th differential of f is ordinarily defined as an element of $\mathfrak{F}_1((E)^{n+1}, F)$. It is shown in this note that if \mathfrak{Z} is the family of all (1) bounded sets, (2) finite sets, then $Mf^{(n)}$, when it satisfies the unusual linearity and continuity conditions, is respectively (1) the n th Fréchet differential when E is normed, (2) the n th Gâteaux differential. Thus this last theorem permits an extension of Fréchet differentiation to non-normed spaces. (Received February 27, 1956.)

457. J. G. Hocking: *A characterization of monotone maps on 2-manifolds*.

The following theorem is established: Let M be a compact 2-manifold (with or without boundary) and f be a map of M onto itself. Then f is monotone if and only if there exists an extension h of f mapping the cone $v^\circ M$ over M at a point v onto itself such that h is a homeomorphism on $v^\circ M - M$: This generalizes a result of Floyd and Fort, *A characterization theorem for monotone mappings*, Proc. Amer. Math. Soc. vol. 4 (1953) pp. 828-830. (Received February 28, 1956.)

458. G. P. Johnson: *Spaces of functions with values in a normed ring*. Preliminary report.

Let R be a commutative normed ring, G a locally compact Abelian group, and $B_1 = B_1(G, R)$ the space of Bochner integrable R -valued functions on G . It is shown that the space \mathfrak{M}_{B_1} of regular maximal ideals of B_1 is homeomorphic to the Cartesian product of the maximal ideal spaces of R and $L_1(G)$ when each of the spaces is given its weak topology. An integral formula for the Fourier transform is obtained; and with some additional restrictions on R , a number of the theorems for $L_1(G)$ are extended to B_1 . B_1 is semi-simple if and only if R is semi-simple, and in this case the linear space generated by the homomorphisms \mathfrak{M}_{B_1} is weakly dense in the conjugate space of B_1 . This leads to an investigation of the form of the general linear functional on B_1 . (Received February 28, 1956.)

459. G. R. Livesay: *On the fixed points of a class of homotopic mappings of a triangulable manifold*.

Let $\{f_t\}$, $0 \leq t \leq 1$, be a class of homotopic mappings of a space M into itself. Define the map $F: M \times I \rightarrow M$ by $F(x, t) = f_t(x)$. Let $P: M \times I \rightarrow M$ be the projection. Then x is a fixed point of f_t if and only if $F(x, t) = P(x, t)$. Let $A = \{(x, t) \in M \times I \mid F(x, t) = P(x, t)\}$. D. G. Bourgin (*On some separation and mapping theorems*, Comment. Math. Helv. vol. 29, 3) has asked for conditions under which A contains a continuum

containing a point of $M \times 0$ and a point of $M \times 1$. A partial answer is given by the theorem: If M is a compact triangulable manifold, and the Lefschetz number of $\{f_i\}$ is not zero, then A contains a continuum C which intersects $M \times 0$ and $M \times 1$. In fact if P is any map of $M \times I \rightarrow M$ such that the coincidence number of $F|_{M \times 0}$ and $P|_{M \times 0}$ is not zero, then A contains such a continuum. The proof consists of two parts: (a) If A contains no such continuum, then there exists a submanifold N of $M \times I$ such that the inclusion $i_*: H(N) \rightarrow H(M \times I)$ is onto, and $N \cap A = \emptyset$. (b) The coincidence number of $F|_N$ and $P|_N =$ coincidence number of $F|_{M \times 0}$ and $P|_{M \times 0}$. Since (b) contradicts $N \cap A = \emptyset$, then A contains a continuum such as C . (Received March 1, 1956.)

460. M. J. Mansfield: *On countably paracompact normal spaces.*

The results of this paper are generalizations of well-known theorems of J. S. Griffen, J. L. Kelley, E. Michael, and A. H. Stone (cf. J. L. Kelley, *General topology*, New York, 1955, p. 156 and pp. 170–171), and of some as-yet-unpublished results of E. Michael. A collection \mathcal{A} of subsets of a topological space is *closure-preserving* if, for each subcollection $\mathcal{B} \subset \mathcal{A}$, the union of the closures is the closure of the union. THEOREM: If X is a normal space, then the following statements are equivalent: (1) X is countably paracompact; (b) each countable open covering of X has a countable, locally finite, closed refinement; (c) each countable open covering of X has a countable, closure-preserving, closed refinement; (d) each countable open covering of X has a σ -discrete closed refinement; (e) each countable open covering of X has a σ -locally finite closed refinement; (f) each countable open covering of X has a σ -closure-preserving closed refinement; (g) each countable open covering of X has a countable, open, star-refinement; (h) each countable open covering of X is even; (i) for each countable locally finite collection $\{A_i\}$ of subsets of X there exists a countable, locally finite, open collection $\{G_i\}$ such that, for each i , $A_i \subset G_i$. (Received February 27, 1956.)

461. J. M. Slye: *Collections whose sums are two manifolds.*

The purpose of this paper is to establish conditions that are both necessary and sufficient in order that an upper semicontinuous collection of arcs (or simple closed curves) filling up a nondegenerate locally connected metric space, and whose decomposition space is an arc, fill a 2-cell (or annulus, or Möbius strip, or Klein bottle). Two additional conditions are needed. First no point (or pair of points) separates the space. The second condition is contained in the following definition. If P is a point of a space S , S has property X at P if and only if there exists a connected open set D containing P such that if Q and Q' are two points of D and B is a compact closed set contained in D that does not contain $Q+Q'$ and is irreducible with respect to separating Q and Q' in S , and P' and P'' are points of B , then $B - (P'+P'')$ is not the sum of three mutually separated sets each of which has $P'+P''$ in its closure. (Received February 27, 1956.)

462*t*. Stephen Smale: *The effect of an open map on the fundamental group.*

Let X and Y be metric spaces where X is locally arcwise connected and Y is semi-locally simply connected. Suppose that $f: (X, x_0) \rightarrow (Y, y_0)$ is an open, onto map and that the inverse images of compact sets are compact. Let g be a map of the unit interval I into Y with $g(0) = y_0$. Then it is proved that there exists a map $\tilde{g}: (I, 0)$

$\rightarrow(X, x_0)$ such that $f\tilde{g}(1) = g(1)$ and $f\tilde{g}$ is homotopic to g with fixed end points. Using this result, by a method of M. L. Curtis (Ann. of Math. vol. 57 (1953) p. 239) it is proved that $\pi_1(Y, y_0)/f_*\pi_1(X, x_0)$ is finite. Hence if $\pi_1(X, x_0)$ is finite, then so is $\pi_1(Y, y_0)$ or if $\pi_1(X, x_0)$ is finitely generated, then so is $\pi_1(Y, y_0)$. (Received February 23, 1956.)

463. R. F. Williams: *A note concerning dimension raising maps.*

An earlier result of the author is generalized to the following: if $f: X \rightarrow Y$ is a map, X compact, metric and n -dimensional, Y m -dimensional, then there is a subcontinuum $M \subset X$ such that $\dim M \geq n - 1/2$ and $\dim f(M) \leq m/2$. As a corollary, if for some integer i , $2^i \leq n$ and $m < 2^i + n$, then there is a nondegenerate continuum $M \subset X$ whose image has dimension at most 1. The proofs are simple. (Received February 27, 1956.)

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