THE APRIL MEETING IN NEW YORK

The five hundred twenty-fourth meeting of the American Mathematical Society was held on Friday and Saturday, April 20–21, 1956 at Columbia University in New York City. The meeting was attended by about 330 persons, including 299 members of the Society.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor J. T. Schwartz of Yale University and New York University addressed the Society at 2:00 P.M. on Friday on *Riemann's method in the theory of special functions*, and Professor Fritz John of New York University addressed the Society at 2 P.M. on Saturday on *Continuation of solutions of partial differential equations*. These sessions were presided over by Professors Nelson Dunford and J. J. Stoker respectively.

Sessions for contributed papers were held at 3:15 P.M. on Friday, and at 10 A.M. and 3:15 P.M. on Saturday. Chairmen at these sessions were Professor G. K. Kalisch, Dr. Lawrence Markus, Professors G. Y. Rainich, Alex Rosenberg, Charles Fox, Emil Grosswald, E. G. Straus. The Employment Register was maintained by Professor W. M. Hirsch.

The Council of the Society met at 5:15 P.M. on Friday, reconvening after dinner.

The Secretary announced the election of the following sixty-five persons to ordinary membership in the Society:

Mr. Sheldon Buckingham Akers, Jr., A C F Electronics, Alexandria, Virginia;
Dr. Allen E. Andersen, University of Massachusetts;
Professor Tommie Marie Anderson, Tougaloo College;
Mr. Americo Aruffo, Philco Corporation, Philadelphia, Pennsylvania;
Mr. Stephen Joseph Barone, Polytechnic Institute of Brooklyn;
Mr. Stephen Barry Behman, State College, Pennsylvania;
Mr. Andrew Angelo Benvenuto, University of Illinois;
Mr. Albert T. Bharucha-Reid, University of California, Berkeley;
Mr. Rupert Dean Boswell, Jr., University of Georgia;
Miss Christine Viola Brannan, University of Houston;
Mr. Kenneth Lawrence Brinkman, Cloud Physics, Dugway Proving Ground, Utah;
Reverend Robert Ignatius Canavan, New York University;
Mr. Michael F. Capobianco, St. John's University;
Mr. Robert Carter, P. O. Box 903, Chicago 90, Illinois;
Mr. George Gilman Chapin, Remington Rand Univac, St. Paul, Minnesota;
Dr. Yun Shick Choi, Seoul National University, Seoul, Korea;
Mr. William Henry Colbert, Jr., University of Nevada;
Professor Samuel D. Conte, Wayne University;

Mr. James Russell Cretcher, Jr., Lockheed Aircraft Corporation, Van Nuys, California;
Mr. Henry Lawrence Crowson, University of Florida;
Professor Joseph U. Crum, Western Carolina College;
Dr. Bogomil Milko Cvetkov, Snowy Mountains Hydro-Electric Authority, Cooma, NSW, Australia;
Lt. Col. Halvor Thomas Darracott, Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey;
Reverend Theodosius Laszlo Demen, St. Louis University;
Professor Augustus Isaac Dhar, Illinois Wesleyan University;
Mr. N. S. D'Andrea DuBois, Jr., Howard University;
Mr. William Preston Eames, Queen's University, Kingston, Ontario, Canada;
Mr. Edwin George Eigel, Jr., St. Louis University;
Dr. Kathryn Powell Ellis, Iowa City, Iowa;
Dr. Carl Clifton Faith, Michigan State University;
Miss Hilda Feist, Pennsylvania State University;
Mr. George David Findlay, McGill University, Montreal, Quebec, Canada;
Mr. Robert Sumner Fishman, University of Vermont;
Mr. Gerald Marvin Fleischner, Polytechnic Institute of Brooklyn;
Mr. Marion McGee Fulk, National Bureau of Standards (CEL), Boulder, Colorado;
Mr. Fernley L. Fuller, Grumman Aircraft Engineering Corporation, Bethpage, New York;
Mr. David Snellenberg Greenstein, University of Pennsylvania;
Mr. Edgar H. Grossman, Grossman and Miller, Vancouver, British Columbia, Canada;
Professor Norman Bray Haaser, University of Notre Dame;
Mr. John Leo Hammersmith, U. S. Naval Research Laboratory, Washington, D. C.;
Mr. Jean-Baptiste François Hocepied, 14 rue des Receveurs, Bruges, Belgium;
Mr. Bernard Ashworth Hodson, Auro Aircraft, Malton, Ontario, Canada;
Dr. Melvin Joseph Jacobson, Bell Telephone Laboratories, Whippany, New Jersey;
Mr. Morton Kupperman, Department of Defense, Washington, D.C.;
Dr. Andre Gilbert Laurent, Michigan State University;
Mr. Carroll Raymond Lindholm, Motorola, Inc.;
Professor Emmet Francis Low, Jr., University of Miami;
Mr. Robert Quinn Macleay, Christian College;
Professor Jean-Marie Maranda, University of Montreal;
Mr. James Walter Moeller, Holloman Air Force Base, New Mexico;
Dr. Jurgen Moser, Institute of Mathematical Sciences, New York University;
Dr. Pauline Mann Nachbar, Lockheed Aircraft Corporation, Van Nuys, California;
Mr. Walter Pressman, Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey;
Dr. Elvira Strasser Rapaport, Stockbridge, Massachusetts;
Professor Robert Davis Richtmyer, Institute of Mathematical Sciences, New York University;
Dr. Shepley Littlefield Ross, University of New Hampshire;
Professor Zevi Walter Salsburg, The Rice Institute;
Mr. Roger Lee Shaw, University of California, Berkeley;
Professor Robert Wesley Sloan, University of New Hampshire;
Mr. Frank Marcellus Staley, Jr., The Fort Valley State College;
Dr. Arthur Steger, University of New Mexico;

Professor Frederick Max Stein, Colorado Agricultural and Mechanical College;
Mr. Ralph Surasky, University of South Carolina;
Mr. Charles Maynard Terry, National Security Agency, Washington, D. C.;
Reverend Clarence Joseph Wallen, St. Louis University.

It was reported that the following two-hundred thirty-six persons had been elected to membership on nomination of institutional members as indicated:

Alabama Polytechnic Institute: Professor Clayton V. Aucoin.

University of Alabama: Mr. Arnold L. Milner.

University of Arizona: Mr. LeRoy Austin Kenna, Mr. Bernard Marcus, and Dr. Arthur Henry Steinbrenner.

Brown University: Dr. Walter F. Freiberger, Professor Albert Edward Green, Dr. Heini Halberstam, Mr. Herbert Meyer Kamowitz, and Mr. Thomas Patrick Mulhern.

California Institute of Technology: Mr. Theodore Kerner Matthes and Mr. Charles Robsom Storey, Jr.

University of California, Berkeley: Dr. Frederick Ellis Alzofon, Mr. John James Harton, Mr. Lee Opert Heflinger, Mr. Jean-Pierre Imhof, Mrs. Carol Ruth Karp, Mr. Melvin Louis Katz, Jr., Mr. Thomas Charles Kipps, Mr. Otto Plaat, Mr. Robert Richard Read, Mr. Edward Scott Robbins, Mr. Lucien Roland Roy, Miss Mandakini J. Sané, and Mr. Aram John Thomasian.

University of California, Los Angeles: Mr. Alfred William Adler, Mr. Richard Lewis Dunn, Mr. Yoichiro Fukuda, Mr. Antranig Vaughn Gafarian, Mr. Tom Alan Magness, Mr. Robert Ralph Phelps, Mr. Kenneth Everett Ralston, and Miss Beatriz Rossello.

Case Institute of Technology: Mr. Donald Wilford Robinson.

University of Chicago: Mr. Hyman Bass, Mr. Robert James Blattner, Mr. Richard Earl Block, Mr. Paul Joseph Cohen, Mr. Platon Constantine Deliyannis, Mr. David James Foulis, Miss Cynthia Freedman, Mr. Leonard Gross, Mr. Laurence Raymond Harper, Jr., Mr. Ray A. Kunze, Mr. Eben Matlis, Mr. John J. McKibben, Mr. Benjamin Muckenhoupt, Mr. Donald Samuel Ornstein, Mr. Donald Lewis Reinken, Mr. Stephen Hoel Schanuel, Mr. Haruo Suzuki, and Miss Mary Catherine Weiss.

University of Colorado: Mr. Robert George Buschman, Mr. James Earl Householder, and Mr. Roy Ben Kriegh.

Columbia University: Mr. Richard Stanley Wasserman.

Duke University: Mr. Waleed A. Al-Salam, Mr. Charles Edward Brooks, Mr. Harry Herbert Corson, III, Mr. Arthur Louis Gropen, Mr. Julius Robert Johnson, Jr., Mr. Nosup Kwak, Mr. Stewart Marshall Robinson, Mr. William Rowley Shawver, Mr. Marvin Gene Sperry, and Mr. Robert Guy Van Meter.

University of Florida: Mr. Dmitri Elias Thoro.

Harvard University: Mr. Robert Daniel Mize Accola, Mr. Glen Eugene Bredon, Mr. Thomas Bulkley Knapp, Mr. Henry Jacob Landau, Mr. Robert Pearlman, Mr. Kenneth Franklin Simpson, Jr., Professor Saffet Süray, Professor Yuki Wooyenaka, and Mr. Mishael Zedek.

College of the Holy Cross: Reverend Raymond Joseph Swords.

University of Illinois: Mr. Walter Scott Bartky, Mr. Hans Peter Dembowski, Mr. Gene Howard Golub, Mr. Vytas B. Gylys, Mr. Frank John Hahn, Mr. James Tomei Joichi, Dr. Klaus Krickeberg, Mr. Indar Singh Luthar, Mr. Chih-Han Sah, Mr. William Mack Sanders, and Mr. Shoji Sato.

Indiana University: Mr. Terence Butler, Mr. Gabriel Margulies, and Mr. Dock Sang Rim.

Institute for Advanced Study: Dr. Michael Francis Atiyah, Dr. Hans Joachim Bremermann, Mr. Georg Kreisel, Professor Sigekatu Kuroda, Professor Shigeo Nakano, Dr. Georges Papy, Dr. Walter Otto Roelcke, Professor Jean-Pierre Serre, Dr. Tsuneo Tamagawa, and Dr. John Hunter Williamson.

The Johns Hopkins University: Mr. Cornelius Bright Baytop, Mr. William Milton Brown, Mr. Vernon E. Derr, Mr. Walter James Douglass, Jr., Professor Jun-ichi Igusa, Mr. Kenneth Frederick Ireland, Mr. Jay Andrew Horton Jacobs, Reverend C. Frederick Koeler, S. J., Mr. Donald Malick, Mr. Richard Ferris Muth, Mr. Frank Vernon Rigler, Dr. Joseph Andrew Silva, Mr. John Clay Stuelpnagel, and Mr. James Howard Young.

Kenyon College: Mr. Trevor Herbert Barker.

Lehigh University: Mr. Francis Clyde Oglesby.

University of Maryland: Mr. Norman William Bazley, Mr. Louis William Ehrlich, Mr. Irving Isadore Glick, Mr. Steven H. Schot, and Mr. Harry Shaw, Jr.

University of Miami: Mr. Ernst Paul Buchtenkirch, Miss Michelle Roberta Harrell, and Mr. Jack Segal.

Michigan State University: Mr. John David Barab, Jr., Mr. Robert Gene Brown, and Mr. Joseph Alphonse Meier.

University of Minnesota: Mr. G. Philip Johnson.

University of Nebraska: Mr. Mervin L. Keedy.

Northwestern University: Mr. S. Forrest Ebey, Mr. Alan John Heckenbach, and Mr. Daniel Saltz.

Ohio State University: Miss Virginia Sheldon Hanley.

Oklahoma Agricultural and Mechanical college: Miss Parsla Kleinhofs and Mr. William Edwin Pruitt.

Oregon State College: Mr. David Linus Clark.

University of Oregon: Mr. Abdur Rahman Ansari, Mr. Larry Clifton Hunter, Mr. Donald Palmer Peterson, and Dr. William Laray Roach, Jr.

The Pennsylvania State University: Miss Thelma Ruth Hobaugh and Professor Mary Lister.

University of Pennsylvania: Mr. Shaul Reuven Foguel and Mr. Thomas Harold MacGregor.

Princeton University: Mr. Franz Bingen, Mr. Peter Nicholas Burgoyne, Mr. Aubert Daigneault, Mr. Arthur Pentland Dempster, Mr. John Edwin Derwent, Mr. Giulio Fermi, Mr. Harry Furstenberg, Mr. Marvin Jay Greenberg, Mr. Frank Bardsley Knight, Dr. Simon Bernard Kochen, Mr. Constantine John Koutsopoulos, Mr. James George Larocque, Mr. James Carlyle Lillo, Mr. Raymond Charles Mjolsness, Dr. Josef Schmid, Mr. Carroll Max Shipplett, and Mr. Stephen Weingram.

Purdue University: Mr. Jack Edwin Forbes, Mr. Joseph Edmund Kist, Mr. Carl William Kohls, Mr. Maynard Joseph Mansfield, Mr. Robert Hull McDowell, and Mr. Theron D. Oxley, Jr.

The Rice Institute: Mr. Howard Benton Curtis, Jr., Mr. Joel Dyne Erdwinn, Mr. James Edward Scroggs, and Mr. Daniel Weiser.

University of Rochester: Mr. David MacGregor Burton, Mr. Robert Allan Hultquist, and Mr. Paul L. Kingston.

Rutgers University: Professor Morton J. Hellman, Professor Richard E. Henry, Professor Robert Edward Luce, Mr. Guy W. Ricker, Professor Francis-Aloysius Charles Sevier, and Mr. Aaron Siegel.

College of St. Thomas: Mr. Lawrence Lee Pinsonneault.

University of Southern California: Mr. John Donald Brooks, Mr. Frank Sidney Cater, Mr. Stanley J. Greif, Mr. Alemu Melaku, Dr. Karl Hans Roth, Mr. Bernard Whitney, and Mr. Abraham Zukerman.

Stanford University: Mr. Gerald Lee Alexanderson, Mr. James Edward Burke, Mr. H. Jay Davis, Mr. Augustus Jerome Fabens, Mr. Donald D. Fisher, Miss Laura Stith Ketchum, Professor Harold Levine, Dr. Johannes Carl Christian Nitsche, Dr. Vikramaditya Singh, Mr. Dale William Swann, Mr. Ju-kwei Wang, and Mr. John Edwin Wetzel.

Syracuse University: Mr. Arnold Grudin, Mr. Simon Hellerstein, Mr. William Jordan Jones, Mr. Norman Samuel Rosenfeld, and Mr. Kapbyung Yoon.

The University of Tennessee: Mr. Ralph David McWilliams.

The University of Texas: Mr. Ben Fitzpatrick, Jr., Professor James Prichard Jewett, Mr. Charles William Leininger, and Mr. James Howard Wells.

University of Toronto: Mr. Frans Handest, Mr. Donald Albert Lyon, Mr. Barry Miller Mitchell, Mr. Frank Arthur Sherk, Mr. Carl Frederick Templin, Mr. James Ray Vanstone, and Mr. Joshua Tsunekatsu Yamada.

Tulane University: Mr. Lee William Anderson and Mr. Arthur Bernard Simon.

University of Utah: Mr. Lawrence Fearnley.

Vanderbilt University: Mr. Harvey P. Carter.

University of Virginia: Mr. John Joseph Costello.

Washington University: Mr. Nai-chao Hsu.

University of Washington: Mr. Billy Lee Foster, Mr. Shoshichi Kobayashi, and Mr. Sam Cundiff Saunders.

Wayne University: Mr. Max Krolik, Dr. Fred Meyer, Mr. Harold Willis Milnes, Mr. David Dean Morrison, and Mr. Sadanand Verma.

Wellesley College: Miss Isabel Stewart Macquarrie.

Yale University: Mr. John Harold Ahlberg, Mr. Robert Allen Bonic, and Mr. Arshag Berge Hajian.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: Deutsche Mathematiker-Vereinigung: Professor Herbert E. L. Bilharz, University of Würzburg, Germany, Dr. Friedrich Claus Huckemann, Harvard University, and Professor Emeritus Eduard Rembs, Technische Universität, Berlin, Germany; Indian Mathematical Society: Mr. Bhagwandas Dube, Government Engineering College, Jabalpur, India; Schweizerische Mathematische Gesellschaft: Dr. Michel Andre Kervaire, Berne University, Berne, Switzerland; Société Mathématique de France: Dr. Marcel Paul Schutzenberger, Centre National Recherche Scientifique, Paris, France; Swiss Mathematical Society: Dr. Urs Walter Hochstrasser, The American University and National Bureau of Standards, Washington, D. C.

The Secretary announced that by mail votes, Professors K. O. Friedrichs and Antoni Zygmund had been elected to the Executive Committee; Professor J. L. Doob had been elected a member of the

Editorial Committee for the Transactions and Memoirs as a replacement for Professor Kac for the period January 1–June 1, 1956; the U. S. Naval Postgraduate School, Worcester Junior College, Brandeis University, and the New Mexico College of Agriculture and Mechanic Arts had been elected to institutional membership.

The Council voted to approve a Summer Institute in 1957 on the topic Mathematical Logic.

The Council approved amendments to the by-laws to make it possible for a member, by paying an appropriate premium in addition to his dues, to receive either *Mathematical Reviews* or the *Transactions* as a privilege of membership in lieu of the *Proceedings*. The amount of the premiums will be set by the Trustees.

The Council voted to recommend the appointment of Professor J. L. Brenner as Executive Editor of *Mathematical Reviews.*

The Council voted to approve a Seminar on Mechanics to be held in the summer of 1957. The major objective of this Seminar will be instructional and it is expected that those who attend will have a background of information substantially equivalent to that of Ph.D.

The Secretary reported that Professor Marshall Stone had accepted an invitation to deliver the Gibbs Lecture in 1956; that Professor L. M. Graves had been appointed Editor of the Proceedings of the Applied Mathematics Symposium held at the University of Chicago, April 12–14, 1956; that the Advisory Board of the Applied Mechanics Reviews had been terminated as of December 2, 1955.

The Council voted to set meetings on April 5–6, 1957, at New York University; October 26, 1957, at the National Bureau of Standards, Washington, D. C.; and approved the dates November 30–December 1, 1956, for the fall meeting of the Southeast section at Lexington, Kentucky; and the dates August 26–30, 1957, for the Summer Meeting at Pennsylvania State University.

The Executive Director reported that the Headquarters of the Society will be moved to a new building, 190 Hope Street, Providence, Rhode Island.

The Council voted to authorize the sale of membership buttons.

The Council elected Professor W. S. Massey a member of the Editorial Committee for the Transactions and Memoirs as a replacement for Professor Samuel Eilenberg for the period June 1, 1956 to September 15, 1957.

The Council voted to request the Trustees to authorize two additional issues of the Proceedings during 1956.

The Secretary reported that Professor Reinhold Baer had submitted his resignation from the Editorial Board of the American Journal of Mathematics effective June 1, 1956.

The Council voted to approve a special session in Applied Mathematics at the Summer Meeting in Seattle, the session to consist of about five invited speakers.

The Council voted to omit the Colloquium Lectures in 1958.

The Council voted to discharge the Committee on the Role of the Society in Mathematical Publications.

Abstracts of the contributed papers follow. Those with "*t*" after the abstract number were presented by title. Where a paper has more than one author, the paper was presented by that author whose name is followed by "(p)". Mr. Akers was introduced by Professor Alonzo Church, Dr. Bareiss by Dr. R. M. Davis, Mr. Kobayashi by Professor C. B. Allendoerfer, Dr. Moser by Professor K. O. Friedrichs, and Professor Rector by Professor D. W. Hall.

ALGEBRA AND THEORY OF NUMBERS

464. S. S. Abhyankar: *Simultaneous resolution for algebraic surfaces.*

Let K be a two dimensional algebraic function field over an algebraically closed ground field of characteristic p and let K^* be a finite algebraic extension of K. Concerning the *simultaneous resolvability* of the pair (K, K^*) the following theorems are proved in this paper: *Theorem* 1. Let v^* be a zero dimensional valuation of K^*. Then there exists a *normal* model of K on whose K^*-normalization v^* has a simple center. *Theorem* 2. Assume that K^*/K is galois and $p=0$. Then there exists a *normal* model of K whose K^*-normalization is *nonsingular*. *Theorem* 3. Assume that K^* is a quadratic extension of K and $p\neq 2$. Then there exists a *nonsingular* model V of K whose K^*-normalization is also *nonsingular* and such that the branch locus on V is a nonsingular (in general reducible) curve. *Theorem* 4. Assume that K^* is a cubic cyclic extension of K and $p\neq 3$. Then we have the same conclusion as in Theorem 3. *Theorem* 5. Assume that K has a minimum model, i.e., that K is not the function field of a ruled surface. Let q be a prime number such that $q>3$ and $q\neq p$. Then there exist (lots of them) cyclic extensions K' of K of degree q such that there *does not exist* any *nonsingular* model of K having a *nonsingular* K'-normalization. (Received February 27, 1956.)

465. George Bachman: *Geometry in groups.*

Suppose that a finite group G is given which is considered as a group of motions of a point set. Let H be the subgroup of G leaving a given point P fixed. In order that G can be considered as a two-dimensional group of motions, it is demanded that the following axioms are satisfied: *Axiom* 1: H determines P uniquely and conversely. *Axiom* 2: Any motion has at most one fixed point unless it is the identity. *Axiom* 3: G is a transitive group of motions. It follows from these axioms that G must contain a subgroup H which is its own normalizer, and such that any two conjugates of H intersect only in the identity. A theorem of Frobenius gives us the existence of a normal subgroup which is interpreted as a subgroup of translations. A three-dimensional group of motions can be defined in a similar but more complicated manner. It is shown that Frobenius' theorem has an analogue in the three-dimensional case if certain additional assumptions are made. Examples can be given showing the necessity of all these conditions and showing that the theorems obtained are not empty. (Received February 28, 1956.)

466. W. E. Baxter: *Lie simplicity of a certain class of associative rings.* II.

Let A be a simple associative ring, characteristic not 2, either with its center $Z=(0)$, or of dimension greater than 16 over Z, and let A have an involution defined on it. Let K be the set of skew elements of A. In this paper, it is proven that if U is a proper Lie ideal of $[K, K]$ then U is contained in Z; that is, $[K, K]/Z\cap[K, K]$ is a simple Lie ring. In particular, if the involution is of the first kind then $[K, K]$ is a simple Lie ring. This generalizes well known results for matrices over fields. (Received February 3, 1956.)

467. David Gale: *The problem of rank in tensor products.*

Let $V_1, \cdots, V_n$ be vector spaces over a field F and let V_i have dimension d_i. Then any element $z\in \otimes_{i=1}^n(V_i)$ can be expressed in the form $z=\sum_{j=1}^k (V_i^j\otimes \cdots \otimes V_n^j)$. The minimum value of k over all such expressions is defined to be the *rank* of z. This agrees with the usual definition of rank for the case $n=2$. The problem of rank is then to determine the maximum of the ranks of all tensors z in $\otimes_{i=1}^n(V_i)$ as a function of the dimensions $d_1, \cdots, d_n$. This maximum is denoted by ρ. For $n=2$, the answer is classical, $\rho=\min(d_1, d_2)$. Some very special answers are obtained for the case $n=3$. (1) If $d_1d_2\leqq d_3$ then $\rho=d_1d_2$. (2) If $d_1=2$, $d_3=2d_2-1$ then $\rho=d_3$. (3) If $d_1=d_2=d_3=2$ then $\rho=3$. (4) The rank of a tensor may decrease if the base field F is extended. An example is given of a tensor having rank three over the real numbers and two over the complex numbers. (Received April 12, 1956).

468*t*. Karl Goldberg and Morris Newman: *Some free groups generated by matrices of order two.*

Let $A=(a_{ij})$ be a rational integral matrix of order two with determinant $d=\pm 1$, trace $t>0$, and $h=|a_{12}|-|a_{11}|-|a_{22}|-|a_{21}|\geqq 0$, such that if $d=1$ then $t\geqq 2$ and if $d=-1$ then $th\geqq 2$. Let B be another matrix with these properties. Then A and B^T generate a free group. (Received February 13, 1956.)

469. Oscar Goldman: *Algebraic functions of several variables.* Part I. *Divisors.*

If K is a field of algebraic functions of n variables with a perfect constant field k, a *temporary constant field* of K/k will mean a subfield L of K, containing k, which is such that K/L is a field of algebraic functions of one variable. If $P(K/k)$ is the set of $(n-1)$-dimensional valuations of K/k, and $P(K/L)$ the subset of $P(K/k)$ of those which are trivial on L, then a *divisor* of K/k is defined as an integer-valued function on $P(K/k)$ whose restriction to $P(K/L)$ is a divisor of K/L for every temporary constant field L of K/k. The group of these divisors is shown to have certain properties upon which one may base an arithmetic theory of algebraic functions of several variables in a manner different from that of algebraic geometry. (Received February 28, 1956.)

470*t*. Edwin Hewitt: *Compact monothetic semigroups.*

A topological semigroup G is *monothetic* if it contains an element a such that $\{a, a^2, a^3, \cdots\}^-=G$. Let G be a compact monothetic (Hausdorff) semigroup. Then

G is of one of the following 3 types. I. $G=H$, where H is the (compact) character group of a subgroup of the reals modulo 1. II. $G=H\cup\{a, a^2, \cdots, a^s\}$, where H is as in I and s is a positive integer. Multiplication in G is: $a^ia^j=a^{i+j}$ for $i+j\leqq s$, $a^ia^j=b^{i+j}$ for $i+j>s$, where b is any element of H such that $\{b, b^2, b^3, \cdots\}^-=H$; $xa^i=a^ix=b^ix$ for $i\leqq s$ and $x\in H$; xy is as in H for $x, y\in H$. All points a^i are isolated and H has its original topology. III. $G=H\cup\{a, a^2, a^3, \cdots, a^n, \cdots\}$, where H is as in I. Multiplication in G is: $a^ia^j=a^{i+j}$ for all i, j; $a^ix=xa^i=b^ix$ for $x\in H$ and all i, where b is as in II; xy is as in H for $x, y\in H$. All points a^i are isolated and a generic neighborhood in G of $x\in H$ is $U(x)\cup\{a^i:b^i\in U(x) \text{ and } i\geqq n\}$, where $U(x)$ is an arbitrary neighborhood of x in H and n is an arbitrary positive integer. Conversely, all topological semigroups of the form I, II, and III are compact and monothetic. (Received March 22, 1956.)

471*t*. Edwin Hewitt: *Compact semigroups with one-sided cancellation.*

Let S be a compact Hausdorff semigroup with the left cancellation law: $ax=ay \Rightarrow x=y$ for all $a, x, y\in S$. Then S is isomorphic and homeomorphic to a direct product $P\times E$, where P is a compact group and E is a compact semigroup in which $xy=y$ identically. E can be taken as the set of all idempotent elements of S. This set E is a compact subsemigroup of S in which $ef=f$ identically. P can be taken as Sa for an arbitrary $a\in E$. Sa is a compact subgroup of S. The mapping $(xa, e)\rightarrow xe$ is then an isomorphic homeomorphism of $(Sa)\times E$ onto S. (Received April 5, 1956.)

472*t*. J. H. Hodges: *Weighted partitions for general matrices over a finite field.*

For $\alpha\in GF(q)$, $q=p^n$, let $e(\alpha)=e^{2\pi i t(\alpha)/p}$ where $t(\alpha)=\alpha+\alpha^p+\cdots+\alpha^{p^{n-1}}$. If $A=(\alpha_{ij})$ is a square matrix with elements in $GF(q)$, let $\sigma(A)=\sum\alpha_{ii}$. Consider the sum $S=S(B, W, R, A)=\sum e\{\sigma(UW+RV)\}$, where B, W, R, A, U and V are matrices having elements in $GF(q)$, A is nonsingular of order m, B is of order t, W and V are m, t, R and U are t, m, and the sum is over all U, V such that $UAV=B$. If $W=0$ and $R=0$, S is the number of solutions U, V of $UAV=B$, which the author has given in *Representations by bilinear forms in a finite field* (Duke Math. J. vol. 22 (1955) pp. 497–510). In the present paper it is shown that the sum S can be expressed in terms of certain Kloosterman sums defined for square matrices over $GF(q)$. A number of the properties of these Kloosterman sums are also given. (Received February 27, 1956.)

473. Arno Jaeger: *Power series representations of rings and modified differentiations.*

The modified theory of differentiations in the sense of Hasse and F. K. Schmidt (cf. J. Reine Angew. Math. vol. 190 (1952) pp. 1–21) is extended to certain types of rings, in particular simple and semi-simple rings, and generalized to include homomorphic mappings. A generalization of the concept of semi-derivations in the sense of Dieudonné is defined, and relations between them and the differentiations established. The totality of all differentiations is constructed by means of formal series, a generalization of the author's expansion theory, and the chain-rule-dependency. (Received February 29, 1956.)

474. G. F. Leger, Jr.: *On cohomology theory for Lie algebras.*

The first three cohomology groups $H^n(L, M)(n=1, 2, 3)$ of a Lie algebra L with respect to an L-module M have well known interpretations. If L is an ideal of a Lie algebra G and M is also a G-module then G operates on $H^n(L, M)$. In this paper we devise operations of G on the interpretations of $H^n(L, M)$ which agree with the operations of G on $H^n(L, M)$. (Received February 29, 1956.)

475. A. B. Lehman: *Free modular lattices and linear graphs.*

Given a linear graph $P(a_1, \cdots, a_n)$ having branches $a_1, \cdots, a_n$, define the polynomials $p^{i,1}(a_1, \cdots, a_n)=a_i^{\bullet}(a_1, \cdots, a_n)$, $p^{i,j+1}(a_1, \cdots, a_n)=p^{i,j}(p^{1,1}, \cdots, p^{n,1})$ where $a_k^{\bullet}$ denotes the product of all sums of branches of P having the property that the elements of each sum constitute a series graph between the vertices of a_k when all other branches are deleted. Define $p_{i,j}$ analogously where a_{*k} is the sum of a_k and at most two product terms whose elements are all the branches other than a_k meeting at a vertex of a_k which is not a vertex of a_i. Theorem 1. $p^{i,j}\geqq p_{i,j}$ in a free modular lattice. If P is series-parallel it has a polynomial representation $p_i(a_1, \cdots, a_n)$ viewed from the vertices of a_i; with $+$ corresponding to series and $\cdot$ to parallel. Lemma. $p^{i,j}\geqq p_i =p^{i,k}$ for $k\geqq n$. Theorem 2. If $P(a_1, \cdots, a_n)$ and $Q(a_1, \cdots, a_n)$ are series-parallel graphs such that Q may be derived from P by a sequence of identifying vertices (shorting) and transpositions of terminals of two-terminal subgraphs (flips) then $p_i\geqq q_i$ in a free modular lattice. As $a(b+c(d+e))\geqq ae(b+cd)$, in a lattice, is equivalent to the modular law, each application of the law may be performed by a single short in a linear graph. Theorem 3. If $r\geqq s$ in a modular lattice, then there exist P, Q and i such that $r=p_i$ and $s=q_i$ in a free lattice, and P and Q satisfy the conditions of Theorem 2. Hence linear graphs may be used to generate free modular lattice inequalities; all free modular lattice inequalities may be computed in terms of linear graphs. Also see Bull. Amer. Math. Soc. Abstract 61-2-311. (Received February 28, 1956.)

476. G. de B. Robinson: *The degree f_λ of an irreducible representation $[\lambda]$ of S_n.*

The original Frobenius-Young formula (FY) for f_λ was recently modified (Frame, Robinson and Thrall, Canadian Journal of Mathematics vol. 6 (1954) pp. 316–324) to yield $f_\lambda=n!/H$, where H is the product of the lengths of all the hooks in $[\lambda]$. Another version of (FY) has been given by Feit (Proc. Amer. Math. Soc. vol. 4 (1953) pp. 740–744) who showed that $f_\lambda=n!|z_{ij}|$, where $z_{ij}=1/(\lambda_j-j+i)!$ This determinantal form can be deduced immediately from the operator equation (Robinson and Taulbee, P.N.A.S. vol. 40 (1954) pp. 723–726) $[\lambda]=(1-R_{ij})[\lambda_1]\cdot[\lambda_2]\cdot\cdots\cdot[\lambda_k]$, where each factor $(1-R_{ij})$ appears once only and R_{ij} raises a node from the disjoint constituent $[\lambda_j]$ to $[\lambda_i]$. Taking the degree on each side we have f_λ expressed as a linear sum $(\pm)$ of the degrees of the permutation representations on the right. We can distinguish "determinantal" terms, which yield the terms in the expansion of $n!|z_{ij}|$, and "nondeterminantal" terms, all of which cancel. This simple direct proof of (FY) emphasizes the importance of the permutation representations of S_n as a basis for the irreducible representations. A similar operator approach is applicable to skew diagrams $[\lambda]-[\mu]$, for which Feit's formula also holds. (Received February 20, 1956.)

477. Alex Rosenberg (p) and Daniel Zelinsky: *Global dimension of tensor products.*

Let A be an algebra over a field K with nilpotent radical N. Set $\bar{A}=A/N$ and suppose further that $[\bar{A}:K]<\infty$. Theorem: If $\bar{A}$ is separable, then for an arbitrary algebra B over K, $\text{l.gl.dim } A\otimes B=\text{l.gl.dim } A+\text{l.gl.dim } B$. If $\bar{A}$ is inseparable then $\text{l.gl.dim } A\otimes B$ is infinite or is equal to $\text{l.gl.dim } A+\text{l.gl.dim } B$. In proving the second half of the theorem we show, under the same hypotheses on A and B: Let M be a left $\bar{A}\otimes B$ module and N a left $A\otimes B$ module. Suppose that for $n>\text{l.dim}_B M$ one has $\text{Ext}^n_{A\otimes B}(M, N)\neq 0$, then also $\text{Ext}^{n+1}_{A\otimes B}(M, N)\neq 0$. (Received February 29, 1956.)

478. G. B. Seligman: *Characteristic ideals and the structure of Lie algebras.*

Let L be a Lie algebra of finite dimension over an arbitrary field F. An ideal I in L is characteristic if $ID\subseteq I$ for every derivation D of L. L possesses a unique maximal solvable characteristic ideal R, and L/R contains no solvable characteristic ideal other than (O). We say that L/R is characteristic semi-simple (css.). Every css. Lie algebra M contains a nonzero unique maximal characteristic ideal S with the property that S is a direct sum of characteristic ideals in S which, in their own right, contain no proper characteristic ideal. The restriction to S of $ad(x)$, x in M, defines an isomorphism of M into the Lie algebra of all derivations of S. The "characteristic simple" summands of S have the property that every proper ideal in such an algebra is nilpotent. An example is known where R does not coincide with the ordinary radical of L. It is possible to develop an analogous theory for restricted Lie algebras. (Received February 10, 1956.)

479. Ruth Rebekka Struik: *A relation between subgroups of a free product.*

A relation between subgroups of a free product is established. Let F be the free product of the groups $A_1, A_2, \cdots, A_p$. Let (R, S) be the commutator subgroup generated by $r^{-1}s^{-1}rs$, $r\in R$, $s\in S$. Let $m_1, m_2, \cdots, m_k$ be non-negative integers. Let ${}_0A_i$ be the normal subgroup in F generated by A_i, and let ${}_{m+1}A_i=({}_mA_i, F)$. Let $c({}_{m_1}A_{i_1}, {}_{m_2}A_{i_2}, \cdots, {}_{m_k}A_{i_k})$ be an arbitrary complex commutator of weight k in the components ${}_{m_j}A_{i_j}$ [see P. Hall; *A contribution to theory of groups of prime power order*, Proc. London Math. Soc. vol. 36 (1933) pp. 29–95]. Let $c_*({}_{m_1}A_{i_1}, {}_{m_2}A_{i_2}, \cdots, {}_{m_k}A_{i_k}) = \prod c({}_{m_1}A_{i_1}, {}_{m_2}A_{i_2}, \cdots, {}_{m_k}A_{i_k})$ where the product is taken over all possible ordered k-tuples $({}_{m_1}A_{i_1}, {}_{m_2}A_{i_2}, \cdots, {}_{m_k}A_{i_k})$, the jth element of which is of the form ${}_{m_j}A_{i_j}$, $1\leqq i_j\leqq p$; however, k-tuples in which $A_{i_1}=A_{i_2}=A_{i_3}=\cdots=A_{i_k}$ are excluded. Let $(A_i)_F$ be the subgroup of F generated by the subgroups (A_i, A_j), $i\neq j$. It is proved that $c({}_{m_1}F, {}_{m_2}F, \cdots, {}_{m_k}F)\cap{}_0(A_i)_F=c_*({}_{m_1}A_{i_1}, {}_{m_2}A_{i_2}, \cdots, {}_{m_k}A_{i_k})$ unless $c({}_{m_1}F, {}_{m_2}F, \cdots, {}_{m_k}F)={}_{m_1}F$ in which case ${}_{m_1}F\cap{}_0(A_i)_F={}_{m_1-1}(A_i)_F$. This relation is useful in computing a large number of the verbal products of S. Moran. (Received February 27, 1956.)

480t. Ruth Rebekka Struik: *On the finiteness of certain products of groups.*

It is shown that certain products of finite groups are infinite. Using the notation of Bull. Amer. Math. Soc. Abstract 61-3-400, let $G=A*B$; let m, n be fixed non-negative integers; let $A(m, n)B = A * B/({}_mA_G, {}_nB_G)({}_nA_G, {}_mB_G)$. Let A, B be finite groups. It is proved that if $A/(A, A)$ and $B/(B, B)$ are both nontrivial groups, then (A, B) in $A(m, n)B$ is infinite for $m \geqq 1$ and $n \geqq 1$; hence $A(m, n)B$ is infinite for $m, n \geqq 1$. If $A = (A, A)$ and $B = (B, B)$, then $A(m, n)B$ is the direct product of A and B for all m, n. If $A = (A, A)$ or $B = (B, B)$, then $A(m, n)B$ is finite for all m, n; in this case (A, B) in $A(m, n)B$ is Abelian. The proof utilizes identities and inequalities of subgroups of free products. Since the nilpotent product of two finite groups is finite [O. N. Golovin, Mat. Sbornik vol. 27 (69) (1950) pp. 427–453], this shows that $A(m, n)B$ for $m, n \geqq 1$ is a different type of product from the nilpotent products. (Received February 27, 1956.)

481. A. W. Tucker: *Dual lattice face structure in polar polyhedral cones.*

Let C be the polyhedral convex cone of all non-negative linear combinations of the vectors of a finite set S in Euclidean vector n-space, and C^* the polar cone of all vectors nonacute-angled to each vector of S. For each subset s of S form the set $F^*(s)$ of all vectors obtuse-angled to each vector of s and orthogonal to each vector of $s' = S - s$. Each vector of C^* belongs to one and only one $F^*(s)$, called an (open) *face* of C^*. Under set-inclusion the system L of those s for which $F^*(s) \neq \emptyset$ constitutes a relatively complemented lattice satisfying the Jordan-Dedekind chain condition, but generally not even semi-modular. For each s' form also the set $F(s')$ of all vectors expressible as fully positive linear combinations of the vectors of s' but not so expressible via any larger subset of S. [$F(\emptyset) = \emptyset$ or 0 according as 0 belongs or not to some other $F(s')$.] Each vector of C belongs to one and only one $F(s')$, called a *face* of C. For each s and $s' = S - s$, $F^*(s)$ and $F(s')$ are both vacuous *or* are open convex sets ($\neq \emptyset$) in complementary orthogonal subspaces. So the system L' of those s' for which $F(s') \neq \emptyset$ is the *dual* lattice of L. (Received April 10, 1956.)

482*t*. P. S. Wolfe: *Games over polyhedra.*

Let A be an m by n matrix, E_m and E_n the sets of all m-component row vectors and n-component column vectors respectively. Let X, Y be (not necessarily bounded) intersections of finite families of half-spaces in E_m, E_n respectively. Let $v_1 = \sup_{x \in X} \inf_{y \in Y} xAy$, $v_2 = \inf_{y \in Y} \sup_{x \in X} xAy$ ($+\infty$, $-\infty$ admitted). (A, X, Y) has the *value* $v_1 = v_2$ iff the equality holds. THEOREM: If (A, X, Y) has a finite value then xAy has a saddle point. If it does not have a value then $v_1 = -\infty$ and $v_2 = +\infty$. There exist (A, X, Y) having: finite value; value $+\infty$; value $-\infty$; no value. The last case does not obtain unless $m, n \geqq 2$. COROLLARY: If either X or Y is bounded, then (A, X, Y) has a value. (Received January 30, 1956.)

Analysis

483. M. G. Arsove: *Mass distributions for products of subharmonic functions.*

The class of all differences of locally bounded subharmonic functions on a region Ω forms an algebra $\mathfrak{A}$ under pointwise multiplication [Trans. Amer. Math. Soc. vol. 75 (1953) p. 347]. Hence, associated with the product $w = w_1w_2$ of two functions in $\mathfrak{A}$ there is a unique mass distribution m. The problem of determining m in terms of the mass distributions m_1 and m_2 for w_1 and w_2 is here studied by means of the Dirichlet integral and approximation techniques based on the work of Brelot, Cartan, Deny, and

Evans. If ω is a subregion bounded by a simple closed curve in Ω, then $m(\omega) = \int_\omega w_1 dm_2 + \int_\omega w_2 dm_1 - (1/\pi) D_\omega(w_1, w_2) = \int_\omega h_1 dm_2 + \int_\omega h_2 dm_1 - (1/\pi) D_\omega(h_1, h_2)$, where D_ω denotes the Dirichlet integral over ω and h_1 and h_2 are the solutions of the Dirichlet problem on ω for boundary functions determined by w_1 and w_2. This and similar results lead to subalgebras of $\mathfrak{A}$. For example the class of all δ-subharmonic functions having mass distributions given by continuous density functions is a subalgebra of $\mathfrak{A}$, and the density functions are related by $\rho_{w_1 w_2} = w_1 \rho_{w_2} + w_2 \rho_{w_1} - (1/\pi)[(\partial w_1/\partial x)(\partial w_2/\partial x) + (\partial w_1/\partial y)(\partial w_2/\partial y)]$. (Received February 27, 1956.)

484t. Stefan Bergman: *On a representation of stream functions of flows of a compressible fluid.*

The integral operator $P_{21}(f)$ of the second kind (see (A) Amer. J. Math. vol. 70 (1948) p. 856, and vol. 74 (1952) p. 444) transforms analytic functions of a complex variable f defined at $Z=a$, Re $a<0$ into solutions $\psi^{\langle\rangle}$ of the differential equation (1) $\psi^{\langle\rangle}_{ZZ^*} + F\psi^{\langle\rangle} = 0$. Here $Z = \lambda + i\theta$, $Z^* = \lambda - i\theta$, λ is a function of the Mach number, θ is the angle which the velocity vector forms with the positive x-direction, $F = \sum_{n=2}^{\infty} C_n \exp(2n\lambda)$ is a function which is singular at $\lambda = 0$. P_{21} is defined for $[|\lambda| < 3^{1/2}|\theta|]$. Replace (1) by (2) $\psi^{\langle\rangle(\alpha)}_{ZZ^*} + F_\alpha \psi^{\langle\rangle(\alpha)} = 0$, $F_\alpha = \sum C_n \exp(2n\lambda)/\Gamma(1+2n\alpha)$ and form the operator $\lim_{\alpha\to 0} \int_{-1} E_\alpha(Z, \overline{Z}, t) f(Z(1-t^2)/2)dt/(1-t^2)^{1/2}$ (see (A), p. 872); then one obtains the analytic continuation of the integral operator $P_{21}(f)$ to the parts of the subsonic region where $\psi^{\langle\rangle}$ is regular and to those segments of the sonic lines where $\psi^{\langle\rangle}(0, \theta)$ and $\{(-\lambda)^{1/3}[\partial\psi^{\langle\rangle}(\lambda, \theta)/\partial\lambda]\}_{\lambda=0}$ are continuous. The operator $P_{22}(f)$ with the generating functions $E_{22}^{(\kappa)} = \sum_{n=0}^{\infty} q^{(n\kappa)} (-t^2 Z)^{-n+1/2-(2/3)\kappa}$, $\kappa = 1, 2$ (see (A), p. 878 and p. 888) yields the continuation of $P_{21}(f)$ to the supersonic region. In this way a representation of the stream function in the whole domain of its existence can be obtained. (Received February 20, 1956.)

485. Jerome Blackman: *Convolutions with rational kernels.*

The convolution equation $f(u) = \int k(u-x)d\alpha(x)$ is considered under the conditions: (i) $k(x) = p(x)/q(x)$ where $p(x)$ and $q(x)$ are polynomials of degree p and q respectively. The distance from the set of zeros of $q(x)$ to the real axis $=d>0$. (ii) $q-p=n\geqq 1$. (iii) $\hat{k}(x)$, the Fourier transform of k, has no real zeros. (iv) $\alpha(x)$ is locally of bounded variation. (v) $\lim \int_{-A}^{B} k(u_0-x)d\alpha(x)$ exists as $A, B\to\infty$ for some u_0, $|\text{Im}(u_0)| > d$. Under these conditions a specific formula for $\alpha(x)$ is obtained, thereby also showing uniqueness. (Received February 16, 1956.)

486. F. E. Browder: *Regularity properties at the boundary of solutions of elliptic boundary value problem.*

Let K be a linear elliptic differential operator of order $2m$ on the domain G of E^n with boundary B of class C^{2m}. If the coefficients of K are of class C^0 on the closure $\overline{G}$ of G, then for $p>1$, there exists a constant $\psi_p>0$ such that for $f\in L^p(G)$, if u is a solution of $Ku=f$ under null Dirichlet boundary conditions, then $\|D^{2m}u\|_{L^p(G)} \leqq \psi_p\{\|f\|_{L^p(G)} + \|u\|_{L^1(G)}\}$, for any derivative $D^{2m}u$ of order $2m$. From a theorem of Sobolev, it follows that if $p>n/2m$, the derivatives of u of order less than $2m$ satisfy Hölder condition on G. More generally if B is of class C^{2m+r} $(r\geqq 0)$ while the coefficients of K are of class C^r on $\overline{G}$, there exist $\psi_{p,r}>0$ such that $\|D^{2m+r}u\|_{L^p(G)} \leqq \psi_{p,r} \cdot \{\|f\|_{H^{r,p}(G)} + \|u\|_{L^1(G)}\}$. The proofs combine theorems of Calderon-Zygmund on singular integral operators with a new integral representation of u near a boundary point. The results are extended to a general class of elliptic boundary value problems. (Received February 28, 1956.)

487. Lamberto Cesari and R. E. Fullerton (p): *A smoothing process for contours.*

Let S be a Fréchet surface defined by a continuous mapping $T: J \rightarrow E_3$ where J is a closed simple Jordan region in E_2 and let f be a real-valued continuous function defined in E_3. For any real number t let $C(t) = [w \in J \mid f(T(w)) = t]$ and $\rho(t)$ the generalized length of the image of the contour $C(t)$ [L. Cesari, *Surface area*, Princeton, 1956]. Let $\tilde{\Gamma}$ be the topological space having as points maximal continua of constancy $g \in \Gamma$ of T in J under the usual hyperspace topology. Let $\tilde{G} \subset \tilde{\Gamma}$ be the set of all elements of $\tilde{\Gamma}$ such that $C(t) = \bigcup_{g \in G} g$ and let $g_1, g_2 \subset C(t)$, $g_1 \neq g_2$. Then there exists in $\tilde{\Gamma}$ at least one irreducible continuum $\tilde{\gamma}$ joining g_1 and g_2 such that $\tilde{g} \in \tilde{\gamma}$ implies $g \subset C(t)$. The continuum $\gamma = \bigcup_{g \in \gamma} g$ in J is called a smoothed contour relative to g_1 and g_2. It is shown that this method of constructing a smoothed contour is equivalent to a second method in which the contour is constructed by eliminating inessential parts of $C(t)$ and also that γ has local properties of uniqueness and γ is an arc in $\tilde{\Gamma}$. Extensions to surfaces defined as mappings from a 2-manifold are considered and other properties of smoothed contours are investigated. (Received February 29, 1956.)

488t. K. T. Chen: *Integration of paths. A unique representation of paths by noncommutative formal power series.*

Let α be an irreducible piecewise smooth continuous path in the n-dimensional real affine space E with coordinates $\langle x_1, \cdots, x_n \rangle$. There are differentials $d\pi_1, \cdots, d\pi_m$ such that $\int_\alpha d\pi_1 \cdots d\pi_m \neq 0$ where $d\pi_i = \sum f_{ij}(x) dx_j$ with each $f_{ij}(x)$ being a polynomial in $x_1, \cdots, x_n$. It follows that some $\int_\alpha dx_{i_1} \cdots dx_{i_p} \neq 0$. Let $\theta(\alpha) = 1 + \sum_{p=1}^{\infty} \sum \int_\alpha dx_{i_1} \cdots dx_{i_p} X_{i_1} \cdots X_{i_p}$ and $\theta^*(\alpha) = 1 + \sum_{p=1}^{\infty} \sum \int_\alpha dx_{i_p} \cdots dx_{i_1} X_{i_1} \cdots X_{i_p}$. Then $\theta(\alpha) \neq 1$ and $\theta^*(\alpha) \neq 1$. It is further proved that if, for two irreducible piecewise smooth continuous paths α and β, one has $\theta(\alpha) = \theta(\beta)$ or $\theta^*(\alpha) = \theta^*(\beta)$, then the path β, subject to change of parameter, can be obtained from α by a translation of E. (Received February 27, 1956.)

489t. K. T. Chen: *Integration of paths in a differentiable manifold.*

Let M be an n-dimensional manifold of class $r \geqq 1$. A path α represented by $\alpha(t)$, $a \leqq t \leqq b$, is required to be piecewise smooth with only finitely many jump discontinuities. The product $\alpha \cdot \beta$ of two paths α and β is the path α followed by β; the inverse α^{-1} is obtained from α by changing the orientation of α. For any differentials $d\omega_1, \cdots, d\omega_p$ in M, set $\int_\alpha d\omega_1 = \int_{t=a}^{t=b} d\omega_1(\alpha(t))$ and, for $p \geqq 2$, $\int_\alpha d\omega_1 \cdots d\omega_p = \int_{t=a}^{t=b} [\int_{\alpha[a,t]} d\omega_1 \cdots d\omega_{p-1}] d\omega_p(\alpha(t))$ where $\alpha[a, t]$ denotes the portion of α with the parameter ranging from a to t. Given m differentials $d\omega_1, \cdots, d\omega_m$ in M, let $X_1, \cdots, X_m$ be noncommutative indeterminates and define $\theta(\alpha) = 1 + \sum_{p=1}^{\infty} \sum \int_\alpha d\omega_{i_1} \cdots d\omega_{i_p} X_{i_1} \cdots X_{i_p}$ and $\theta^*(\alpha) = 1 + \sum_{p=1}^{\infty} \sum \int_\alpha d\omega_{i_p} \cdots d\omega_{i_1} X_{i_1} \cdots X_{i_p}$, where $i_1, \cdots, i_p$ run over $1, \cdots, m$. By straightforward verification, $\theta(\alpha \cdot \beta) = \theta(\alpha)\theta(\beta)$ and $\theta^*(\alpha \cdot \beta) = \theta^*(\beta)\theta^*(\alpha)$. Furthermore, both $\log \theta(\alpha)$ and $\log \theta^*(\alpha)$ are Lie elements. A path α is irreducible if α is not in the form $\alpha = \alpha' \cdot \beta \cdot \beta^{-1} \cdot \gamma$, where α', β, γ are paths. It is shown that, given any irreducible path α, there exist differentials $d\omega_1, \cdots, d\omega_m$ in M for which $\theta(\alpha) \neq 0$ and $\theta^*(\alpha) \neq 0$. (Received February 6, 1956.)

490. W. F. Eberlein: *Approximation and functional analysis.* Preliminary report.

The prototype of the problems considered is that of best uniform approximation of a function f by an exponential sum $\sum_1^n A_m e^{-t_m s}$ $(0 \leqq s < \infty)$ where *both* the coefficients (A_m) and the exponents (t_m) are indeterminate—in contrast to the classical theory in which the n-exponents are assigned in advance. Existence and uniqueness theorems are obtained. There are specializations to the theory of numerical integration and generalizations to abstract harmonic analysis on groups. (Received February 27, 1956.)

491*t*. Albert Edrei: *A density theorem for entire functions possessing deficient values.*

Let $f(z) = \sum c_n z^{\lambda_n}$ be an entire function of finite order ρ and let $M(r)$ and $n(r)$ denote, respectively, the maximum modulus and the number of zeros of $f(z)$ in the disk $|z| \leqq r$. If (1) $n(r) = O(\log M(r))$, then (2) $\Delta(1) \geqq 1/(2\rho)$, where $\Delta(1)$ is the maximum density of the sequence $\{\lambda_n\}$. Using a result of Pfluger [Comment. Math. Helv. vol. 19 (1947) pp. 91–104], the above theorem implies the following corollary. Let σ denote the sum of the deficiencies of the finite deficient values of $f(z)$. The inequality (2) remains true if the condition (1) is replaced by the assumption $\sigma = 1$. It would be interesting to know whether (2) is still true if it is only known that $f(z)$ is of finite order and possesses some finite deficient value. (Received March 1, 1956.)

492. Charles Fox: *A classification of kernels which possess integral transforms.*

If $K(s) = \int_0^\infty k(x)x^{s-1}dx$, $K(s)$ is the Mellin transform of $k(x)$. It is well known that if $K(s)K(1-s) = 1$ then $k(x)$ can be the kernel of a generalized Fourier transform. In this paper kernels for which $K(s)K(1-s) \neq 1$ are considered. If $K(s) = H(s)/(m+ns)$ where $H(s)H(1-s) = 1$, m, n are constants it is proved that (1) $f(x) = \int_0^\infty k(ux)g(u)du$, (2) $g(x) = a\int_0^\infty k(ux)f(u)du + b(d/dx)\int_0^\infty xk(ux)(d/du)\{uf(u)\}du$, $a = m^2 + mn$, $b = n^2$. An example of such a kernel is $k(x) = x\int_x^\infty (\cos u/u^2)du$. It is then shown that kernels with Mellin transforms given by $K(s) = H(s)/(m+ns)P\{s(1-s)\}$ and $K(s) = H(s)/P\{s(1-s)\}$, $H(s)H(1-s) = 1$ and P denotes a polynomial form, also have transforms analogous to (1), (2) but more complicated. In the second case the transforms are symmetrical and a Parseval theorem exists of the form $\int_0^\infty f(x)^2dx = \int_0^\infty g(x)^2dx$. It is then possible to form a classification in which the type of transform for which $k(x)$ is a kernel is related to the properties of its Mellin transform $K(s)$. Thus Fourier kernels form class 1, kernels for (1) (2) form class 2 etc. One can frequently go from one class to the next by convolution. (Received February 17, 1956.)

493. Alvin Hausner: *Group algebras of vector-valued functions.*

Let $B = B(G, X)$ denote the set of all strongly measurable functions defined over a locally compact abelian group, G, with values in a complex commutative Banach algebra with or without an identity, X, which are such that $|f| = \int_G |f(a)| da < \infty$. B is a complex commutative Banach algebra under natural addition, scalar-multiplication, and multiplication defined by $f * g(a) = \int_G f(b)g(a-b)db$; the integral is a Bochner integral with respect to Haar measure. The following is proved: (1) The radical of B consists of functions with values in the radical of X a.e. so that B is semi-simple if and only if X is semi-simple; (2) The space of regular maximal ideals in B, topologized in the Gelfand sense, is homeomorphic with the topological product of the dual group of G and $\mathfrak{M}(X)$, the space of regular maximal ideals in X in the Gelfand topology; (3) B is regular if and only if X is regular; (4) Every proper closed ideal in B is contained in a regular maximal ideal if X is semi-simple, regular, and if the set of $x \in X$

with $\phi_M(x)$ having compact support in $\mathfrak{M}(X)$ is dense in $X(\phi_M$ denotes the canonical homomorphism from X onto the complex numbers associated with $M\in\mathfrak{M}(X))$. (4) generalizes Wiener's tauberian theorem from $L(G)$ to $B(G, X)$. The paper also considers kernels and hulls, representations of B, and certain homomorphisms from B to $L(G)$. (Received February 20, 1956.)

494*t*. Edwin Hewitt: *Invariant means for compact semigroups.*

Let S be a compact Hausdorff semigroup, $\mathfrak{C}(S)$ the linear space of all continuous real functions on S. For $\phi\in\mathfrak{C}(S)$ and $a\in S$, ${}_a\phi$ and ϕ_a are the functions such that ${}_a\phi(x)=\phi(ax)$ and $\phi_a(x)=\phi(xa)$, respectively, for $x\in S$. A non-negative linear functional M on $\mathfrak{C}(S)$ of norm 1 is a left [right] invariant mean if $M({}_a\phi)=M(\phi)$ $[M(\phi_a)=M(\phi)]$ for all $\phi\in\mathfrak{C}(S)$ and $a\in S$. Let K be the least [compact] ideal in S, and E the [nonvoid and compact] set of idempotents in K. Theorem 1. $\mathfrak{C}(S)$ admits an invariant mean M iff K is a group. In this case M is unique, and has the form $M(\phi)=\int_K\phi(x)d\lambda(x)$, where λ is normalized Haar measure on K. Theorem 2. If M is a right invariant mean and N is a left invariant mean, then $M=N$, and Theorem 1 applies. Theorem 3. If K is not a group, $\mathfrak{C}(S)$ may still admit a left invariant mean. This happens iff $eK=K$ for all $e\in E$. In this case Ke is a group for all $e\in E$ and E is a compact semigroup in which $xy=y$ identically. K is isomorphic to $(Ke)\times E$. The left invariant means on $\mathfrak{C}(S)$ are just the functionals M of the form $M(\phi)=\int_E\{\int_{Ke}\phi(x)d\lambda_e(x)\}d\mu(e)$, where λ_e is normalized Haar measure on the group Ke and μ is any non-negative Borel measure on E such that $\mu(E)=1$. (Received April 5, 1956.)

495. F. C. Huckemann: *On the strength of indirectly critical points of Riemann surfaces.*

The structure of the critical points of Riemann surfaces is related to the distribution of values of the generating function. It is an old problem [which is yet far from a solution] to determine this relation. Results so far available in this direction concern mostly directly critical points. It seems, therefore, useful to consider the simplest types of indirectly critical points and to investigate their contribution to the order as well as to the exceptional values of the generating function. After the introduction of a certain measurement for the strength of directly and indirectly critical points of a certain class, it is seen that indirectly critical points of positive strength on parabolic Riemann surfaces behave in some sense similarly to (the directly critical) logarithmic winding points. On surfaces of bounded characteristic, on the other hand [where directly critical points outside the closure of the boundary values of the generating function are always exceptional (Lehto)], this is not so; there are, however, also (strong) indirectly critical points on such surfaces which make the corresponding value exceptional. (Received February 28, 1956.)

496. Robert C. James: *A characterization of reflexivity.*

It is known that a reflexive Banach space has the property that each linear functional attains its sup on the unit sphere. It is shown that the converse of this is true for separable Banach spaces. Also, that a Banach space is reflexive if each linear functional attains its sup on the unit sphere of each separable subspace which has a basis. These results are related by the theorem that any nonreflexive Banach space has a nonreflexive subspace with a basis. It follows that a separable Banach space is reflexive if and only if for any bounded closed convex set U which does not intersect the unit sphere S there is a hyperplane which separates S and U and is disjoint from each of S and U. (Received March 1, 1956.)

497. W. T. Kyner: *Stability of invariant manifolds.*

J. McCarthy has generalized the periodic surface problem that has been studied by S. P. Diliberto and his students. Let $T:(W, V)\rightarrow(W, V)$ be a differentiable homeomorphism where V is a compact differentiable submanifold of the manifold W. V is called an invariant manifold of T and is said to be stable if for any differentiable family $T_\gamma: W\rightarrow W$ such that $T_0=T$, there is a continuous family $V_\gamma(|\gamma|<c)$ of submanifolds such that $T_\gamma: V_\gamma\rightarrow V_\gamma$ and $V_0=V$. McCarthy [Bull. Amer. Math. Soc. Abstract 61-2-264] has given sufficient conditions for stability. His result includes the (nondegenerate) periodic surface results of M. D. Marcus, but not those of G. Hufford and the author. The assumption that a certain pair of linear transformations associated with the homeomorphism T have uniformly bounded inverses implies stability. This extension of McCarthy's theorem includes the above-mentioned periodic surface results. (Received February 28, 1956.)

498. R. E. Lane: *Linear operators on quasi-continuous functions.*

Of the linear transformations from the set of all quasi-continuous functions to the set of all functions, the author considers those which satisfy a special boundedness condition, and shows that the transform of a quasi-continuous function is quasi-continuous and is the sum of two integrals; he gives a condition on the transformation which is necessary and sufficient for the transform of each quasi-continuous function to be an integral. He gives conditions sufficient for the transform of each quasi-continuous function to have various properties, such as continuity, bounded variation, continuous first derivative, etc., and gives examples of transformations for smoothing data and obtaining derivatives of the smoothed data. (Received December 22, 1955.)

499. P. D. Lax: *An abstract stability theorem.*

The following stability theorem for abstract differential equations is proved: Let H be a Hilbert space, D a nonpositive self-adjoint operator, $u(t)$ a solution of $du/dt=(D+K(t))u$, where $K(t)$ is bounded and $\int_0^\infty\|K(t)\|dt<\infty$. Conclusion: $u(t)$ unless identically zero tends to zero no faster than some finite exponential. Take H to be the space of harmonic functions in a solid sphere under L_2 scalar product, D to be $-\sum x_i\partial/\partial x_i$ and $u(t)$ the harmonic function whose boundary values are $w(rx)$, $r=e^{-t}$, and where $w(x)$ is a solution of $\Delta w=Nw$, N of first order. One can show that $\|du/dt-Du\|\leqq O(r)\|u\|$; hence, application of the abstract stability theorem gives a new proof of Claus Müller's theorem on the finiteness of zeros of solutions of such elliptic equations (see also E. Heinz, Hartman-Wintner, and Aronszajn for general second order equations). (Received March 29, 1956.)

500. W. S. Loud: *Periodic solutions of the equation $x''+cx'+g(x)=\epsilon e(t)$.*

Let $x_0(t)\not\equiv 0$ be a solution of (1) $x''+g(x)=0$ of period L. Using a technique given by Coddington and Levinson (*Theory of ordinary differential equations*, 1955, Chapter 14) it is shown that under appropriate conditions (2) $x''+g(x)=\epsilon e(t)$, with $e(t)$ of period L and small, has a unique solution of period L in some neighborhood of $x_0(t)$. $e(t)$ must be orthogonal over a period to $x_0'(t)$ and not orthogonal to $x_0''(t)$. The variation equation (3) $y''+g'(x_0(t))y=0$ must have no solution of period L linearly independent of $x_0'(t)$. The same results hold for the damped equation (4) $x''+cx'+g(x)$

$=\epsilon e(t)$ where c is small compared to ϵ. The stability properties of the solutions of (3) and (4) can be deduced from the algebraic sign of $\int_0^L e(t)x_0''(t)dt$ and the hardening or softening character of $g(x)$. ($g(x)$ is hardening if the periods of solutions of (1) decrease with amplitude, softening if they increase.) Existence and stability properties of numerous subharmonic solutions of (2) and (4) can be deduced. Results are obtained in cases in which some of the above conditions are not satisfied. The research for this paper was supported in part by the Office of Ordnance Research. (Received February 28, 1956.)

501*t*. F. I. Mautner: *Geodesic flows on symmetric Riemann spaces.*

Let M be a locally symmetric Riemann space (in the sense of E. Cartan) of negative curvature and finite volume. Denote by B the manifold of all tangent vectors of unit length to all points of M. The ergodic parts of B under the geodesic flow are determined explicitly, in terms of the group G of isometries of the universal covering space $\tilde{M}$ of M. The geodesic flow is ergodic in B if and only if the universal covering space $\tilde{M}$ is of rank one in the sense of E. Cartan. It follows that there exist compact complex analytic locally symmetric Kaehler manifolds of finite volume and negative curvature for which the geodesic flow is not ergodic in B. The spectrum of the geodesic flow is studied and it is proved to be absolutely continuous under certain conditions. The method used is the reduction of the problem to that of unitary representations of semi-simple Lie groups and properties of invariant measures on homogeneous spaces. (Received February 23, 1956.)

502*t*. B. C. Meyer and H D Sprinkle: *Two nonseparable complete metric spaces defined on I.*

Two non-separable, complete metric spaces are defined on certain equivalence classes of the set of all subsets of $[0, 1]$. For each of the spaces, the subspace consisting of the equivalence classes of measurable sets has cardinality $\mathfrak{c}$, is perfect, and is nowhere dense in the space; its complement has cardinality $\mathfrak{f}$, is open, and is dense in the space. (Received February 7, 1956.)

503. Josephine Mitchell: *Orthogonal systems in matric spaces.* Preliminary report.

Let the domain $D=E\{z|I-zz^*>0\}$, where z is an n by n matrix of complex numbers, z^* its conjugate transpose, I the identity and 0 the zero matrix. A proper part of the boundary of D is the set $B=E\{z|zz^*=I\}$. For $n=2$ a complete orthonormal system with respect to the inner product $(f, \bar{g})=\int_D f\bar{g}dV_D$ (dV_D being Euclidean volume element on D) for analytic functions on D with $(f, \bar{f})<\infty$ is (1) $\{\phi_{pqr}^{(\nu)}(z) \equiv Nz_{ab}^p z_{cd}^q z_{ef}^r F(-\nu, -\nu-p; q+r+4-2\nu; Z)\}$ ($\nu=0, \cdots, m=\min (q, r)$; $p, q, r=0, 1, 2, \cdots$), where $Z=z_{ab}z_{gh}/z_{cd}z_{ef}$, the z's are distinct elements of the matrix z with $c\neq e$ and $d\neq f$ and F is the hyper geometric function. The normalizing factor N equals $\{(4(\nu+1)/\pi^4)C_{r-\nu+2,2}C_{q-\nu+2,2}C_{p+q+r+3-\nu,p+\nu}C_{q+r+3-\nu,\nu+1}\}^{1/2}$. The system (1) with a different normalizing factor is also a complete orthonormal system for functions f, analytic on D and defined on B, with respect to the inner product $\int_B f\bar{g}dV_B$. For $n>2$ and the inner product $(f, \bar{g})$ the system (1) with appropriate modifications is orthonormal but not necessarily complete. Let $s(z)=\sum_{p,q,r=0}^{\infty}\sum_{\nu=0}^{m} a_{pqr}^{(\nu)}\phi_{pqr}^{(\nu)}(z)$ be an orthogonal series formed from (1). If $s(I)$ exists, then $s(rI)$ is uniformly convergent for $0 \leqq r \leqq 1$ and $\lim_{r \to 1-} s(rI) = s(I)$. This is a special case of a more general theorem which assumes that $s(I)$ is Cesàro or Abel summable. (Received February 28, 1956.)

504. Jürgen Moser: *The analytic invariants of an area-preserving unstable mapping.*

The statement presented is concerned with area-preserving mappings M: $x_1 = f(x, y) = \lambda x + \cdots$, $y_1 = g(x, y) = \lambda^{-1} y + \cdots$, $f_x g_y - f_y g_x = 1$; $\lambda^2 > 1$, in the neighborhood of an unstable fixed point which we assume to be $x = y = 0$. The functions $f(x, y)$, $g(x, y)$ are assumed to be real analytic functions in some neighborhood of the origin. Then there exists an area-preserving transformation $x = \phi(\xi, \eta)$, $x_1 = \phi(\xi_1, \eta_1)$, $y = \psi(\xi, \eta)$, $y_1 = \psi(\xi_1, \eta_1)$, which transforms M into the "normal form" $\xi_1 = \xi U(\xi\eta)$, $\eta_1 = \eta U^{-1}(\xi\eta)$, where $U(\xi\eta) = \lambda + U_1 \xi\eta + \cdots$ is a function of the product $\xi\eta$. Furthermore ϕ, ψ, U are real analytic functions for sufficiently small ξ, η. In a weaker form this theorem was known to G. D. Birkhoff (Acta Math. vol. 43 (1920) pp. 1–119) namely for *formal* power series ϕ, ψ, U of ξ, η. The present theorem guarantees the *convergence* of these series. The proof uses Cauchy's majorant method. The coefficients of $U(\xi\eta)$ turn out to be the full system of invariants of M under the group of real analytic area-preserving transformations. (Received March 8, 1956.)

505. Dr. G. O. Peters: *Boole polynomials and numbers of the second kind.*

Charles Jordan in his book *Calculus of finite differences* (Budapest, 1939) defines Boole polynomials of order one. Nörlund in his book *Verlesungen über differenzenrechnung* (Berlin, 1924) defines Bernoulli and Euler polynomials of higher order. The author (Bull. Amer. Math. Soc. Abstract 62-1-24) defined the Boole polynomials of higher and negative order. Interchanging the roles of Δ and D and replacing factorials of X by powers of X, the author calls a new set of polynomials, Boole polynomials of the second kind. The author (Bull. Amer. Math. Soc. Abstract 62-2-213) defined the operator ℸ (Dalet) where $ℸ = 1 + 1/2\, D$. The author defines the Boole polynomials of the second kind of degree ν and order n, as the polynomial $Z_\nu^n(X)$ satisfying the equation $ℸ^n Z_\nu^n(X) = X^\nu$. The Boole polynomials of the second kind of degree ν and order $-n$, is defined as the polynomial $Z_\nu^{-n}(X)$ satisfying the equation $Z_\nu^{-n}(X) = ℸ^n X^2$. The Boole numbers of the second kind are defined by the equation $Z_\nu^n(X) = \sum_{i=0}^{\nu} \nu(i(Z_i^n/2^i) X_\nu^{-i} = (Z^n/2 + X)^\nu$ where n can be either positive or negative. Some properties of these Appell polynomials are found and the values of the Boole numbers are determined. The symmetry of the polynomials is also found. An interesting polynomial relation is found relating Euler, Boole and Bernoulli polynomials of the second kind. (Received February 29, 1956.)

506. Alexander Peyerimhoff: *On a class of matrix transformations.*

In a recent paper (J. London Math. Soc. vol. 29 (1954) pp. 459–476) H. C. Chow proved that a triangular matrix transformation $y_n = \sum_{v=0}^{n} a_{nv} x_v$ which transforms every sequence $x_n = O(1)$ into $\sum |y_n| < \infty$ has the property $\sum |a_{nn}| < \infty$. This is a special case of the following theorem: A matrix transformation $y_n = \sum_{v=0}^{\infty} a_{nv} x_v$ transforms every sequence $x_n = o(1)$ into $\sum |y_n| < \infty$ if and only if there exists a constant K with $\sum_{n=0}^{\infty} | \sum_{v \in \mathfrak{M}_n} a_{nv} | \leqq K$ for every sequence $\{\mathfrak{M}_n\}$ of sets of non-negative integers such that $\mathfrak{M}_n \cap \mathfrak{M}_m = 0$ if $\mathfrak{M}_n \neq \mathfrak{M}_m$ (this formulation does not exclude the case $\mathfrak{M}_{n_1} = \mathfrak{M}_{n_2} = \mathfrak{M}_{n_3} = \cdots$), and K independent of $\{\mathfrak{M}_n\}$. This condition cannot be replaced by $\sum_{n,v} |a_{nv} < \infty$. (Received February 29, 1956.)

507. V. C. Poor: *Residue of a complex function, a contour integral.*

This paper contains a mathematical deduction of a logarithmic residue formula from the fundamental results obtained in previous papers. It reduces the two formulae for residues over an area to a single contour integral. Its application to some special residues is included. (Received February 27, 1956.)

508*t*. V. C. Poor: *The logarithmic residue.*

This paper contains a brief development of the formula for the residues of a subclass of the class of complex functions. The formula is a contour integral of the logarithmic type, which completes the theory of residues of complex functions. Applications to complex functions are briefly but adequately discussed. (Received February 13, 1956.)

509. J. J. Price: *Certain classes of orthonormal step functions.*

Let $n_1, n_2, n_3, \cdots$ be a sequence of positive integers, $n_i \geqq 2$, and let G_i be a cyclic group of order n_i. There exists on the unit interval an orthonormal system of step functions $S(n_1, n_2, n_3, \cdots)$ which is essentially the set of characters of the group $G_1 \times G_2 \times G_3 \times \cdots$. $S(2, 2, 2, \cdots)$ is the Walsh system. $S(\alpha, \alpha, \alpha, \cdots)$ has been studied recently by Chrestenson. When $n_1, n_2, n_3, \cdots$ is unbounded, the behavior of the Fourier series of an integrable function with respect to $S(n_1, n_2, n_3, \cdots)$ is considerably different from the behavior in the above cases. The following theorems are proved for $n_1, n_2, n_3, \cdots$ unbounded. (1) There exist functions in Lip 1 whose Fourier coefficients are not $O(1/n)$. (2) If the n_i do not increase too rapidly, the Fourier coefficients of any nonconstant absolutely continuous function are not $O(1/n)$. (3) There exists a continuous function whose Fourier series is not $(C, 1)$ summable at a given point. (February 29, 1956.)

510*t*. R. T. Prosser: *Homogeneous W^* algebras.*

A W* algebra is homogeneous if it is isomorphic with each of its direct summands. If A is a homogeneous W* algebra with center Z, then A contains a W* subalgebra F such that F is a factor algebra of the same type as A, and A is the W* tensor product of F and Z. Moreover, F is unique, up to equivalence. If it is true that every W* algebra is the W* direct sum of homogeneous algebras, then these results effectively reduce the classification problem of general W* algebras to that of simple factor algebras. (Received February 29, 1956.)

511. R. T. Prosser: *Representations of C^* algebras.*

Let A be a C* algebra. We recall that the second adjoint A'' of A forms a W* algebra under the operations inherited from A. We show here that every C* representation of A may be lifted to a W* representation of A'' essentially by taking weak closures. This process generates a natural one-one correspondence between the C* representations of A and the W* representations of A''. It follows that the C* representations of A may be characterized (up to equivalence) by suitable multiplicity functions defined on the set of central projections of A''; or, equivalently, by multiplicity functions defined on the set of norm-closed central supports of the state space Ω of A. These results include most of the standard theorems on representations of C* algebras, and settle a number of open questions. In particular, the multiplicity theory recently developed by R. V. Kadison is a consequence. (Received February 29, 1956.)

512. R. A. Raimi: *Mean values and Banach limits.*

Let E be a Banach space of essentially bounded measurable real-valued functions on the real line, $\|f\| = \text{ess sup } |f(x)|$. Let E contain all the uniformly continuous bounded functions, and all translates of functions in E. Let $T_\alpha : f(x) \to (1/2\ \alpha)\int_{-\alpha}^{\alpha} f(x+t)dt$, and let $V = \{f \in E \mid T_\alpha f$ converges in $E\}$. Then $\lim_\alpha T_\alpha f$ is a constant, $m(f)$, defining a continuous linear functional on V. The class M of all norm-preserving extensions of $m(f)$ to all of E is studied. ϕ is in M if and only if ϕ is linear and $\lim_\alpha \inf_x T_\alpha f(x) \leqq \phi(f) \leqq \lim_\alpha \sup_x T_\alpha f(x)$ for all f in E (the limits exist). Alternate expressions, involving the geometry of E and its conjugate space, are given. All the elements of M are *Banach Limits* (translation-invariant positive functionals of norm one); if E contains only uniformly continuous functions, the converse is true. (Received February 16, 1956.)

513. B. L. Reinhart: *Harmonic integrals on almost product manifolds.*

An almost product manifold V is a compact, connected C^∞ manifold provided with a C^∞ direct sum decomposition of the tangent bundle $T(V)$ into integrable subspaces $PT(V)$ and $QT(V)$; local coordinates $(x^1, \cdots, x^p, y^{p+1}, \cdots, y^n)$ exist consistent with the structure. A natural bigrading is induced on the differential forms. This decomposition induces a splitting of the exterior derivative $d = d' + d''$ and its adjoint $\delta = \delta'\delta''$; and special Laplacians $\Delta' = d'\delta' + \delta'd'$, $\Delta'' = d''\delta'' + \delta''d''$, and $\tilde{\Delta} = \Delta' + \Delta''$. The operators d', δ', and Δ' differentiate with respect to x only. Examples show that the Green's operator for Δ' is not in general everywhere defined on $\mathfrak{L}_2(V)$, and that the analogue of Hodge's theorem is false. It is here proved that the Green's operator is defined for every C^∞ section of the sheaf $E = D_x^r \wedge D_y^s$, where D_x^r is the sheaf of germs of r-forms depending on x alone, provided V has a torsionless metric. The proof employs a method of Morrey and Eeels (Ann. of Math. vol. 63 (1956) pp. 91–128) applied to the Hilbert space generated by the C^∞ sections of a sheaf coherent in the sense that locally there exists a finite number of forms $\gamma(y)$ generating each stalk as a D_x^r-module. The necessary parametrix is constructed by a method of Kodaira (Ann. of Math. vol. 50 (1949) pp. 587–665). The operator $\tilde{\Delta}$ has the property that its solutions are precisely the harmonic forms of pure type; in the case of a torsionless metric, $\Delta = \tilde{\Delta}$. (Received February 29, 1956.)

514*t*. Walter Rudin: *Boundary values of continuous analytic functions.*

Let K and C denote the closure and boundary, respectively, of the open unit disc U. The following extension theorem is proved: If E is a closed subset of C, if the Lebesgue measure of E is 0, if g is a complex-valued continuous function on C, and if T is a closed Jordan region such that $g(E) \subset T$, then there exists a function f which is continuous on K and analytic in U, such that $f(K) \subset T$ and $f(z) = g(z)$ for all $z \in E$. (Received February 23, 1956.)

515. Robert Schatten: *On the "trace-class" of operators.*

Consider the Banach space of completely continuous operators on a Hilbert space $\mathfrak{H}$. Its conjugate space may be interpreted as the "trace-class," i.e., a Banach space of some operators A on $\mathfrak{H}$ where the norm of A is given by the trace of $(A^*A)^{1/2}$. It is natural therefore to consider the space $\mathfrak{C}$ of completely continuous operators on a given Banach space $\mathfrak{B}$ and identify its conjugate space $\mathfrak{C}^*$ with the corresponding "trace-class." Under some reasonable assumptions a nontrivial evaluation of the norm of an operator in $\mathfrak{C}^*$ is given. (Received February 17, 1956.)

516*t*. V. L. Shapiro: *On Green's theorem.*

Let D be a bounded domain in the (x, y)-plane, let C be its boundary, and let E be a closed set of logarithmic capacity zero contained in $D+C$. Then by means of the Riemannian theory of multiple trigonometric series, it is shown in this paper that Green's theorem $\int_C Ady-Bdx=\int_D(A_x+B_y)dxdy$ holds under the following four assumptions: 1. The boundary C consists of one or several simple closed rectifiable curves. 2. The functions $A(x, y)$, $B(x, y)$ are defined and continuous in the closure $\overline{D}=D+C$. 3. The functions A and B have total differential in $D-E$. 4. The sum $\Phi(x, y)=A_x+B_y$ is Lebesgue integrable over D. Furthermore, it is shown in this paper that this theorem cannot be extended to the case where E is of measure zero and positive logarithmic capacity. This theorem extends a result recently obtained by Bochner (Math. Zeit. 1955). Analogous results also hold in n-dimensional Euclidean space $(n\geqq 2)$. (Received February 23, 1956.)

517. J. R. Shoenfield: *Infinite dimensional manifolds.* Preliminary report.

Let B be a Banach space. A B-manifold of class C^k is a space S with a class of mappings $[\phi_\alpha | \alpha\in A]$ of B into S such that each ϕ_α is one-one; the ranges of the ϕ_α cover S; and each $\phi_\alpha^{-1}\phi_\beta$ is differentiable of class C^k in the sense of Frechêt. Many elementary properties of differentiable manifolds hold for such manifolds. There is a natural definition of the tangent space at a point x of S; it is a Banach space isomorphic to B. Affine connections and Riemann tensors may be defined for such manifolds. Sample theorem: In a Riemannian B-manifold, the geodesics issuing from a point simply cover a neighborhood of that point. The definition of Lie groups may be extended to such manifolds. The basic theorems of Lie groups hold, except that it is necessary in some cases to add the following hypothesis: Every closed subspace of B has a complementary closed subspace. (Received February 29, 1956.)

518. Seth Warner: *Equivalence of a problem in topological algebra with Ulam's measure problem.*

An *Ulam ultrafilter* $\mathfrak{F}$ on a set A is an ultrafilter such that the intersection of a countable family of members of $\mathfrak{F}$ is again a member of $\mathfrak{F}$. Call the following assertion Axiom U: For any set A and any Ulam ultrafilter $\mathfrak{F}$ on A, there exists $\alpha\in A$ such that $\mathfrak{F}$ is the class of all subsets of A containing α. (Equivalently, for any set A and any nonzero measure μ defined on all subsets of A taking on only the values 0 and 1, there exists $\alpha\in A$ such that $\mu(X)=1$ if and only if $\alpha\in X$.) A Hausdorff locally m-convex algebra E is *i-bornological* if any homomorphism from E into any other locally m-convex algebra carrying bound, idempotent subsets of E into bound sets is continuous. Axiom U is equivalent to the following: If $\{E_\alpha\}$ is any family of i-bornological algebras, each with identity, $\prod_\alpha E_\alpha$ is i-bornological. Recent advances in logic show that if the usual axioms of set theory are consistent, Axiom U may be added without destroying consistency. Recent advances in topology show that Axiom U is equivalent with any of the following: (1) Every Hausdorff topological space admitting a compatible, complete uniform structure is a Q-space in the sense of Hewitt; (2–3) Every paracompact [respectively, metrizable] space is a Q-space. (Received February 27, 1956.)

519. Albert Wilansky: *Banach algebra and summability.*

The set C of conservative matrices is a semi-simple Banach algebra, $\|A\| = \sup_n \sum_k |a_{nk}|$; a closed subalgebra of the space of endomorphisms of m (bounded sequences). Trivial fact: a maximal linear subspace of an algebra is an ideal if it is a subalgebra. Corollaries (also provable directly). 1. *The set of conull matrices is an ideal in C.* Proof. The product of conull matrices is conull. 2. $\chi(A) \equiv \lim_n \sum_k a_{nk} - \sum_k \lim_n a_{nk}$ *is a multiplicative function of A.* For a sequence x, let (x) be the class of matrices in C which sum x. Theorem. (x) *is a left ideal iff* x *is bounded,* (x) *is a right ideal iff* x *is convergent,* (x) *is closed iff* (x) *is bounded,* $I \in \overline{(x)}$ *if* x *is unbounded.* It is known that $(x) \subset (y)$, y divergent, imply $(x) = (y)$. *Each* a_{nk} *is a continuous function of* A, *so also is each* $\lim_n a_{nk}$. Corollary: *The subset of multiplicative matrices is closed.* Let F be Banach algebra of matrices of finite norm. Theorem. *C has inverses in F* (i.e. $A \in C$, $A^{-1} \in F \Rightarrow A^{-1} \in C$). A conull matrix may have a right inverse in F (compare Corollary 1). Mercer's theorem implies that the $(C, 1)$ matrix lies in closure of maximal group. (Received February 27, 1956.)

520. Y. K. Wong: *On unbounded infinite matrices and their inverses.*

Let the columns of A consist of a sequence of Hilbert vectors such that every finite subset is (right) linear independent. The elements of A lie in a locally compact, connected topological field satisfying the second axiom of countability. By the orthogonalization process, $B = AT$ where $T(i, j) = 0$ for $i > j$ and $T(i, i) > 0$. Let B^* be the conjugate-transpose of B. Then $B^*B = I$ and $BB^* = E$ which is idempotent, hermitian, nonnegative-definite, and dependent only on A. There exists a unique matrix L_0 such that $L_0E = L_0$ and $L_0A = I$ if and only if all columns of T^* are Hilbert vectors. Then $L_0 = TB^*$ which is independent of T and B. L_0 is bounded if and only if T is bounded. Similarly if the columns of A^* are Hilbert vectors which are finitely (right) linearly independent, $C = SA$, $CC^* = I$ and $C^*C = F$ which is idempotent, hermitian and nonnegative-definite. There exists uniquely R_0 such that $FR_0 = R_0$ and $AR_0 = I$ if and only if all columns of S are Hilbert vectors. Then $R_0 = C^*S$. R_0 is bounded if and only if S is bounded. If the bounded matrices L_0, R_0 exist such that $L_0A = I = AR_0$ and $AL_0 = E$, $R_0A = F$, then $A^{-1} = L_0 + R_0(I - E) + Z = R_0 + (I - F)L_0 + Z$, where $ZA = AZ = 0$ and Z is bounded. (Received February 21, 1956.)

APPLIED MATHEMATICS

521. E. H. Bareiss: *On the numerical solution of the Boltzmann transport equation.* I.

An iterative method for the numerical solution of the Boltzmann transport equation for solving neutron transport problems is described. Several "Flexible Transport Theory Routines" have been coded for the UNIVAC and NORC and are currently used by the Westinghouse Atomic Power Division in their nuclear reactor design program. The problems are identified by the following common characteristics: One-dimensional, slab geometry; arbitrarily many regions per reactor cell; single energy with isotropic and anisotropic scattering and sources. The reactor cells may have reflecting and free boundaries. In the first code, acceleration of convergence may be obtained by Aitken's δ^2-process applied on the entire field of the solution. In the other codes, there is no provision for the extrapolation of the approximate solutions. (February 27, 1956.)

522. G. F. Carrier and R. C. DiPrima (p): *On the unsteady motion of a viscous fluid past a semi-infinite flat plate.*

The flow past a semi-infinite flat plate is considered when the flow at infinity is of the form $U_0+U_1 \exp (i\omega t)$, where U_0 and U_1 are constants. The equations of motion governing a viscous incompressible fluid are linearized by a modification of the Oseen linearization. Two fourth order partial differential equations are obtained governing the steady flow and the time dependent flow. These equations are solved by the use of Fourier transforms and the Wiener-Hopf technique. Results valid near the leading edge and far down the plate are obtained. (Received February 21, 1956.)

523*t*. Nathaniel Coburn: *A class of compressible flows.*

The purpose of this paper is to express the characteristic relations of an irrotational, isentropic, supersonic fluid in intrinsic form for the case when one family of of characteristic surfaces is ∞^1 parallel planes and to study the resulting flows of the fluid. Two types of flow are investigated: (1) the bicharacteristics are concentric circles; the stream lines are helices; (2) again, the bicharacteristics are concentric circles; the stream lines lie on cones and are a type of generalized helix. In each case, the Mach number varies with the radius of the bicharacteristic. (Received April 9, 1956.)

524. Ruth M. Davis (p) and Elizabeth Cuthill: *Improving convergence of the successive over-relaxation method.*

In solving numerically either eigenvalue or boundary value problems involving a system of n elliptic partial differential equations of the form (1) $\nabla\cdot D_i\nabla\phi_i - A_i\phi_i + B_i\phi_{i-1}=0$, $\nabla\cdot D_1\nabla\phi_1 - A_1\phi_1+\eta B_n\phi_n=0$, $i=2, 3, \cdots, n$, one is led to consider systems of linear equations which can be written in matrix notation as (2) $A\boldsymbol{\phi}_i=B\boldsymbol{\phi}_{i-1}$, $C\boldsymbol{\phi}_1=\eta D\boldsymbol{\phi}_n$, $i=2, 3, \cdots, n$. The linear iterative method of successive overrelaxation developed by D. Young has been adapted for use in solving the system (2). An optimum over-relaxation factor ω_b is computed, using a Rayleigh Quotient, at each iteration which can be incorporated into succeeding calculations to aid convergence. To further hasten the convergence of slowly convergent sequences of solution approximations for each individual equation of (2) use is made of Aitken's δ^2 process. Additional extrapolatory techniques are applied to the successive approximations to the entire solution of the system (2) with favorable results. (Received February 27, 1956.)

525. I. J. Epstein: *A study of integrals and integral equations arising in diffraction theory.*

The problem of the diffraction of a normally incident plane wave by a circular aperture in a plane screen is treated here. The development is according to Bouwkamp and Magnus. The field in the aperture is determined by an infinite system of linear equations. The coefficients of these equations are shown to have a simple asymptotic behavior as the wavelength λ of the incident wave goes to zero. The system of linear equations is shown to be equivalent to an integral equation which is solved explicitly in the limit case $\lambda=0$. A method for determining correction terms for small values of $\lambda\neq 0$ is given and the order of magnitude of the correction term is investigated. Some of the incidental results are listed separately since they are of some interest as cosine transforms. (Received February 21, 1956.)

526. B. A. Fleishman: *The random convection model of turbulent diffusion.*

The fundamental problem of turbulent diffusion is to find the mean mass density distribution, $\langle s(x, y, z, t)\rangle$, of matter inserted in a turbulent velocity field, in terms of the statistical characteristics of the field. By combining a purely kinematical approach with the velocity-ensemble approach of the mathematical theory of turbulence, it is possible to formulate such problems as initial value problems for partial differential equations with random coefficients. If some dispersible matter (assumed not to be subject to molecular diffusion) is convected by a vector velocity field $\bar{v}(x, y, z, t)$, its mass density, $s(x, y, z, t)$, must satisfy the convection (or continuity) equation $s_t = -div(s\bar{v})$. Upon the choice of appropriate initial and/or boundary conditions an initial value problem is formulated, for a particular convection field $\bar{v}(x, y, z, t)$. Now let $\bar{v}(x, y, z, t)$ belong to an ensemble of functions (which is identified with the real turbulent velocity field). Then the expected mass density distribution, $\langle s(x, y, z, t)\rangle$, is found by solving the initial value problem with an arbitrary member $\bar{v}(x, y, z, t)$ of the velocity ensemble, then taking an ensemble average of the result. In general, the procedure is to solve by iteration an integral equation equivalent to the initial value problem, and then average. (Received February 27, 1956.)

527. H. M. Gurk: *Finite solutions of simple games.*

An n-person *simple game* is defined (using the (0, 1) normalization) by a characteristic function v taking on only the values 0 and 1. The desirable coalitions of such a game are the *minimal winning coalitions*, i.e., the members of the collection $W^m = \{S/v(S)=1, \text{ but } v(T)=0 \text{ if } T\subset S,\ T\neq S\}$. von Neumann and Morgenstern have defined a natural, finite solution for such games, the *main solution* (*Theory of games and economic behavior*, Princeton, 1947, Chap. 10). Such solutions are based on the desire of players to form minimal winning coalitions and represent success for player i ($i=1, \cdots, n$) by $x_i \geqq 0$, failure by zero. The question has been asked whether these are the only finite solutions (i.e., only a finite number of imputations) for simple games. A *natural solution* of a simple game is defined as a finite solution V such that $\alpha \in V \Rightarrow$ there exists $T\in W^m$ such that $\mathfrak{J}_\alpha = \{i/\alpha_i > 0\} \subset T$. Natural solutions are exhibited for some simple games which have no main solutions, and some lemmas are given which are useful in finding such solutions. Other finite solutions are also exhibited. All known finite solutions are shown to belong to a class of *naturally-derived solutions*. The conjecture is offered that all finite solutions of simple games are naturally-derived. A simple game is given which has no such solution. (Received February 29, 1956.)

528. J. R. Isbell and F. J. Wagner (p): *Military evaluation and statistical decision.*

This is an essay toward a military decision theory analogous to thermodynamics rather than to statistical mechanics. Such a theory would presumably turn on state variables. This paper gives a clear definition of one state variable already shadowily recognized and achieves a shadowy recognition of another variable or complex of variables which are so new that they may be meaningless. The former variable is termed *confidence*, occupies the same place in the formal theory as a priori bounds on the strategies of nature, but has a clear operational definition. The latter variable has no name nor formal status but seems to be recognizable in military contexts as that dimension which stretches between pure statistical uncertainty and pure game-theoretic competition. These two ideas are employed in axiomatic and heuristic comparisons of several proposed decision rules. (Received March 12, 1956.)

529. C. C. Lin: *On uniformly valid asymptotic solutions of the equation of hydrodynamic stability and other differential equations involving a turning point.* Preliminary report.

The asymptotic solution of a class of fourth order differential equations containing a large parameter, including the equation of hydrodynamic stability for parallel flows, is treated in a manner different from those of Wasow (Ann. of Math. vol. 58 (1953) pp. 222–253) and Langer (private communication). The equation is first transformed into the standard form (2) $L(\psi)=\lambda^2 M(\psi)$ where $L(\psi)=\psi^{iv}+\lambda^2(z\psi''+\alpha\psi'+\beta\psi)$ (α, β being constants), and $M(\psi)=Z\{a(z)\psi'+b(z)\psi\}+\lambda^{-1}\{c_0(z,\lambda)\psi''+c_1(z,\lambda)\psi'+c_2(z,\lambda)\psi\}$. In $M(\psi)$, $c_i(z,\lambda)=\sum_{j=0}^{\infty}c_{ij}(z)\lambda^{-j}$ ($i=0, 1, 2$), and the functions $a(z)$, $b(z)$, and $c_{ij}(z)$ are regular in the neighborhood of $z=0$. The asymptotic solutions of (1) are then expressed in terms of the solutions of $L(u)=0$, which can be explicitly solved by the method of Laplace transformation. It is shown that if ψ is written in the form (2) $\psi=C_0u+C_1u'+C_2u''+C_3u'''$, then $C_i(z,\lambda)$ can be formally obtained in the form $C_i(z,\lambda)=\sum_{j=0}^{\infty}C_{ij}(z)\lambda^{-j}$, where $C_{ij}(z)$ are regular in the neighborhood of $z=0$. Proof of the existence of actual solutions of (1) in the form (2) are being carried out. (Received February 21, 1956.)

530*t*. Cathleen S. Morawetz: *On the nonexistence of continuous transonic flows past profiles.* II.

The following result is proved: There cannot exist two steady two-dimensional irrotational continuous transonic symmetric flows, with the same Mach number at infinity, past two symmetric convex profiles which differ from each other (by a finite amount) only on an arc cut out by two intersecting characteristics of one of the flows and containing the point of maximum velocity for that flow, if it is assumed that the flows are close to each other. The latter condition amounts to imposing a certain natural bound on the differences of the velocities and accelerations. For an earlier partial result see C. S. Morawetz, *On the non-existence of continuous transonic flows past profiles* I, Communications on Pure and Applied Mathematics vol. 9 (1956) no. 1. (Received April 4, 1956.)

531. G. Y. Rainich: *Conditional covariance and theories of light.*

In the *general* cases the electromagnetic field at a point is given by a six-vector and a particle by a four-vector. Here we consider *singular* cases when both invariants of the six-vector and the invariant of the four-vector are zero. A singular four-vector is shown to be a conditional covariant of a singular six-vector; in fact, in this case the square roots of the diagonal elements of a matrix derived from the six-vector are the components of the four-vector. The four-vector in turn determines the six-vector but only up to an arbitrary angle. When we consider the situation in a neighborhood of a point and take into account Maxwell's equations a *constant* field of four-vectors determines a *periodic* electromagnetic field. This seems to establish a mathematical connection between the corpuscular and the wave theory of light. (Received February 28, 1956.)

532. Domina E. Spencer: *The foundations of universal time.*

In accordance with Einstein's theory of relativity, it is generally believed today that the concept of simultaneity is untenable for moving observers. This paper reexamines the concept of simultaneity and the synchronization of clocks. The possibility of establishing universal time is approached from a postulational point of view. Two types of synchronization are defined: quasi-synchronization and complete synchronization. The conditions under which each type of synchronization is possible are derived. It is shown that quasi-synchronization is always possible but that complete synchronization is permissible only if certain limitations are imposed on the law of transformation of the velocity of light and on the relative motion of the observers. The results are compared with the work of Einstein, Milne, and Grünbaum. Practical discrepancies betwen quasi-synchronization and complete synchronization are also evaluated. (Received February 28, 1956.)

GEOMETRY

533. Theodore Frankel: *The homology class of an isometric flow.*

Let v be a vector field on a compact, orientable Riemannian manifold M. If v generates isometries of the Riemannian metric (i.e. if v is a Killing vector) and if there are any closed integral curves of v then we have the following. Theorem: (i) all closed integral curves lie in dependent homology classes, (ii) the homology class of a closed integral curve is determined (up to a nonzero multiple) by de Rham's theorem, (iii) if v vanishes at some point of M then all closed integral curves bound. Corollary: if $M=G/g$, G a compact Lie group, g a closed subgroup, then all closed orbits of a one parameter subgroup of G are in dependent homology classes. (Received March 26, 1956.)

534*t*. Shoshichi Kobayashi: *Nondegenerate pseudo-Kaehlerian spaces.*

A pseudo-Kaehlerian space M is said to be nondegenerate if the restricted homogeneous holonomy group contains the operator J of the almost complex structure. The following results are obtained. (1) An irreducible pseudo-Kaehlerian space with nonvanishing Ricci curvature is nondegenerate. (2) A simply connected and complete nondegenerate pseudo-Kaehlerian space is the direct product of irreducible nondegenerate pseudo-Kaehlerian spaces. (3) Every affine transformation of an irreducible nondegenerate pseudo-Kaehlerian space preserves the almost complex structure or gives the conjugate structure. (4) Every element of the connected component of the unit of the affine transformation group of a complete nondegenerate pseudo-Kaehlerian space is an isometry and preserves the almost complex structure. (Received January 20, 1956.)

LOGIC AND FOUNDATIONS

535*t*. S. B. Akers, Jr.: *A full disjunctive normal form for the monadic functional calculus of first order.*

In using machine techniques for the processing and analysis of logical expressions, it is often desirable to be able to assign binary numbers to the logical expressions involved so that a complete isomorphism exists between these binary designation numbers and the classes of logically equivalent expressions. Such an assignment is immediately possible with expressions in the propositional calculus because of the existence of a full disjunctive normal form for such expressions. For the monadic functional calculus of first order, a corresponding full disjunctive normal form is obtained by adapting the normal form of Herbrand (*Recherches sur la théorie de la démonstration*, published in Warsaw in 1930 in the Series, Travaux de la Société des Sciences et des Lettres de Varsovie, Class III, No. 33) and of Quine (*O Sentido da Nova Lógica*, São Paulo, Brazil, 1944) to the case that propositional variables and free individual variables are present; and it is shown that this normal form leads to an assignment of binary designation numbers in a way that is useful for machine techniques. The rules for expanding expressions into this normal form are given, and the satisfiability of the resulting disjuncts is shown. This work was sponsored in part by the USAF, Contract AF19(604)-1582. (Received February 6, 1956.)

536*t*. M. O. Rabin: *Construction of test groups.*

When studying recursive solvability questions for groups one needs the following algebraic construction: Given a group G by a presentation $\Pi=(x_i, \cdots, x_n: r(x))$ and a word $w(x)\in G$, to construct a presentation Π_w defining a *test group* G_w such that (1) if $w(x)=1$ in G then G_w reduces to the trivial group, (2) if $w(x)\neq 1$ then G is imbedded in G_w. G_w may thus serve to test whether $w(x)=1$. Let Π_0 be $(x_1, \cdots, x_n+_1: r(x))$ (i.e. add one free generator to π), then the following statements hold. Lemma 1: Let H_0 be presented by Π_0 and let $u(x)$ have infinite order in H_0; then H_0 is isomorphically imbedded in the group H_1 defined by $\Pi_1=(x_1, \cdots, x_{n+1}, t, a, s, b: r(x), u(x)t=t^2u(x), ta=a^2t, u(x)s=s^2u(x), sb=b^2s)$, moreover the subgroup of H_1 generated by $a, b, x_1, \cdots, x_{n+1}$ is isomorphic to $(a, b) * \mathrm{H}_0$ ($*$ denotes free product). Lemma 2: In (a, b) the elements b^iab^{-i}, $i=0, \cdots, n+1$, $b^{n+2}aba^{-1}b^{-n-2}$ satisfy no nontrivial relation. Lemma 3: H_0 is imbedded in H_2 obtained from $(c, d) * H_1$ by adding the relations: $a=c$, $x_ib^iab^{-i}=d^icd^{-i}$, $i=1, \cdots, n+1$, $b^{n+2}aba^{-1}b^{-n-2}=d^{n+2}cdc^{-1}d^{-n-2}$. Note that the presentation Π_2 of H_2 can be obtained explicitly from Π_0 and $u(x)$. *Theorem: The presentation Π_w of the test group G_w is obtained from Π_2 by specializing $u(x)$ to be $x_{n+1}w(x)x_{n+1}^{-1}w(x)$. I.e. the group G_w thereby defined has the properties* (1) *and* (2) *required of a test group.* (Received April 12, 1956.)

537*t*. M. O. Rabin: *Recursive unsolvability of group theoretic problems.*

Combining Novikov's theorem on the recursive unsolvability of the word problem for groups (*Doklady Akad. Nauk SSSR. vol.* 85 (1952) *pp.* 709–712) with the construction of test groups given in the previous abstract, one can prove the following: *Theorem: Let P be a group theoretic property such that there exists at least one finitely presented group which has P and at least one finitely presented group which does not possess P and which cannot be imbedded in a group having P, then there does not exist any generally effective procedure for deciding for every presentation Π whether the group G_Π defined by it has the property P.* As corollaries one obtains the following results: *There does not exist a generally effective procedure for recognizing from a presentation Π whether the group G_Π defined by it is a* (this statement should be prefixed to each of the following) (1) *trivial group,* (2) *cyclic group,* (3) *finite group,* (4) *locally infinite group,* (5) *free group,* (6) *commutative group,* (7) *solvable group,* (8) *group expressible as nontrivial free product,* (9) *group expressible as nontrivial direct product,* (10) *group for which the word problem is solvable,* (11) *group which can be presented using only one relation.* (Received April 12, 1956.)

Statistics and Probability

538t. Charles Fox: *Some applications of Mellin transforms to the theory of bivariate statistical distributions.*

If $F(r, s) = \int_0^\infty \int_0^\infty f(x, y) x^{r-1} y^{s-1} dx dy$ then $F(r, s)$ is the double Mellin transform of $f(x, y)$. This can be inverted to (1) $f(x, y) = (1/(2\pi i)^2) \int_{h-i\infty}^{h+i\infty} \int_{k-i\infty}^{k+i\infty} F(r, s) x^{-r} y^{-s} dr ds$. The following results are proved: let (ξ, η) be a pair of random bivariate variables with frequency function $f(x, y)$, assumed symmetrical about the x and y axes. Then $E(|\xi|^{r-1} |\eta|^{s-1}) = 4 F(r, s)$, where E is the expectation function. Hence, if the expectation function is known one can, by (1), compute the frequency function. If (ξ_1, η_1) and (ξ_2, η_2) are two pairs of bivariate random variables (with symmetrical frequency functions) then the frequency function of the pair $(\xi_1\xi_2, \eta_1\eta_2)$ has Mellin transform $4F_1(r, s)F_2(r, s)$ where $F_1(r, s)$ is the Mellin transform of the frequency function of (ξ_1, η_1) and $F_2(r, s)$ is that of (ξ_2, η_2). Thus, on inverting $4F_1(r, s)F_2(r, s)$ by (1) we can compute the frequency function of the distribution of $(\xi_1\xi_2, \eta_1\eta_2)$. Similarly the frequency function of $(1/\xi_1, 1/\eta_1)$ has Mellin transform $4F_1(2-r, 2-s)$ and of $(\xi_1/\xi_2, \eta_1/\eta_2)$ has Mellin transform $4F_1(r, s)F_2(2-r, 2-s)$ and so the frequency functions of these distributions can be computed by (1). Results of this nature can be obtained for various products and ratios of the random variables (ξ_1, η_1) and (ξ_2, η_2). The case of unsymmetrical frequency functions is also considered. (Received February 17, 1956.)

539. Eugene Lukacs: *A question raised by D. Dugué.*

The rectangular distribution has the characteristic function $(\sin t)/t$; it is not infinitely divisible. For every positive integer n it has the factor $[\sin (t/n)]/(t/n)$. Therefore it provides an example for a distribution which is not infinitely divisible but has an enumerable infinity of different factors which depend on n. D. Dugué [Annales de l'Institut Henri Poincaré vol. 12 (1951) pp. 159–169] raised the question whether there exists a distribution which is not infinitely divisible but a non-enumerable set of factors. A characteristic function is constructed which has this property so that Dugué's question can be answered in the affirmative. (Received February 27, 1956.)

Topology

540. R. D. Anderson: *A characterization of the universal curve.*

The universal curve is a one-dimensional continuous curve which is an analog of the Cantor middle third set and is obtainable by punching rectangular "holes" out of a cube in a particular regular fashion. The author gives a characterization of the universal curve in terms of a sequence of finite coverings of a one-dimensional continuum M by continua. The essential conditions may be considered to be strong forms of the statements that M have no local cut points and that M be locally nonplanar, in fact, in a sense that M have a local anti-Jordan curve property. The characterization is used in another paper (abstract for February 1956 meetings) to establish continuous collections of universal curves. The technique of the argument is also used to show the n-point homogeneity of the universal curve as previously announced by the author (Bull. Amer. Math. Soc. Abstract 59-2-249). (Received February 28, 1956.)

541t. J. D. Baum: *P-recurrence in topological dynamics.*

The notion of recurrence [cf. W. H. Gottschalk and G. A. Hedlund, *Topological dynamics*, Providence, 1955, Chap. 7] in a "direction," the analog of positive or negative recurrence is generalized to transformation groups whose phase groups have a structure more general than the reals or integers. The notion of regional recurrence is similarly treated. Inheritance theorems are proved; and the relation between recurrence and asymptoticity and incompressibility is explored. Applications of these notions to other topics in topological dynamics are given. (Received February 27, 1956.)

542. E. E. Floyd: *Closed coverings in Čech homology theory.*

If α is a finite closed covering of a compact space X, there is a projection homomorphism π_α of the Čech group $H_n(X)$ into the group $H_n(\alpha)$ of the nerve. We say that α, β determine $H_n(X)$, α and β finite closed coverings with $\alpha<\beta$, if and only if π_α maps $H_n(X)$ isomorphically onto the image of the projection $\pi_{\beta\alpha}:H_n(\beta)\rightarrow H_n(\alpha)$. Sufficient conditions are given that α, β determine $H_n(X)$. These conditions are in terms of the groups of the intersections of elements of finite closed coverings. Applications are made to homology local connectedness, regular convergence, and the Vietoris mapping theorems. In the proof of the fundamental theorem, the Kelley-Pitcher theory of coverings of complexes [Ann. of Math. vol. 48 (1947) pp. 682–709] is generalized to pairs (X, α) where X is a compact and α is a finite closed covering of X. (Received December 12, 1956.)

543. O. H. Hamilton: *Fixed points for certain noncontinuous transformations on N-cells, a counterexample.*

An example is given of a noncontinuous transformation T of a closed N-cell I into itself such that the graph mapping, $g(x)=(x, T(x))$ transforms locally connected subsets of I onto connected subsets of $I\times I$, but such that T leaves no point of I fixed. There is given also a necessary and sufficient condition on the transformation T in order that g shall transform connected subsets of I onto connected subsets of $I\times I$. (Received February 27, 1956.)

544. Mary-Elizabeth Hamstrom and Eldon Dyer (p): *Certain completely regular mappings.*

An open mapping f of a compact metric space K onto a space M is *regular* iff for each positive number ϵ, there is a positive number δ such that if p and q are points in the inverse under f of a point of M at a distance apart of less than δ, there is an arc in that inverse containing p and q and having diameter less than ϵ. The mapping f is *completely regular* iff for each positive number ϵ, there is a positive number δ such that if p and q are points of M at a distance apart of less than δ, there is a homeomorphism of $f^{-1}(p)$ onto $f^{-1}(q)$ moving no point as much as ϵ. In this paper it is shown that if f is a completely regular mapping of a compact metric space K onto a closed interval I such that (1) for each point x of I, $f^{-1}(x)$ is an n-cell, and (2) $\bigcup_{x\in I} Bdf^{-1}(x)$ is the product of an $(n-1)$-sphere with I such that f is the projection map of this product onto I, then K is an $(n+1)$-cell. If $n\leqq 4$, the conclusion holds even when condition (2) is omitted. And if f is a regular mapping of K onto an i-cell, $i=1$ or 2, such that each $f^{-1}(x)$ is a 2-cell, then K is an $(i+2)$-cell. An application of the preceding theorem is that if f is a mapping of Euclidean 3-space, E^3, onto a metric space

K such that for each point x of K, $f^{-1}(x)$ is a compact continuum lying in a horizontal plane and not separating that plane, then K is E^3. (Received February 28, 1956.)

545. L. J. Heider: *Generalized G_δ spaces.*

Let X denote a completely regular space, while νX denotes the Hewitt Q-space extension of X, and $C^*(X)$ denotes the class of all real-valued, bounded functions, defined and continuous on X. The space X is called a generalized G_δ space if, for each point p in X which is not a G_δ point, some element of $C^*(X-p)$ fails to be a restriction to $X-p$ of any element of $C^*(X)$. It may be shown that if X and Y are generalized G_δ spaces such that νX and νY are homeomorphic, then X and Y are homeomorphic. Moreover, if X is a generalized G_δ space and Y is a completely regular space with νY homeomorphic to νX, then $X \subset Y \subset \nu X$. (Received February 27, 1956.)

546. J. F. Nash: *Point set topology based on connectivity.*

Instead of basing point set topology on open sets one can use connected sets. Open sets can be defined from connected sets in a canonical manner, but this canonical open set topology derivable from a connectivity topology is usually not the only one which yields the same set of connected sets. The approach via connectivity might give a more natural insight into some questions. (Received February 23, 1956.)

547*t*. C. D. Papakyriakopoulos: *On loops on the boundary of 3-manifolds.*

The following theorem is proved: *Let M be a 3-manifold, finite or not, with boundary N, not necessarily connected or finite. Let N' be a closed orientable component of N, and let q be a point of N' such that there exists a loop on N' based at q which is $\simeq 0$ in M, and not homotopic to* 0 *on N' (the symbol $\simeq$ means homotopic to). Then there exists a system G_i, $i=1, \cdots, n(<\infty)$ of simple (=without multiple points) loops on N' based at q, $\simeq 0$ in M, not homotopic* to 0 *on N' and such that: Any loop on N' based at q, and $\simeq 0$ in M, is homotopic on N' to a product of transforms of the G_i^ϵ ($\epsilon = \pm 1$) by loops on N' based at q.* The proof of this theorem is based on the theorem contained in the author's abstract *On a lemma of Kneser* [Bull. Amer. Math. Soc. Abstract 62-2-279]. Applications of this theorem will be given in a subsequent abstract. (Received February 10, 1956.)

548. C. D. Papakyriakopoulos: *On solid toruses of higher genus.* Preliminary report.

The following theorem is proved: *Let M be a finite* 3*-manifold with nonvacuous orientable boundary N such that $\pi_1(M, N)=1$. Then M is aspherical, acyclic in dimension >1, and $\pi_1(M)$ is a free group on g generators, where g (≥ 0) is the genus of N.* The proof of this theorem is based on the theorem contained in the author's abstract *On loops on the boundary of* 3*-manifolds* [Bull. Amer. Math. Soc. Abstract 62-4-548], and on Hopf's paper *Fundamentalgruppe und zweite Bettische Gruppe* [Comment. Math. Helv. vol. 14 (1941–1942)]. (Received February 14, 1956.)

549*t*. R. W. Rector: *Fundamental linear relations for the seven ring.*

Birkhoff and Lewis in their comprehensive paper, *Chromatic polynomials*, (Trans. Amer. Math. Soc. vol. 60 (1946) pp. 355–451) conclude their attack on the Four Color Problem with a partial analysis of the n-ring with special attention to the

6-ring and 7-ring. The present paper extends the list of linear relations for the 7-ring to 1505 equations and further demonstrates the presence of the predicted 126 linearly independent relations for the 7-ring. (Received March 12, 1956.)

550. Mary E. Rudin: *A property of the countable ordinals.*

Let W be the set of all countable ordinals with the usual topology. It is proved that there is a set E in W such that both E and $W-E$ intersect every uncountable closed subset of W. More precisely, for each limit ordinal w, let $f_n(w)$, for each positive integer n, be a countable ordinal less than w such that $\lim_{n\to\infty} f_n(w)=w$. Let $S(n, z)$ be the set of all w such that $f_n(w)=z$ (n is a positive integer and z is a term of W). Then at least one of the sets $S(n, z)$ has the desired property. (Received February 27, 1956.)

551. Walter Rudin: *Homogeneity problems in the theory of Čech compactifications.*

A topological space X is homogeneous if to every pair of points p, q in X there is a homeomorphism of X which carries p to q. If X is completely regular, the Čech compactification βX of X is a compact space in which X is dense, such that every continuous bounded real function on X can be extended to a continuous function on βX. The following question arises: Does the homogeneity of X imply the homogeneity of $\beta X-X$? Under the assumption of the continuum hypothesis, it is shown in this paper that the answer is negative even if X is a countable discrete space. This in turn leads to the result that $\beta X-X$ fails to be homogeneous whenever X is a locally compact Hausdorff space which is not sequentially compact. (Received February 23, 1956.)

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