817t. G. T. Whyburn: *Dimension and nondensity preservation of mappings.*

The property of a mapping $f(X)=Y$ to preserve nondensity for compact subsets of X is characterized in terms of the lightness kernel L_f of f consisting of all $x \in X$ such that $f^{-1}f(x)$ is totally disconnected. These results are applied to show that if $w=f(z)$ is continuous in a region X of the z-plane and differentiable at all points of a dense set $f^{-1}(Y_0)$ in X which is the inverse of an open subset Y_0 of $Y=f(X)$, then $\dim f(K) \leqq 1$ for each compact set K in X of dimension $\leqq 1$. Under the same conditions it is shown that f is strongly quasi-open. (Received July 5, 1956.)

818. R. E. Zink: *A note concerning regular measures.*

Let X be a topological space and let (X, S, μ) be an associated measure space. Let $f(x)$ be non-negative and measurable (S). Define the measure ν on S by means of the formula: $\nu(E)=\int_E f(x)d\mu(x)$, for all E belonging to S. Using the ancient techniques of measure theory, one can easily establish the following remarks: (1) If μ is inner regular, so also is ν. (2) If μ is outer regular, then ν is outer regular if and only if for every set E on which f is integrable: (i) There exists a measurable open set containing E on which f is also integrable. (ii) There exists, for every pair of positive numbers ϵ, δ, a measurable open set U containing $E-N(f)$ such that $\mu(\{x: f(x)>\delta\} \cap U)<\epsilon$. (Received July 5, 1956.)

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RESEARCH PROBLEMS

15. Richard Bellman: *Approximation theory.*

Consider the differential equation (1) $dx/dt=\Phi(x)$, $x(0)=c$ where $\Phi(x)$ is a continuous function of x satisfying additional conditions ensuring the existence of a unique solution over the interval $0 \leqq t \leqq T$ for $a \leqq c \leqq b$.

Let (2) $dy/dt=\sum_{k=0}^{K} a_k(c, t)y^k$, $y(0)=c$, represent an approximation to (1) where the coefficients $a_k(c, t)$ are functions to be determined so as to minimize one of the following functionals: (3) (a) $J_1=\mathrm{Max}_{0\leqq t\leqq T}\ |x-y|$, (b) $J_2=\int_0^T(x-y)^2dt$, (c) $J_3=\mathrm{Max}_{a\leqq c\leqq b}\ \mathrm{Max}_{0\leqq t\leqq T}\ |x-y|$ (d) $J_4=\mathrm{Max}_{a\leqq c\leqq b}\ \int_0^T(x-y)^2dt$, (e) $J_5=\int_a^b[\int_0^T(x-y)^2dt]dc$ under the following alternatives: (4) (a) $a_k(c, t)$ depends only upon c, for $k=0, 1, \cdots, K$. (b) $a_k(c, t)$ are polynomials of degree M in t with coefficients dependent upon c. (The case $K=1$ is the most interesting.)

Consider the analogous problem for systems of the form (5) $dx_i/dt=\Phi_i(x_1, x_2, \cdots, x_n)$, $x_i(0)=c_i$, $i=1, 2, \cdots, n$, in the particular case where the approximating equation is (6) $dy_i/dt=a_i(c)+\sum_{j=1}^{n} b_{ij}(c)y_j(t)$, $y_i(0)=c_i$, and we wish to minimize the functional (7) $\int_{a_1}^{b_1}\cdots\int_{a_n}^{b_n}[\int_0^T\sum_{i=0}^{n}(x_i-y_i)^2dt]\Pi_i dc_i$. (Received August 17, 1956.)