caré before either Birkhoff or Witt was born; they proceed to call it the Poincaré-Witt theorem. The facts of the matter are these. The theorem asserts that if the Lie algebra L over K is free as a K-module, then the natural map of L into its enveloping associative algebra L^e has kernel zero. Birkhoff and Witt both found proofs of this theorem in 1936; Birkhoff's was received by the editors 29 days earlier than Witt's, but there is every reason to suppose that the two were independent and that the theorem was then "in the air." Birkhoff makes a (partial) reference to Poincaré; Witt does not. Birkhoff defines the enveloping algebra, not as a quotient of the tensor algebra, but by "straightening" elements of that tensor algebra. His proof is carelessly done, but with some little trouble (which the reviewer has taken) his proof can be made complete and correct. The world would be happy to honor Poincaré, who was well ahead of his time on this, but not at the expense of a manifest injustice to Birkhoff. Since a three-handled theorem is clumsy, it will doubtless remain Birkhoff-Witt.

This book is very carefully prepared and well proof-read; the reviewer noted only one troublesome misprint: on page 185 the η in the top row of the square diagram and in the next line of the text should be ρ (notation from p. 168). The letter Λ is overworked; it appears variously as a ring, as an augmented ring, or as an algebra. More application of the usual (unexpressed) conventions about different letters for different notions would have helped the reader. The index might have the following additions: complete resolution 240, derivation 168, direct family of homomorphisms 4, direct product 4, direct sum 4, exterior ring 146, free ring 146, homology of differential module 54, image 3, kernel 3, Lie algebra 266, negative graded module 58, normal map 349, normalized standard complex 176, polynomial ring 146, positive graded module 58, 60, $\varnothing$-projective module 30, standard complex 175, syzygies 157.

SAUNDERS MACLANE

Methods in numerical analysis. By K. L. Nielsen. Macmillan, New York, 1956. 13+382 pp. $6.90.

Introduction to numerical analysis. By F. B. Hildebrand. New York, McGraw-Hill, 1956.+10 511 pp. $8.50.

Numerical analysis, with emphasis on the application of numerical techniques to problems of infinitesimal calculus in single variables. By Z. Kopal. New York, Wiley, 1955. 14+556 pp. $12.00.

The demand for computer personnel has been made abundantly clear—in the advertisement pages of our newspapers, and at formal conferences.[1] Universities have a hard time finding enough capable teachers for courses planned to produce mathematicians qualified in this area; there has also been some difficulty about suitable text material. The second difficulty may be the easier to overcome, for the present three volumes are undoubtedly the forerunners of more.

None of these show much trace of the influence of automatic computers; this, however, is no defect for we have found that the discipline of a spell with desk machines is an excellent preparation for all phases of work with automatic computers, and courses on numerical analysis without access to computers are for the expert not for the novice. Moreover, it was apparent at the conference and has become clearer since, that the need for computer personnel with a broad training in mathematics is more urgent than for those with less mathematics and a narrow training in numerical analysis. This is because there has been an admirable tendency to use the computer to help itself in certain more routine phases of problems, and because more powerful computers bring deeper problems into range.

The books are listed above in advancing order. Nielsen's is admittedly an elementary textbook for the practical man. Hildebrand's is for an introductory course in the subject and Kopal's aims to provide an advanced undergraduate textbook as well as a research handbook on certain topics. The last two are based on courses given at M.I.T.

All three books cover the basic topics: Difference Operators, Interpolation, Differentiation and Integration, Differential Equations, Trigonometrical Approximation. In the first two there is some account of least squares and the solution of systems of linear equations, and of nonlinear equations. There is a chapter on integral and integro-differential equations in the third. All have collections of examples; that of Hildebrand is most impressive. All could well have been supplemented by a detailed discussion of a rigorous "digital" calculation, like the evaluation of a square root or a polynomial, such as have been given by Householder, as an introduction to an essential part of modern numerical analysis.

In Nielsen's book, worked examples, naturally, play a large part and there is a detailed account of schemes for the solution of least-square problems, due to the author and L. Goldstein. This book includes a collection of tables. There are some theoretical uncertainties, e.g. the usual one about the θ in an error term of the form $f^n(a+\theta h)$, not depending on h, and the determination of the error term in the Lagrangian interpolation is at least notationally obscure.

[1] E.g., A. W. Jacobson, editor, *Proceedings of the First Conference on training personnel for the computing machine field*, Detroit, Wayne University Press, 1955.

In both Hildebrand and Kopal there seems to be an overemphasis on approximate quadratures. Kopal has interesting historical notes, a detailed account of the choice of interval in numerical differentiation and some discussion of boundary value problems (in the one-dimensional case) which was not readily accessible in English. The mathematician reading Kopal will be disturbed from time to time by such phrases as "In general, we may expect . . . ," which he can, often, rightly question.

Certainly the second two books will be of value to the teacher and research worker, but they seem to be far too extensive for the ordinary student, whom they may discourage. There is no doubt that there are many principles in numerical analysis which can be incorporated at various places in regular courses and teachers can find suitable material in all three volumes. For instance, an efficient method for solving linear equations, with checking devices, can be discussed at the beginning of algebra courses. The concept of differences and Lagrangian ideas can be introduced when polynomials are being studied. Then, when calculus is being started, the ideas of numerical differentiation and integration can be added. When differential operators are being discussed in connection with differential equations with constant coefficients, an account of difference operations can be added. There are many opportunities in courses on matrix theory, to introduce ideas useful in numerical analysis. In analytical geometry, too, the extremal properties of the principal axes of conics leads to the ideas of Rayleigh, Ritz, and others. All this can be covered with the use of a desk calculator or two, and a few books of tables as a source for examples. These principles could be later consolidated in a short course, and there would seem to be a need for an appropriate text, of a fraction of the size of any of the three. A shortened version of Hildebrand could be very useful. This would be supplemented in due course by a similar account of the principles of programming—as soon as these are established—and of what we can call "modern" numerical analysis, the type of numerical analysis relevant in the use of the high speed automatic computers.

JOHN TODD

Algèbre. By R. Ballieu and F. Simonart. Louvain, Librarie Universitaire and Paris, Gauthier-Villars, 1955. xvi+355+1 pages.

In this book, classical algebra is treated, to quote the senior author, Simonart, "en langage moderne." The central themes are those aspects of linear algebra which pertain most directly to the solution of simultaneous linear equations over a field, and the study of poly-