Whether one agrees that the axiomatic approach is a good one for beginning students or not, there is much to recommend the book for use as a text (most suitable, perhaps, for a second year graduate course). The treatments of the singular and Čech theories are modern, complete, and quite readable by themselves. Diagrams of homomorphisms, which are used so frequently today, were first systematically used in this book, both to motivate proofs and to assist the reader in following arguments. Each chapter of the book begins with an introduction stating what the chapter covers and how the material fits into the general scheme of the book. Notes are at the end of the chapter. These discuss the historical development of the subject and its relations to other topics. References to the literature are also found in these notes. Each chapter is followed by a set of exercises. Some of these are easy and some more difficult but most of them are interesting, and the student who works his way through them will learn a great deal.

Since its publication the terminology and notation of the book has been almost universally adopted by topologists. The axioms have led to cleaner proofs of many theorems and increased their generality at the same time. In addition, the axioms have been applied to prove new results. One of the most recent of these applications is the theorem proved by Dold and Thom (C. R. Acad. Sci. Paris vol. 242 (1956) pp. 1680–1682) to the effect that the qth homotopy group of the infinite symmetric product of a polyhedron X is isomorphic to the qth homology group of X. They prove this by showing that the homotopy groups of the infinite symmetric product of X, regarded as functions of X, satisfy the axioms, whence the result follows from the uniqueness theorem for polyhedra.

The book contains no discussion of cup products or cross products. This was to be included in a projected second volume, which was to contain also a treatment of cell complexes and the practical calculation of the homology groups of such spaces. It is to be hoped that the authors have not abandoned their plan to write this second volume. Such a continuation of the present useful book would be a welcome and worthwhile contribution to the mathematical literature.

E. H. SPANIER

Vorlesungen über Himmelsmechanik. By Carl Ludwig Siegel. Springer, Gottingen, 1956. 9+212 pp. DM 29.80. Bound DM 33.

The appearance of this remarkable book is certainly one of the great mathematical events of the century. Written on the subject matter which is the mother field of modern mathematics and sparkling with new theorems, new proofs and the polished finesse for which its author is famous, it is a fitting monument to his genius.

This book concerns itself primarily with two central problems of celestial mechanics: Newton's three body problem (T.B.P.) and the related simpler problem of Jacobi, the restricted three body problem (R.T.B.P.). The problem in each case is to establish significant mathematical information about a set of solutions. An example of a question whose answer would be mathematically significant for the T.B.P. of sun, earth, and moon is this: Is the orbit of the moon around the earth stable?[1]

This question is not completely posed until one makes precise the definition of stability. Many different types of stability have been introduced in studies of the T.B.P., but none has had more than a very limited success. The studies centering around the stability for T.B.P. may be classified into two categories according to the methods employed: in the one the methods are purely geometric and in the other they are analytic and employ series expansions. The analytical techniques are definitely the more successful. The failure of the geometric techniques would appear to stem from their inability to exclude the topological complexities which may occur in global situations. The relative success of the analytical techniques appears to be due to the fact that they have been applied to problems which are topologically simple in that they refer to a neighborhood of a singular point or periodic solution and within that neighborhood they allow only such decompositions (by trajectories) of the phase space as can be achieved by some type of linear characterization. Although the number of topological possibilities thus allowed is quite small the number of different series expansion techniques introduced to study them is very large.

The successes of the analytical studies have always been limited. In every case where an attempt has been made to obtain conclusive results about stability by analytic means the convergence of the series employed could not be proved. It is characteristic that one could not determine whether this failure was due to a limitation of the technique employed or whether the geometric implications (which would follow from convergence) were false. Siegel's papers on celestial mechanics, prior to the publication of this book, are an attack on the T.B.P. which re-examine (at considerable depth) a large number of analytical devices introduced earlier—especially ideas of Poincaré

[1] The field is dominated by pessimists whose aim is to show that the orbit of the moon is unstable. The pessimists have a substantial lead both in the number and quality of their results, but the final outcome is hardly in sight.

and Sundman. Siegel's more recent results relate to the determination of the limitation of some of the techniques introduced by earlier workers on the T.B.P. This present book broadens studies already initiated by Siegel and others; it introduces new methods; and it sheds considerable light on G. D. Birkhoff's studies of area preserving surface transformations.

Two new results in Siegel's book deserve special mention: Siegel's Center Theorem and the Birkhoff-Lewis-Siegel Fixed Point Theorem.

Siegel's Center Theorem. Given an analytic manifold M of dimension two, a point $p \in M$, and a system of analytic ordinary differential equations defined in a neighborhood of p we shall say p is a *center* if p is an isolated singular point and in a neighborhood of p every solution, except p, is periodic. Furthermore, for an analytic arc through p with parameter σ the period $\tau = \tau(\sigma)$ of the periodic solutions depends analytically on σ. A system of analytic ordinary differential equations is said to have a *two dimensional center* at a singular point, q, if there exists a regularly imbedded two dimensional manifold through q tangent to the vector field such that the induced differential equation on the manifold has q as a center. *Theorem. Let $x=0$ be a singular point of $dx/dt = Ax + f(x)$ an analytic Hamiltonian system (A constant, $f = O(\|x\|^2)$) with n degrees of freedom. Let A have a purely complex root $\lambda_1 = i\omega$. If the $2n$ roots of A $\pm\lambda_1, \pm\lambda_2, \cdots, \pm\lambda_n$ are distinct and λ_R/λ_1 is not an integer for $R = 2, 3, \cdots, n$ then $x=0$ is a two dimensional center.* The attempts to characterize solutions near a singular point by the characteristic roots of the linear part of the equation extend back over a hundred years. The literature on this problem is vast and it includes the names of Briot and Bouquet, Poincaré, Picard, Liapunov, Dulac and Perron. Before 1951 the hypotheses of all the results included: The smallest convex polygon in the complex plane containing the characteristic roots should not contain the origin. In 1952 Siegel showed the origin could lie inside this polygon provided the c.g. of integer valued masses distributed at the characteristic roots did not get too close to the origin *Uber die Normalform . . .* , Nachr. Akad. Wiss. Gottingen. Math.-Phys. Kl. IIa (1952) pp. 21–30. In particular the characteristic roots had to be linearly independent over the rationals. This new result is really quite surprising for it allows the c.g. of an integer valued mass distribution to be right on the origin.

The Birkhoff-Lewis-Siegel fixed point Theorem. Let T be an analytic area preserving transformation of the plane into itself for which the origin is fixed. One may determine from T a certain infinite set of real numbers $\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_R, \cdots$. Suppose $\Upsilon_1 = \Upsilon_2 = \cdots = \Upsilon_{l-1} = 0$ and $\Upsilon_l \neq 0$. Let λ and λ^{-1} be the characteristic roots of the linear part of T (expanded near $x=0$). *Theorem. If* $|\lambda| = 1$, *and* $\lambda^R \neq 1$ *for* $R = 1, 2, \cdots, 2l+2$ *then for any neighborhood* G *of the origin there is an integer* $n_0(G)$ *such that for any* $n > n_0(G)$ *there is a* $z \in G$ $(z \neq 0)$ *such that* $T^m z \in G$ *for all* m *and* $T^n z = z$.

If $\Upsilon_i = 0$ for all i the convergence problems that arise have not been settled, if they were the transformation would be analytically equivalent to a rotation, and in that event there would be a periodic point $(\neq 0)$ if and only if $\lambda^R = 1$ for some R.

The history of this beautiful theorem is strange: Siegel refers to Birkhoff's Pontifical Memoir *Nouvelle recherches sur les systems dynamiques* Memoria Pont. Acad. Sci. Novi. Lyncaei S. 3. vol. 1 (1935) pp. 85–216 (pages 132–146 are the pertinent ones). The proof there is far from complete. Moser has pointed out to us a more detailed version of this proof (a version which we believe to be also incomplete), occurs earlier in the paper of Birkhoff and Lewis *On the periodic motions near a given motion of a dynamical system.* Ann. Mat. Pura Appl. vol. 12, ser. 4, 1933 pp. 117–133. The main object of the proof of this theorem is to establish certain sharp approximations. Since Siegel has done this and Birkhoff and Lewis have not we feel the theorem should be called the Birkhoff-Lewis-Siegel fixed point theorem. That M. Morse in his mathematical biography of G. D. Birkhoff (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 357–391) does not mention this theorem as one of Birkhoff's great results is not surprising to us for although Birkhoff wrote many papers on area preserving surface transformations in dynamics the style in which they were written was such as to discourage readers. In addition, the lack of a convergence proof raised the question of whether or not these studies had any real significance. Siegel deserves a great deal of credit for his development of Birkhoff's pioneering efforts.

The material in Siegel's book is divided into three parts. I. The T.B.P., II. Periodic Solutions, and III. The Stability Problem (singular points).

The first sections of *Part* I have a short but sound introduction to Lagrangian derivatives, canonical transformations, and Hamilton-Jacobi theory, all treated in a style to which they are not accustomed. Cauchy's existence theorem is given an attractive new proof. The remaining sections are devoted to a proof of Sundman's theorem concerning the existence of a global uniformizing variable for solutions of the T.B.P. (p. 61). The discussion has been simplified by