The difficulties above can be easily overcome by an alert lecturer, especially with the aid of all the references given to van der Waerden's big, young cousin, Bourbaki.

DANIEL ZELINSKY

Real analysis. By E. J. McShane and T. A. Botts. Princeton, Van Nostrand, 1959. 9+272 pp. $6.60.

As stated in the Preface, "The aim of this book is to present, in a form accessible to the mature senior or beginning graduate student, some widely useful parts of real function theory, of general topology, and of functional analysis." If a mature student, in this context, is understood to be one who has already enjoyed and profited from a substantial introduction to real variables—preferably including some topology and Lebesgue theory—the authors have achieved their objective well. Although material of considerable generality is handled in a style that is frequently quite compact, the proofs and discussion are sufficiently clear and carefully presented to enable the interested reader to follow the argument and to complete any gaps that have been left for him to fill. In the compass of 250 pages the authors lead their audience through the impressive totality of material outlined below.

The book contains eight chapters—numbered 0 through VII—and three appendices. Chapter 0—Preliminaries—sets the stage, with a brief and informal presentation of some of the notation and languages of sets, functions, integers, and the principle of inductive proof. Chapter I—Real Numbers—characterizes the real number system as a complete ordered field (completeness by means of suprema), and introduces partially ordered sets and the maximality principle (a more extended discussion of which is given in Appendix II). Chapter II—Convergence—develops a highly comprehensive limit theory based entirely on the concept of a "direction," that is, a nonempty family of nonempty sets any two of which contain a third, inspired by Moore-Smith generalized convergence. Topological spaces are studied, with uniqueness of limits established for Hausdorff spaces. Compact sets receive special attention. Order-convergence for lattice-valued functions is defined in terms of upper and lower limits (limits superior and inferior). The real number system R and the extended real number system R^* lead to the product spaces R^n and $(R^*)^n$. The Cauchy criterion for convergence of a function from any domain with a direction to a range in R^n is proved. In Chapter III—Continuity—the directions under consideration are specialized either to the family of all relative neighborhoods of a point or (for a nonisolated point) the family of all deleted relative neighborhoods of the point. Continuity is defined in terms of composition of functions, with an arbitrary meaningful background parameter (continuity *means* that the limit of the function is the function of the limit), and then reformulated in a variety of ways. Semicontinuity, uniform continuity, uniform limits, and iterated and double limits, including the Moore uniform convergence theorem, receive generous treatment. Also included in the chapter are the Stone-Weierstrass theorem, Ascoli's theorem on equicontinuity, and a special case of the Tietze extension theorem. Chapter IV—Bounded Variation, Absolute Continuity, Differentiation—discusses bounded variation and absolute continuity for both point and interval functions. Also included are derivatives and differentials of real-valued functions, mean-value theorems, and the implicit function theorem. In Chapter V—Lebesgue-Stieltjes Integration—the Daniell approach to integration is followed. A leisurely heuristic introduction is followed by a streamlined development that produces the Lebesgue dominated convergence theorem on the fifteenth page of the chapter. Such topics as Fubini's theorem, Baire functions, Borel sets, and the Riemann-Lebesgue theorem are included. Measure-theoretic considerations remain secondary to those of the integrals, but are given fair treatment. Outer and inner measure are nowhere considered. The Riemann and the Riemann-Stieltjes integrals conclude the chapter. Chapter VI—The Integral as a Function of Sets—presents the case for abstract measure theory, and includes such items as the Lebesgue decomposition of a countably additive set function, the Radon-Nikodym theorem, differentiability of a Lebesgue integral, metric density, integration by parts, and transformations of multiple integrals. Chapter VII—The L_p Spaces—contains in its 49 pages a surprising amount of the theory of L_p spaces in general and Hilbert space in particular, including representation theorems for linear functionals on L_p and on the space of real-valued continuous functions on a compact subset of R^n. The Riesz-Fisher theorem for general complete orthonormal systems and the Fourier and Fourier-Plancherel transforms are included, and the chapter ends with a brief treatment of bounded hermitian operators on Hilbert space, including the spectral-resolution theorem.

The first of the book's three appendices treats the principle of inductive definition, the second establishes the equivalence of six formulations of the maximality principle (including the axiom of choice, Zorn's lemma, and the well-ordering theorem), and the third gives a proof of Tychonoff's theorem on the compactness of the product of compact spaces.

The book is remarkably free of errors, and those that are present are principally typographical and unlikely to disturb the reader. For example, although juxtaposition (fg) is used for composition of functions and dotted separation ($f \cdot g$) for products, there are a few occasions (Theorem IV. 3.4 and twice on page 116) when multiplication is indicated fg. In Exercise 7, page 80, both factors should be assumed bounded.

It might be regretted by some that although arcwise connectedness is defined in an exercise, connectedness is omitted. Also missing is the theorem concerning the continuity of the inverse of a one-to-one continuous mapping from a compact set into a Hausdorff space. In this connection it is hoped that readers of this book will not infer from Theorem 2.5 of Chapter III that for a continuous image of a compact set to be compact the image space must be Hausdorff. The preceding Theorem 2.3, on which Theorem 2.5 is based, can easily be proved in full generality without the restriction that the image topological space be Hausdorff if proper interpretations of limit statements (including $\mathrm{L}f(a) = f(a)$) to permit nonuniqueness are made. This form permits a more general setting also for the succeeding corollary relating continuity with openness (closedness) of inverse images of open (closed) sets. Such details as these can readily be brought to the attention of a class by the teacher. Another role that a teacher using this book can fill is to supplement the text by sidelights and examples. He can, for instance, show how the uniqueness of an implicit function can be guaranteed by continuity in a neighborhood of the domain of that function, rather than by a restricted neighborhood in a product space. He can point out how a measurable function can be characterized in terms of measurability of inverse images of Borel sets. He may wish to discuss Cantor's classical set of measure zero, and its related continuous monotonic function that is not absolutely continuous. He may decide to exhibit an example of a measurable function of a Baire function that is not measurable. But responsibility for including such items in a course rests more properly with the instructor than with the authors of a text.

Some potential readers who seek an introduction to the Lebesgue integral may object to being forced to assimilate the full generality of the Lebesgue-Stieltjes integral, with the attendant complexities related to intervals of continuity. It should be pointed out to such individuals that the particularization to the Lebesgue integral is immediately available if the proper omissions and simplifications are made.

There are 134 exercises, of which 121 are requested proofs. The remaining 13 call for discussion (2), generalization (2), counter examples (2), verification (3), and finding an answer (4). No answers are given. There are no diagrams. A bibliography and an index of symbols are included. The typography is handsome, and the use of boldface and italic type helpful.

The authors are to be congratulated on an ambitious but successful undertaking. Their book is certain to be recognized as a valuable contribution to mathematical literature as both text and reference.

JOHN M. H. OLMSTED

Grundlagen und Anwendungen der Informationstheorie. By W. Meyer-Eppler. Kommunikation und Kybernetik in Einzeldarstellungen, vol. 1. Berlin-Göttingen-Heidelberg, Springer, 1959. 18+446 pp. DM 98.

Information theory has provided mathematics with a number of interesting problems and with at least one new idea (the Kolmogorov-Sinai invariant in ergodic theory). These matters will doubtless be of interest for some time to come; they may even find a permanent place on the mathematical scene. The volume under review, the first in Springer's new series on information and control, edited by Mr. Meyer-Eppler, covers most of the topics usually associated with the phrase *information theory* in its broadest meaning. The first six chapters treat Fourier analysis of signals, channels, communication in the presence of noise, and coding theory. The last four chapters deal with the sense organs as links in a communication channel, with optics and acoustics, and with structural linguistics. Since the mathematical treatment is casual, the reader hoping to come to grips with the analytic and probabilistic problems involved will be disappointed. On the other hand, he will find a clear account of the pre-mathematical ideas, together with a vast amount of illustrative material and a large number of interesting applications of information-theoretic notions to such subjects as phonetics and phonemic analysis, morphology, semantics, and the theory of vision. For those of us who are used to reading German in just one field, the book is hard to read because of the breadth of the subject-matter; a translation would find a wide audience.

PATRICK BILLINGSLEY

An introduction to the geometry of numbers. By J. W. S. Cassels. Grundlehren der mathematischen Wissenschaften, vol. 99. Berlin-Göttingen-Heidelberg, Springer, 1959. 7+344 pp. DM 64.50; bound, DM 69.

The geometry of numbers deals essentially with an arithmetical