RESEARCH PROBLEM

39. Mok-Kong Shen and Tsen-Pao Shen: Number theory.

The first 2n natural numbers will be grouped into n pairs (a_1, b_1) , $(a_2, b_2), \dots (a_n, b_n)$ with $b_i > a_i$ and the sum and difference of the elements of each pair formed: $c_i = a_i + b_i$, $d_i = b_i - a_i$. Can we prove that for $n \ge 3$ it is always possible to form the above mentioned pairs such that the 2n numbers c_i and d_i are all different? Examples:

n = 3: (1, 5), (2, 3), (4, 6). n = 6: (1, 10), (2, 6), (3, 9), (4, 11), (5, 8), (7, 12).n = 8: (1, 10), (2, 14), (3, 16), (4, 11), (5, 9), (6, 12), (7, 15), (8, 13).

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