BOOK REVIEWS

Wahrscheinlichkeitsrechnung mit einem Anhang über Informationstheorie. By A. Rényi. VEB Deutscher Verlag der Wissenschaften, Berlin, 1962. 11+547 pp. DM 55.

Until recently, probability theory has been called "calculus of probabilities" in the French and German languages. This book is so called and deserved the name in the best sense of the word. It is essentially a textbook of mathematical analysis as applied to the field of probability. By this it is not implied that the measure-theoretic foundations are not given adequately and rigorously. Indeed, the book begins with axiomatic Boolean algebra including a proof of M. H. Stone's isomorphism theorem, albeit in fine print. Kolmogorov's extension theorem is also given its full treatment while the Radon-Nikodym theorem, though not proved, is discussed in some detail with examples—which is probably more helpful than reproducing a standard proof. However, the unmistakable flavor of this book is the abundance of classical analytic techniques vigorously and interestingly employed to calculate the probabilities.

The author states in the Foreword that besides a careful exposition of the basic theory he has treated especially those topics that "lie near him" and about which he has more to say. Among the less usual material may be mentioned: characterizations of the normal law, Linnik-Singer's sharpening of Cramér's theorem, central limit theorems with densities, with a random number of summands, and without replacement in sampling, mixing sequences, conditional and ratio limit theorems, the author's neat treatment of order statistics, Smirnov's theorem and its variants, and an entire appendix (64 pages) devoted largely to the definitions and analytic properties of various concepts of "information content." The numerous exercises are also rich in interesting topics such as: Euler's ϕ function, the Borel-Cantelli lemma and the strong law of large numbers for pairwise independent random variables, Laplace's method of steepest descent, Post-Widder inversion formula, the Maxwell gas law, infinite series and continued fractions expansions of real numbers.

Some sacrifices of usual or fashionable topics have to be made for these unusual ones. For example, infinitely divisible laws are only mentioned and stable laws are treated only so far as Lévy's earlier derivation of the quasi-stable case. There is no mention of martingales and Markov chains are discussed only briefly in the finite case. Stochastic processes with a continuous time parameter, or special cases thereof, have not been included. One may also wonder if so many pages spent on the preliminaries of information theory are not a bit extravagant. The book is also expansive on the pedagogic side. There are many examples, computations, tables and interesting pictures. Some important results appear in successive stages with increasing generality and depth, and variations on a theme are often presented. All this should make the content of the book easier to digest and to appreciate. The current tendency to conciseness of exposition, even on the textbook level, is not followed here, to the benefit of the reader.

This book can well serve as a basic course in probability theory, before a serious involvement with measure theory as required in any worthwhile study of stochastic processes. It has been said that probability theory is just a chapter of measure theory. This statement is not so much false as it is fatuous, as it would be to say that number theory is just a chapter of algebra. However, for a student of mathematics who wants to learn probability, there is need for a book which is not trivial on the one hand, and does not resemble chapters on measure and integration on the other. Of course Feller's well known *Introduction to probability theory and its applications*, Volume 1, can fit the bill, except for those who are eager for a general probability space and keep wondering about his Volume 2. (It may be disclosed here that Volume 2 will be a surprise to them.) There are also some mature readers who have no use for coins and dice, or even genes and particles. This book by Rényi may be what they have been looking for.

KAI LAI CHUNG

Introduction to differentiable manifolds. By Serge Lang. Interscience, New York, 1962. 10+126 pp. $7.00.

From modest beginnings in the eighteenth century, differential calculus has had a continuous increase of power and scope, culminating recently with the global theory of differential manifolds and mappings. This theory, basic to modern differential topology and geometry as well as classical physics, emerges from two decades of semi-secret existence with the publication of this definitive *Introduction.*

In addition to organizing in a report to the public fragments collected since 1936 (from original articles by H. Whitney, mimeographed notes by S. S. Chern, and J. Milnor, books by C. Chevalley, and G. de Rham, and many others), this text extends the global calculus to the infinite-dimensional case, and constitutes a natural sequel to the *Foundations of modern analysis* by J. Dieudonné [Aca-