EXTENSIONS OF BRANDT SEMIGROUPS

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Communicated by O. G. Harrold, February 21, 1966

The purpose of this note is to announce the determination of all (ideal) extensions of a Brandt semigroup by an arbitrary semigroup with zero and to give two applications of this result. We use the terminology and notation of [1].

THEOREM. *Let* $(V, \circ)$ *be an extension of a Brandt semigroup* S *by an arbitrary semigroup* T *with zero,* $0'$. *Let* S *be given the Rees representation* $S = M^0(G; I, I; \Delta)$. *Then there exists a partial homomorphism* $w: A \rightarrow w_A$ *of* $T^* = (T \backslash 0')$ *into* $\mathcal{I}_I$ *the full symmetric inverse semigroup on* I. *Let* s_A *and* t_A *denote the domain and range of* w_A *respectively. If* $AB = 0'$ *(juxtaposition denoting multiplication in* T*) either* $s_A \cap t_B = \square$ *or* $s_A \cap t_B$ *is a single element* $d_{A,B}$. *For each* $A \in T^*$ *there exists a mapping* ψ_A *of* s_A *into the group* G *such that for* $AB = 0'$

$$(i\psi_A)(iw_A\psi_B) = i\psi_{AB} \quad \text{for all } i \in s_{AB}.$$

The products in V *are given by*

$$(1)\quad \text{(a)}\ A \circ B = AB \quad \textit{if } AB = 0' \textit{ in } T,$$

$$\text{(b)}\ A \circ B = 0 \ (\textit{in } S) \quad \textit{if } AB = 0' \ (\textit{in } T) \textit{ and } t_A \cap s_B = \square,$$

$$\text{(c)}\ A \circ B = (d_{A,B}\overset{-1}{w_A}\psi_A)(d_{A,B}\psi_B);\ d_{A,B}\overset{-1}{w_A},\ d_{A,B}w_B)$$
$$\textit{if } AB = 0' \ (\textit{in } T) \textit{ and } t_A \cap s_B = d_{A,B}.$$

$$(2)\quad (a; i, j) \circ A = \begin{cases} (a(j\psi_A); i, jw_A) & \textit{if } j \in s_A, \\ 0 & \textit{if } j \overline{\in} s_A, \end{cases}$$

$$0 \circ A = 0$$

$$(3)\quad A \circ (a, i, j) = \begin{cases} ((i\overset{-1}{w_A}\psi_A)a;\ i\overset{-1}{w_A}, j) & \textit{if } i \in t_A, \\ 0 & \textit{if } i \overline{\in} t_A, \end{cases}$$

$$A \circ 0 = 0.$$

Conversely let S *be a Brandt semigroup and* T *be a semigroup with zero such that* $S \cap T = \square$. *If we are given the mappings* w *and* ψ_A *described above and define product* $\circ$ *in the class sum of* S *and* T^* *by* (1)–(3), *then* V *is an extension of* S *by* T.

REMARK. An extension of S by T always exists [1].

COROLLARY 1. *An extension V of a Brandt semigroup S by a regular 0-bisimple semigroup T is given by a partial homomorphism* [2, p. 522] *if and only if there exists an idempotent E in T^* such that there is at most one idempotent of S^* under E.*

REMARK. Warne [2] gave a similar result for the case where S and T are both completely 0-simple semigroups.

If A is a set, $|A|$ will denote the cardinality of A.

COROLLARY 2. *If S is a finite Brandt semigroup and T^* is a simple group with $|T^*| > \max(|I|!\,|G|)$, then there are $2^{|I|}$ extensions of S by T.*

These results will appear in [3].

REFERENCES

1. A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*, Vol. 1, Math. Surveys No. 7, Amer. Math. Soc., Providence, R. I., 1961.

2. R. J. Warne, *Extensions of completely 0-simple semigroups by completely 0-simple semigroups*, Proc. Amer. Math. Soc. **17** (1966), 522–523.

3. ———, *Extensions of Brandt semigroups and applications* (to appear).

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