studied where the quadratic form involves only the final value of the physical process over a fixed bounded time interval. The solution of this problem is applied to a control problem where the physical system $X(t)$ is affected by the sum of two controls. The control functions are regarded as the player strategies in a two person zero sum game and the stochastic system is described as a differential game with imperfect information.

The book by Arnold may serve as a textbook or reference work. Its substantial bibliography contains reference lists for topics such as Markov and diffusion processes, stochastic differential equations, stability, filtering, control, and probability theory. There is a good index and each section of the book is well organized. The book is especially valuable for nonexperts on stochastic differential equations who wish to deal with models for processes affected by noise. One can learn the limitations of the theory as well as recent results on a variety of problems. The notes of Balakrishnan are valuable to anyone who desires to master the mathematical techniques involved in modern stochastic control theory.

VICTOR GOODMAN

Optimal control theory, by L. D. Berkovitz, Applied Mathematical Sciences, 12, Springer-Verlag, New York, 1974, ix+304 pp., \$9.50

The term "mathematical theory of optimal control" has come to refer to the optimization of a certain class of functionals of state and control variables for dynamical systems whose evolution with time is described by ordinary differential equations. Such problems are similar to the Bolza problem in classical calculus of variations, with the important difference that inequality constraints may be imposed. A large literature on optimal control theory developed during the 1960's, stimulated by the slightly earlier work of Bellman, Pontryagin, and their associates. Most of the questions with which that literature was concerned have by now been resolved. It is the task of authors of books on control theory to preserve the essential aspects, for those interested in the applications and as a foundation for students entering an area of active current research (e.g. control of systems governed by partial differential equations, control systems with time delays, and stochastic control).

In this book Berkovitz gives a readable account not only of the standard Pontryagin necessary conditions for a minimum but also of the problem of existence. The proof given for the Pontryagin necessary conditions follows Gamkrelidze, SIAM J. Control (1965). Like other proofs, it depends on the idea of convex set of variations (due to McShane in 1939) and the Brouwer fixed point theorem.

The traditional method in calculus of variations for proving existence of a minimum is to show precompactness of minimizing sequences and lower semicontinuity. A nicer technique was found in 1959 by Filippov; it avoids lower semicontinuity but uses a theorem about measurable selections. This technique was extended by Cesari in 1966, who found existence theorems for optimal controls which are more widely applicable.

The first of two chapters on existence theorems is devoted to the Filippov-Cesari technique, and to further results of Berkovitz. These results all depend on certain convexity assumptions. When these assumptions do not hold, there may be no optimal control in the usual sense of the term. The second chapter on existence is devoted to existence of generalized optimal controls without convexity assumptions. The basic idea of generalized solutions to problems in calculus of variations first appeared in L. C. Young's work during the 1930's, under the name "generalized curves". It reappeared in control theory under the names "sliding regimes" or "relaxed controls", in work of Gamkrelidze, Warga, McShane, and others.

In summary, this book gives a well-presented treatment of a well-selected package of topics. It is a valuable addition to the control theory literature.

W. H. FLEMING

Einige Klassen singularen Gleichungen, by S. Prössdorf, Akademie-Verlag, Berlin; Mathematische Reihe, Band 46, Birkhäuser, Verlag, Basel and Stuttgart, 1974, 353 pp.

An important early chapter in operator theory was concerned with the study of various classes of integral equations with singular kernel. Chief among these were F. Noether's investigation of integral equations with Cauchy kernel and the study by N. Wiener and E. Hopf of the integral equation which now bears their name. The results of these studies were not only of considerable practical importance but were instrumental in the development of the abstract notions of Fredholm operator (called Noether operator in the book under review) and index. The study of various spaces of analytic functions was stimulated, as well as the problem of factoring functions on the real line into factors analytic on the upper and lower half-planes. In addition, the important notion of the symbol of an operator first occurred in this context.

In the book under review the author presents the theory of various classes (including systems) of one-dimensional singular integral operators including those with Cauchy or Hilbert kernels as well as the study of Wiener-Hopf and Toeplitz operators. In the definition of these operators, a function called the symbol can be identified which determines the behavior of the operator up to "smooth" operators. In particular, the operator with symbol equal to the inverse of the symbol of the given operator defines an inverse for the operator modulo the compact operators; thus the problem of deciding when an operator is Fredholm is reduced to inverting the symbol. The techniques are drawn largely from functional analysis and the methods show the strong contribution that Soviet mathematicians have made to this subject, especially I. C. Gohberg and his collaborators.

In addition to the more common results on operators of normal type which