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Murray Rosenblatt
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Adventures of a mathematician, by S. M. Ulam, Charles Scribner's Sons, New York, 1976, xi + 317 pp., $\$ 14.95$.
I. Ulam is a magic name in modern mathematics. One thinks of Leonardo's letter to the Duke of Milan:
"Most Illustrious Lord;
... Item: In case of need I will make big guns, mortars, and light ordnance of fine and useful forms, out of the common type.

Item: I can carry out sculpture in marble, bronze, or clay, and also I can do in painting whatever may be done, as well as any other, be he who he may. . . ."

And so he could.
In Ulam's writing, as in Leonardo's, scarcely a mention of mother and father. At eleven Ulam began to be known as a bright child who understood the special theory of relativity. He was an A student but did not study much, active in sports, played bridge, poker, and chess. At 15 he absorbed the calculus, number theory, and set theory. At 18, when he matriculated from gymnasium, the choice of profession presented difficulties. His father wanted him to join his successful law practice, while Ulam longed for a university career. But university positions in Poland were almost impossible to obtain if one's family, however wealthy and culturally assimilated, had a Jewish background. As a compromise, Ulam entered Lwów, Polytechnic Institute to study engineering.

From the first, mathematics took complete possession of him. Kuratowski quickly recognized the young student's gifts and took special pains with him. The names of Mazur, Lomnicki, Borsuk, Kacmarz, Nikliborc, Tarski, Schauder, Averbach, Schreier, Steinhaus, and above all Banach dominated a euphoric period of feverish activity. At 23 Ulam was sufficiently well known to be an invited speaker at the Zürich congress. Meeting foreign mathematicians for the first time, he found them nervous and given to facial twitches, or short and old, like Hilbert; certainly less impressive than his fellow Poles. Returning to Lwów, Ulam wrote a master's thesis which among other things outlined what is now category theory, and at 24 won his doctorate with a thesis in measure theory. But still there were no prospects of a university position for him in Poland.

Financed by his parents he visited Menger in Vienna, Hopf in Zürich, Cartan in Paris, and Hardy in Cambridge. Returning to Poland, he began a correspondence with von Neumann who invited him to visit the Institute at Princeton. In December 1935, Ulam sailed on the Aquitania for New York.

It was von Neumann whom Ulam came to admire above all others as a mathematician and kindred spirit. (The book was originally intended as a biography of von Neumann.) Things really began to happen when Ulam met G. D. Birkhoff at von Neumann's house and was in due course invited to Harvard as a Junior Fellow for three years. But soon after Ulam's return in 1939 from his customary three month visit to Poland, Hurewicz telephoned to say in somber tones "Warsaw has been bombed, the war has begun."

Next spring, when things looked darkest, it was Birkhoff who came to the rescue again by securing for Ulam an instructorship at Madison. This was no easy matter, for there were many emigrés by then and even modest positions were hard to find. At Madison he was promoted quickly to assistant professor, a position which held good hope for the future. He became an American citizen, married, and in 1943 rejoined von Neumann at Los Alamos, ignorant until he arrived of just what was going on there.

For Ulam, the transition from pure mathematics to applied physics was remarkably easy. (Not so for von Neumann, who had little physical intuition.) The physicist Otto Frisch in his first visit to Los Alamos from embattled Britain wrote "I also met Stan Ulam early on, a brilliant Polish topologist with a charming French wife. At once he told me that he was a pure mathematician who had sunk so low that his latest paper actually contained numbers with decimal points!"
II. Although Ulam's three intellectual heroes were Banach, von Neumann, and Fermi, none of them is portrayed so vividly in the book as Birkhoff. The Ulam-Birkhoff relationship seems to have been somewhat ambiguous on both sides.
"He liked the way I got almost furious when-in order to draw me out-he attacked his son Garrett's research on generalized algebras and more formal abstract studies of structures. I defended it violently. His smile told me that he was pleased that the worth and originality of his son's work was appreciated."
"In discussing the general job situation, he would often make skeptical remarks about foreigners. I think he was afraid that his position as the unquestioned leader of American mathematics would be weakened by the presence of such luminaries as Hermann Weyl, Jacques Hadamard, and others. He was also afraid that the explosion of refugees from Europe would fill the important academic positions, at least on the Eastern seaboard. He was quoted as having said, 'If American mathematicians don't watch out, they may become hewers of wood and carriers of water. ${ }^{1 "}$

Even after Birkhoff's death the American suspicion of foreigners-even those who as Ulam describes himself were "not unpresentable"-continued to cause trouble. When the war ended in 1945 and Ulam wanted to return to Madison, chairman R. E. Langer answered when Ulam inquired about his

[^0]chances for promotion and tenure: "No reason to beat around the bush, were you not a foreigner, it would be much easier and your career would develop faster."

At the time Ulam was 36 , by any standards an outstandingly creative mathematician, pleasant and courteous in manner, and well supplied by now with friends in high places. How is it that all this did not suffice to overcome the Wisconsin xenophobia, nor to secure for him then or later a position commensurate with his talents from some leading American university? Surely there is a mystery here.

Before 1945 mathematicians were about as numerous in the academic world as professors of French literature, and their importance in the military-industrial-intellectual complex about as great. During the next twenty years American mathematics was a growth industry, since mathematicians had contributed essentially to making the weapons on which our safety now depended and would be needed in the future to keep ahead of possible rivals. Contrary to Birkhoff's fear, the refugees had created several jobs for American mathematicians for every one they occupied. Only the German rocket engineers imported after the war had a comparable effect.
III. Turned down by Wisconsin, Ulam spent an unhappy year at U. S. C., interrupted by a mysterious illness which brought him close to death, and in 1946 returned to Los Alamos. There he proposed the Monte Carlo method in a conversation with von Neumann. "Little did we know in 1946 that computing would become a fifty-billion-dollar industry annually by 1970." Teller and von Neumann were emotionally committed to constructing an H bomb at all costs. Ulam was not so obsessed, but it was he who thought of a way to make it work. "Contrary to those people who were violently against the bomb on political, moral, or sociological grounds, I never had any questions about doing purely theoretical work . . . I I sincerely felt it was safer to keep these matters in the hands of scientists and people who are accustomed to objective judgments rather than in those of demagogues or jingoists, or even wellmeaning but technically uninformed politicians."

In 1967 Ulam returned to university life at Boulder and became an elder statesman of government science.
IV. Some readers will be put off by the frequent examples of mathematical humor characteristic of Ulam and his friends. Thus of Erdös: "Once he stopped to caress a sweet little child and said in his special language: 'Look, Stan. What a nice epsilon.' A very beautiful young woman, obviously the child's mother, sat nearby, so I replied 'But look at the capital epsilon.' This made him blush with embarrassment.' "In fact, these episodes provide almost the only evidence of the humanity of the characters portrayed in this book. Erdös apart, they are preoccupied with seeking recognition of their precise rightful place in the official pecking order. It is a pity that this aspect of the world of mathematicians is so much emphasized in a book for the general reader; the more pity if indeed the emphasis is justified. The appearance of being thinking machines on the make, without discernible relation to parents, spouses, or children, and oblivious to the human concerns of our times, may be due in part to foreign systems of higher education that were devised to
turn out idiot savants in the sciences as being more likely to be useful to the state. But if mathematical intelligence is strongly associated with emotional deprivation and social alienation, then even we earthy, super-honest, solid, and simple native Americans-the qualities that Ulam admires in us-are in for trouble.
H. E. Robbins

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Interpolation spaces, an introduction, by Jöran Bergh and Jörgen Löfström, Springer-Verlag, Berlin, Heidelberg, New York, 1976, x + 204 pp., \$24.60.

We don't want to get involved in the current, often heated, debate as to what constitutes pure, as opposed to applied, mathematics. We just want to start this review by saying that the theory of interpolation of operators is an impressive application of pure mathematics to pure mathematics. The purists (?) are welcome to wrangle over the semantics.

The subject has its origins in classical Fourier analysis, where it was conceived as an elementary means of finding $L^{p}$-estimates. The very nature of interpolation theory, however, is functional-analytic: typically, a linear operator $T$ is bounded between spaces $X_{\alpha}$ and $Y_{\alpha}$ when $\alpha=0$ and $\alpha=1$, and one wants to conclude that $T$ carries $X_{\alpha}$ to $Y_{\alpha}$ whenever $0<\alpha<1$. Such problems arise in many areas of analysis, and the abstract theory has always been influenced, even guided, by the potential applications to such areas as harmonic analysis, approximation theory, and the theory of partial differential equations. As a result, interpolation has no one place to call home; it is, quite simply, interesting mathematics.

Its success, like that of a good executive, stems from its ability to handle specifics while operating on a generally higher plane. Consider, for example, the thorny Fourier and Hilbert transforms. A great deal of highly-specialized information is known about these operators, and it rarely comes for free. Yet, remarkably enough, the "correct" $L^{p}$-estimates can be derived from general interpolation theorems valid for all linear operators.

Such examples show that it is worthwhile to solve the interpolation problem simultaneously for all operators. It also changes the face of the problem, because the operators themselves, since only their linearity is important, tend to fade into the background. Interpolation theorems then are more properly construed as statements about the underlying system of spaces. This observation, simple as it is, represents the point of departure from classical $L^{p}$-interpolation (the Riesz-Thorin and Marcinkiewicz theorems) into the abstract theory of interpolation spaces and interpolation methods.

Suppose $\left\{X_{\alpha}: 0 \leqslant \alpha \leqslant 1\right\}$ is a family of Banach spaces for which an interpolation theorem is desired. The idea is to construct, using only the extremal spaces $X_{0}$ and $X_{1}$, an intermediate space $\left(X_{0}, X_{1}\right)_{\alpha}$, say, for which the interpolation property automatically holds. Several of these constructions, called interpolation methods, are known. What remains, and this is often the hard part, is to identify the interpolation space $\left(X_{0}, X_{1}\right)_{\alpha}$ with the original space $X_{\alpha}$.


[^0]:    ${ }^{1}$ Birkhoff's statement on the subject can be found in American Mathematical Society Semicentennial Publications, Vol. II, New York, 1938, pp. 276-277.

