theorem immediately and to an existence theorem in various possible ways. A comprehensive discussion of energy methods, and further references, are given in Courant-Hilbert, vol. 2 [2, pp. 652–661, 668–671].

Many important questions about hyperbolic equations are being studied today besides regular linear initial-value problems in space-time regions near a space-like (or characteristic) initial surface, to which our discussion and Friedlander's book for the most part have been confined. Among the other questions are, for instance, boundary-value problems, effects of nonlinearity, the long-range behavior of solutions, and scattering. In the extensive field thus evidenced, Friedlander's monograph is an outstanding instance of unified, detailed treatment of advanced ideas, worthwhile to anyone who makes the preparation called for, indispensable to specialists.

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1. *Integer programming and network flows*, by T. C. Hu, Addison-Wesley, Reading, Mass., 1969, xii + 452 pp., $17.50.

2. *Integer programming*, by Harold Greenberg, Academic Press, New York, 1971, xii + 196 pp., $16.50.

3. *Integer programming*, by R. S. Garfinkel and G. L. Nemhauser, John Wiley and Sons, New York, 1972, xiv + 427 pp., $23.25.

4. *Mathematical programming*, by T. C. Hu and Stephen M. Robinson (editors), Academic Press, New York, 1973, x + 295 pp., \$13.00.

5. *Integer programming*, by Harvey M. Salkin, Addison-Wesley, Reading, Mass., 1975, xx + 537 pp., \$17.95.

6. *Integer programming: Theory, applications and computations*, by Hamdy A. Taha, Academic Press, New York, 1975, xii + 380 pp., \$19.50.

The subject of integer programming is of interest both for its potential practical value and for its position at the intersection of several mathematical areas. Practical problems come from a great diversity of applied subjects; for example, economic planning, engineering design, and combinatorial optimization. However, many problems of even modest size (say fifty variables) may be virtually intractible computationally. This fact adds importance to the theory of the subject, which up to now has been mainly concerned with polyhedra, linear algebra, graph theory, and convexity. There has been little contact with other apparently relevant subjects such as geometry of numbers and algebraic topology.

Here, we shall sketch the principal research areas, discuss computational considerations, and summarize the coverage in the six selected books.

The integer programming problem is to minimize a linear objective function $z = cx$ subject to both linear constraints, $Ax = b$, $x \geqslant 0$, and integer constraints, x_j equal to an integer for $j \in J \subseteq \{1, \ldots, n\}$.

One of the main research areas has resolved around the idea of replacing the integer restrictions with linear restrictions $A'x \geqslant b'$ so that the linear program

$$\begin{aligned} &\text{minimize } z = cx \\ &\text{subject to } x \geqslant 0, \\ &\qquad\quad Ax = b, \quad \text{and} \\ &\qquad\quad A'x \geqslant b' \end{aligned}$$

will have an integer optimum. In principle, this linear program can be formed by taking $A'x \geqslant b'$ to be the facets of the convex hull of integer solutions. In general, it is very difficult to find this A' and b'.

In certain cases, the original linear program $x \geqslant 0$, $Ax = b$ will always have an integer optimum. This case arises in the assignment problem and, more generally, in network flows. There is, however, a larger class of matrices, called totally unimodular, for which the linear program has an optimum integer solution. Much work has gone into characterizing these matrices and extending the notion. For other problems, the facets $A'x \geqslant b'$ are known. This area could be termed polyhedral combinatorics; that is, the study of polyhedra arising from combinatorially described objects. Two high points here are the characterization of the matching polytope by Edmonds and the proof of the perfect graph theorem by Fulkerson and Lovasz.

Another research area has grown out of the cutting plane work of Gomory. He first described convergent algorithms for generating enough constraints $A'x \geqslant b'$ in order for the resulting linear program (for a particular objective) to have an integer optimum. One of the principal computational methods, the cutting plane method, uses this approach. It is applied only to the pure problem (x_j integer for all $j = 1, \ldots, n$) and has mixed success.

A theoretical area which grew partly out of the cutting plane work is the study of Gomory's group problem. This problem is the relaxation of the integer programming problem obtained by replacing $Ax = b$ with $Ax \equiv b$ (mod 1). The significance of this relaxation is that when b is large enough, in a certain sense, the answer to the group problem solves the integer problem. Extensive work has been done to characterize the facets of the convex hull of solutions to the group problem, and this work has led to a general subadditive approach to integer programming.

Turning to computation, the method most used, especially for mixed problems (x_j integer for $j \in J$, a proper subset of $\{1, \ldots, n\}$), is branch-and-bound. This method is simple in principle and consists of sequentially solving linear programs and branching on an integer variable $x_j^0, j \in J$, with x_j at a noninteger value by fixing it at each of its possible integer values. A related method is enumeration, which keeps the integer variables at integer values and tries many possible values. Both of these methods rely on some type of bounds in order to terminate unpromising possibilities. Both rely on heuristics to choose variables to branch on and to guide the search.

We turn now to the six books and the material present in each.

The book edited by Hu and Robinson has a paper by Fulkerson on the perfect graph theorem, an eighty page survey by Garfinkel and Nemhauser stressing computation, and three papers developing the group problem with a stress on cyclic groups, cutting planes, and subadditive methods.

The other five books are basically texts. Probably the best coverage is that of Garfinkel and Nemhauser. They have a chapter including some polyhedral combinatorics, a little about totally unimodular matrices, and Edmonds' method to solve the maximum matching problem. They also cover cutting plane theory, the group problem, and computational methods including some test problems along with some idea as to what methods seem to work satisfactorily at least on classes of problems.

Their tendency toward brevity is vexing to students and may be irritating even to experienced integer programmers. However, the book is, in this reviewer's opinion, the best available on the subject.

The book by Salkin has a very good chapter on the group problem, including recent work. In general, the book has well-written coverage of the basic elements of integer programming. There is, however, almost no discussion of combinatorial problems or topics such as unimodularity. Some attempt is made to give computational results. The chapter by Woolsey on "Integer programming in the real world" provides some amusement and useful discourse, but the remarks and section on "what kind of real integer programming problems are presently being formulated and solved" seem to indicate a lack of experience rather than a true picture of the question.

Overall, Salkin has a good presentation of the basic material and enough advanced material for the book to be useful as a reference.

The books by Greenberg and Taha are more elementary. They may be useful in teaching, but both limit their coverage to presentation of methodology.

The book by Hu, *Integer programming and network flows*, was the first in this area and remains a useful reference. The areas covered well are network flows, cutting planes, and Gomory's group problem. In particular, Gomory's original, ground-breaking papers on the group problem are reproduced here. Of the other books, only Salkin gives an adequate survey of this work and some of its present directions.

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Rings with involution, by I. N. Herstein, Univ. Chicago Press, Chicago and London, 1976, x + 247 pp., $5.50.

For me, a "week-end" associative ring theorist, reading this book is like reading a letter from a not-too-distant relative who writes periodically to inform us (with a certain amount of pride and joy) of what his branch of the family has been doing. The recent work of Professor Herstein's immediate family (among them Baxter, Lanski, Martindale, and Montgomery) as well as the work of older family members (Jacobsen, Kaplansky, and Herstein himself) play a central role in this book. (This list of names is not meant to be a complete family tree.) The author states "I have tried to give in this book a rather intense sampler of the work that has been done recently in the area of rings endowed with an involution. There has been a lot of work done on such rings lately, in a variety of directions. I have not attempted to give the last minute results, but, instead I have attempted to present those whose statements and proofs typify." Such a "letter" must perforce have its main interest with those already familiar with the "family" and no effort is made to interest outsiders (aside from a careful and lucid presentation which highlights the intrinsic interest of the material). Applications and motivation from outside associative ring theory (from Jordan and quadratic Jordan algebras, and from operator and Banach algebras) are purposefully omitted in order to achieve the author's goal efficiently. Indeed, one really should be familiar with the letter of several years ago, *Topics in ring theory* [2] (= TRT in the remainder of the review) in order to read the present one. The general theme of the current letter is: Given a ring R with involution $*$: $R \to R$, define the subsets $S = \{x \in R | x = x^*\}$, $K = \{x \in R | x^* = -x\}$, $T = \{x + x^* | x \in R\}$, $K_0 = \{x - x^* | x \in R\}$ and then try to (1) determine what effect on R the imposition of certain hypotheses (e.g. regularity, periodicity, or the satisfaction of a polynomial identity) on the elements of S, K, T, or K_0 will have, and (2) characterize (or extend) mappings on R (or on S, K, T, or K_0) which preserve properties of, or operations on S, K, T, or K_0. (Beware "linear" reader! The sets S and T are not necessarily the same since $\frac{1}{2}$ may not be present. Similarly R is not necessarily the span of its selfadjoint elements. In fact, one of the lessons that a nonspecialist, such as myself, can learn from this book is how nice it is to have linearity instead of just additivity.) In the absence of further restrictions, these questions usually cannot be answered so that R is almost always assumed to be simple (no two-sided ideals), prime