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GENERALIZED ALBANESE VARIETIES FOR SURFACES

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In this paper we announce a solution to the generalized Albanese problem for smooth projective surfaces. More precisely, for such a surface X over a field k and for each modulus m (see next paragraph) we show the existence of a pair (G, α), where G is a commutative algebraic group over k (or more generally a principal homogeneous space under such a group), $\alpha\colon X \to G$ is a rational map, and any rational map with modulus m factors through α.

Let X be such a surface and let $U = X \sim \bigcup D_j$ be the complement of a finite number of integral divisors on X. In [**2**, Chapter 3, Proposition 1] it was shown that for a rational map $\alpha\colon X \to G$ into an algebraic group we get a homomorphism $\gamma_m\colon C_m(X) \to G(k)$ for some modulus m, where $C_m(X)$ denotes the K-theoretic idele class group of X. When domain$(\alpha) = U$ we have $m = \sum m_j D_j$ with $m_j \geq 1$. In this situation we say that α admits m as modulus.

It is clear that by usual descent arguments we may assume that k is algebraically closed and work with algebraic groups rather than principal homogeneous spaces.

Let Cat_m denote the category of maps $\alpha\colon X \to G$ which admit m as modulus.

THEOREM 1. *In* Cat_m *there exists* $\alpha\colon X \to G_{um}$ *with the universal mapping property described above.*

SKETCH OF THE PROOF. By [**5**, Corollary to Theorem 2] it suffices to show that the dimension of algebraic groups G with $\beta\colon X \to G$ in Cat_m and β maximal [**5**, Definition 2] is bounded. For this by blowing up points in U we reduce to the case of a Lefschetz pencil $\pi\colon X' \to \mathbf{P}^1$ with m flat over $\mathbf{P}^1$.

Then by using [**2**, Chapter 3, Lemma 1] we see that $(\beta, \pi)\colon X'' \to G \times S$ admits m as a modulus in the sense of [**6**, Definition 1] ($X'' \to S \subset \mathbf{P}^1$ is the smooth part of the pencil). Hence it factors through the relative generalized jacobian J_m of X'' [**6**, Theorem 1]. Then it is easy to see that the dimension of the group generated by β is equal to the dimension of the image of the composite map

$$J_m \to G \times S \overset{\text{proj}}{\to} G.$$

Therefore if β generates G then $\dim(G) \leq \dim(J_m)$.

REMARK. We can give an alternate proof of Theorem 1 by applying [**7**, §3, Proposition 4] to show that α admits m as modulus iff

$$\alpha^*(\Omega_G^{\mathrm{inv}}) \subset (H^0(U, \Omega_U)^{d=0} \cap H^0(X, \Omega_X(-m))).$$

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