presupposes a working knowledge of basic topics in dynamics such as sinks and sources, saddle nodes and period doublings, none of which are precisely defined. There is a 12 page refresher course on manifold theory, tangent bundles, manifolds with boundary and the like, and a 6 page section devoted to transversality, structural stability, and genericity, but I'm afraid that the reader will need to be previously exposed to these topics to fully appreciate them. One can learn the preliminary material elsewhere, as for example in the book of Guckenheimer and Holmes [GH]. Holmes was Wiggins' thesis advisor and, because of this, their books are naturally complementary and provide a good "one-two punch" in applied dynamics.

Wiggins' book is aimed primarily at the practicing applied scientist who has encountered chaos in his or her work. It will undoubtedly give these scientists an excellent bag of tricks necessary to recognize chaos and, more importantly, to analyze it. In this endeavor, the book succeeds admirably.

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Stochastic equations for complex systems, by A. V. Skorohod. Translated by L. F. Boron, D. Reidel Publishing Company, Dordrecht, 1988, xvii + 175 pp., $69.00. ISBN 90-277-2408-3

The modern theory of Markov processes in $\mathbf{R}^d$ developed out of the pioneering work of Kolmogorov, Feller, Lévy, Itô, and Dynkin, and many deep properties of the basic processes such as Brownian motion and Lévy processes have been investigated. Moveover based on the seminal work of Doob, the general theory of processes and stochastic calculus of semimartingales were systematically developed during the 1960s and 1970s (cf. Dellacherie and Meyer [**1**] for a complete exposition). On the other hand, the study of complex stochastic systems is still in its infancy. The book under review contains two chapters each devoted to an important aspect of this subject. The first chapter is devoted to the construction of continuous Markov processes in locally compact state spaces, including for example manifolds of variable dimension and manifolds with boundary.

This chapter is based on the theory of semimartingales and stochastic calculus and in particular is based on a representation of continuous Markov processes in terms of weak solutions to infinite systems of stochastic differential equations. The second chapter "Randomly interacting systems of particles" is a contribution the study of the collective behavior of large interacting systems and has its origins in statistical physics. In particular the limiting dynamics of the empirical measures of n-particle systems are studied as the number of particles $n \to \infty$.

1. Stochastic equations for continuous Markov processes. One of the most fully developed chapters of the theory of stochastic processes is that concerned with continuous strong Markov processes in $\mathbf{R}^d$. Such a process has an associated Feller-Dynkin semigroup $\{P_t\}$ of contraction operators on $C_0(\mathbf{R}^d)$, the Banach space of continuous functions vanishing at ∞, furnished with the supremum norm. If we assume that $C_K^2(\mathbf{R}^d)$ (functions of compact support with continuous second derivatives) is contained in the domain of the infinitesimal generator, L, of such a semigroup, then L has the form

$$(1.1)\qquad L\varphi(x) = \frac{1}{2}\sum\sum_{i,j=1}^{d} b_{ij}(x)\partial^2\varphi/\partial x_i\partial x_j + \sum_{i=1}^{d} a_i(x)\partial\varphi/\partial x_i$$

where the coefficients are continuous (cf. Dynkin [**2**]). On the other hand Itô showed that if the matrix b has a nonnegative square root σ and both σ and a satisfy Lipschitz and growth conditions then the corresponding process exists and is characterized as the unique solution of the following system of stochastic differential equations

$$(1.2)\qquad dx^i(t) = a_i(x(t))\,dt + \sum_{j=1}^{m} \sigma_{ij}(x(t))\,dw_j(t)$$

where $x^i(t)$ is the ith coordinate of the vector $x(t)$ and $w_1, \dots, w_m$ are independent standard Brownian motions. In this approach, which was originally proposed by Lévy, the Brownian motions serve as fundamental building blocks from which other processes are constructed. It is particularly important because it shows that locally such a process looks like a Brownian motion with drift and in fact many sample path properties such as nondifferentiability are similar to those of Brownian motion. On the other hand, it is clear that these approaches do not exhaust all interesting continuous strong Markov processes in $\mathbf{R}^d$. Moreover it is desirable to have an approach which could be extended not only to locally compact metric spaces but also to infinite dimensional spaces including for example Hilbert spaces. Such an approach has been developed over the past 20 years based on the idea of martingale problem and this is described below.

Before going on to general locally compact spaces we should mention that the "building block" approach was extended to processes with jumps in the pioneering 1961 monograph of Skorohod [**14**]. In this monograph he introduced equations of the form

$$(1.3)\qquad dx^i(t) = a_i(x(t))\,dt + \sum_{j=1}^{m} \sigma_{ij}(x(t))\,dw_j(t) + \int f(t, x(t), u)p(du \times dt)$$

where p is a Poisson measure. The solution to equations (1.3) yields a family $\{P_x : x \in \mathbf{R}^d\}$ of probability measures on $D_{\mathbf{R}^d}$ where D_E denotes the collection of right continuous functions having left limits from $[0, \infty)$ into E, furnished with the Skorohod topology.

Now consider a continuous Markov process $\{x_t\}$ on a locally compact metric space E. Such a process is given by a family $\{P_x : x \in E\}$ of probability measures on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,\infty)})$ where $x : \Omega \to C_E$ (the space of continuous functions from $[0, \infty)$ into E) is $\mathcal{F}_t$-adapted and $P_x(x_0 = x) = 1$. The function $\varphi \in C_0(E)$, the space of continuous functions vanishing at infinity, is said to be in the domain $\tilde{D}$ of the *quasi-infinitesimal operator* $\tilde{A}$ if there exists a bounded measurable function $g(x)$ such that

$$(1.4)\qquad \zeta_t(\varphi) := \varphi(x_t) - \int_0^t g(x_s)\,ds$$

is a $\mathcal{F}_t$-martingale (with respect to P_x for all $x \in E$). The process is said to be a *quasi-diffusion* if for $\varphi_1, \dots, \varphi_n \in \tilde{D}$ and $F \in C^2(\mathbf{R}^n)$ then

$$F(\varphi_1(x), \dots, \varphi_n(x)) \in \tilde{D}.$$

In this case $\varphi^2 \in \tilde{D}$ and the quadratic variation of $\zeta_t(\varphi)$ is given by

$$(1.5)\qquad \langle \zeta(\varphi) \rangle_t = \int_0^t [\tilde{A}\varphi^2(x_s) - 2\varphi(x_s)\tilde{A}\varphi(x_s)]\,ds$$

that is, $\zeta_t(\varphi)^2 - \langle \zeta(\varphi) \rangle_t$ is also a continuous martingale. Then $\varphi(x_t)$ satisfies the stochastic differential equation

$$(1.6)\qquad d\varphi(x_t) = \tilde{A}\varphi(x_t)\,dt + d\zeta_t(\varphi).$$

The main idea is now to attempt to characterize the probability laws $\{P_x : x \in E\}$ of processes of interest as solutions to the *martingale problem* (1.4–1.6). Note that the martingale problem can be viewed as a collection of integral equations for the measures $\{P_x : x \in E\}$; for example, if $s < t$, and g is bounded and $\mathcal{F}_s$-measurable,

$$E_x((\zeta_t(\varphi) - \zeta_s(\varphi))g) = 0$$

where E_x denotes expectation with respect to P_x.

In a striking demonstration of the power of the martingale problem approach, Stroock and Varadhan **[15]** established the existence of diffusions with generators of the form (1.1) under the assumption that the coefficients are bounded and continuous but without the Lipschitz conditions imposed by Itô. Since then their approach has been successfully applied to infinite particle systems, infinite dimensional processes and various limit theorems.

The main idea of Chapter I is to reformulate a solution to such a martingale problem as the solution to a system of stochastic differential equations. In particular, it is established that there exists a Hilbert space H-valued Wiener process, $w(\cdot)$, defined on some extension of the original probability space and a $\mathscr{F}_t$-adapted measurable function $b_t(\varphi)\colon \mathbf{R}_+ \times \tilde{D} \to H$ such that

$$d\varphi(x_t) = a(\varphi, x)\,dt + (b(\varphi, x_t), dw(t))_H \tag{1.7}$$

where $a(\varphi, x) := \tilde{A}\varphi(x)$. The proof of (1.7) is based on a general representation theorem for families of continuous martingales in terms of an H-valued Wiener process.

A solution to equation (1.7) is called *weakly unique* if any two processes satisfying it have the same probability laws on C_E. Clearly weak uniqueness is essential in order to characterize a process using either the martingale problem or (1.7). Unfortunately in many problems the weak uniqueness is difficult to establish (see Ethier and Kurtz **[4]** for a discussion of some approaches to this problem). On the other hand, in certain cases reminiscent of Itô's theory it is possible to establish "strong uniqueness" which in turn proves weak uniqueness. For example assume that for $\varphi \in D_0$, a subset of D that separates points, the following Lipschitz type condition is satisfied

$$E(|a(\varphi,\xi) - a(\varphi,\eta)|^2 + |b(\varphi,\xi) - b(\varphi,\eta)|_H^2) \\ \leq c \sup_{\psi \in D_0} E|\psi(\xi) - \psi(\eta)|^2.$$

In this case it is proved that the solution satisfies strong uniqueness, that is, given any two solutions x_1, x_2 of (1.7) with $x_1(0) = x_2(0)$, then $x_1(t) = x_2(t)$ for all t a.s. Furthermore, in this case the solution x_t is adapted with respect to the filtration generated by w.

Chapter I also includes some convergence and Girsanov-type results in this setting as well as a number of applications of the above ideas to diffusions on manifolds with boundary and manifolds of variable dimension.

2. Interacting Markov systems. One of the most active areas of current research in the theory of stochastic processes is the study of the collective behavior of systems having a large number of degrees of freedom. For example lattice systems with nearest neighbor interactions have been intensively studied—refer to the recent book of Liggett **[10]** for an excellent exposition of this direction. However many problems of lattice systems remain unsolved and in statistical physics many systems have been studied in the simpler *mean-field approximation* in which the subsystems are assumed to be exchangeable. This is the type of interaction to be discussed below.

In statistical physics there is a long history of attempts to derive the Boltzmann equation of the kinetic theory of gases from the molecular level. Boltzmann derived his equation based on an assumption of molecular chaos, namely, that the velocities of particles at fixed times are independent. In his 1956 Berkeley symposium paper Kac **[8]** studied a model

n-particle system and proved that in the $n \to \infty$ limit Boltzmann's molecular chaos hypothesis propagates in time. This in turn stimulated a series of important papers including those by McKean **[11]** and Tanaka **[12]**. These authors studied symmetrically interacting stochastic particle systems in the limit as the number of particles tends to infinity. In Chapter II this circle of questions is studied in the context of the following system of stochastic differential equations of Skorohod type

$$(2.1) \quad dz_i(t) = \left[A(z_i) + \sum_{j=1}^{n} a_n(z_i, z_j) \right] dt + \sum_{j=1}^{n} \int f(\theta, z_i, z_j) p_{ij}^{(n)}(d\theta \times dt), \qquad i = 1, \ldots, n,$$

where z_i defines the state of the ith particle in phase space Z, A is the external field and $a_n(z_i, z_j)$ is a nonrandom force of interaction between the ith and jth particles. In (2.1) the Poisson measure $p_{ij}^{(n)}$ represents an impulsive random force that causes the particles to move via random jumps. The solution of (2.1) induces by a family of probability measures on D_{E^n}.

In studying the limiting behavior as $n \to \infty$ the main object of interest is the *empirical distribution process*

$$\mu_t^{(n)}(A) = \frac{1}{n} \sum_{i=1}^{n} \chi_A(z_i(t))$$

which is described by a family of probability measures on $M_1(E)$, the space of probability measures on E. (Here χ_A denotes the indicator function of the set A.) To characterize the random measures $\mu_t^{(n)}$ it suffices to determine the collection of *moment measures*

$$m_t^{(k)}(dz_1, \ldots, dz_k) = E\mu_t^{(n)}(dz_1) \cdots \mu_t^{(n)}(dz_k), \qquad k \in Z^+.$$

In fact the moment measures of $m_t^{(k)}$ satisfy the analogue of the BBGKY (Bogoliubov, Born, Green, Kirkwood, Yvon) *hierarchy of equations* which plays an important role in kinetic theory. Although the rigorous study of these equations is notoriously difficult, they were used for example in Lanford's **[9]** derivation of the Boltzmann equation for a classical gas.

It turns out that the collective behavior of large systems of this type can be best understood by first studying the asymptotic behavior as the number of particles, n, tends to infinity. Let us first describe the *law of large numbers limit.* The limiting behavior of the system (2.1) is obtained under the natural scaling conditions that $a_n = a/n$ and

$$E(p_{ij}^{(n)}(d\theta \times dt)) = \frac{1}{n} m(d\theta) dt$$

where m is a finite measure on Θ. Then under certain additional smoothness assumptions on the coefficients, the measures $\mu_t^{(n)}(\cdot)$ converge weakly to a deterministic dynamical system in the space of probability measures, denoted by $\lambda_t(\cdot)$. The dynamical system $\{\lambda_t\}$ satisfies the following system of nonlinear integro-differential equations

(2.2)
$$\frac{d}{dt}\int \lambda_t(dz)\varphi(z) = \int \left(\varphi'(z), A(z) + \int a(z,z')\lambda_t(dz')\right)\lambda_t(dz) + \iiint [\varphi(z+f(\theta,z,z')) - \varphi(z)]m(d\theta)\lambda_t(dz')\lambda_t(dz).$$

In the special case in which $a(\cdot,\cdot)$ and $A(\cdot)$ are identically zero, it is also proved that the solution of this nonlinear equation is unique. Equation (2.2) is the analogue of the classical Boltzmann equation and $\lambda_t(\cdot)$ also represents the limiting distribution of a tagged particle. The class of Boltzmann-type equations (also called McKean-Vlasov equations in some settings) which arise in this way is of considerable current interest (cf. Sznitman [**16**]). They also describe the evolution of the marginal distribution, λ_s, as a function of the time s, of a special class of *nonlinear Markov processes*, namely, non-time-homogeneous Markov processes whose transition functions $P(s,x;s+t,dy)$ depend on s only through λ_s, that is, they have the functional form $P(\lambda_s,x;t,dy)$.

Let us now consider the joint behavior of k tagged particles whose initial positions $z_i^{(n)}(0),\dots,z_k^{(n)}(0)$ are assumed to converge to $z_1(0),\dots,z_k(0)$ as $n\to\infty$. Then it is proved that the joint distribution of the process $(z_1^{(n)}(t),\dots,z_k^{(n)}(t))$ converges as $n\to\infty$ to the joint distribution of k independent processes $(z_1(t),\dots,z_k(t))$ each of which is a Markov process that satisfies the stochastic differential equation

$$dz_i(t) = \tilde{a}(t,z_i(t))\,dt + \int f(\theta,z_i(t),z')\tilde{p}(d\theta\times dz'\times dt)$$

where $\tilde{a}(t,z) = A(z) + \int a(z,z')\lambda_t(dz')$ and $\tilde{p}$ is a Poisson measure on $\Theta\times Z\times[0,\infty)$. This is precisely *Kac's propagation of chaos property for this system.*

This asymptotic independence property would suggest that the fluctuations around the law of large numbers limit should satisfy a *central limit theorem.* There are actually two perspectives from which to view fluctuations from the law of large numbers limit. In the first the n-particle system is viewed as an empirical measure on D_E, and a central limit theorem is derived for functionals F on D_E. This is carried out in §6 and has also been studied for interacting diffusions in Sznitman [**17**] and for pure jump processes in Shiga and Tanaka [**13**].

In the second perspective one considers the time evolution of the signed measure-valued fluctuations

$$\nu_t^{(n)}(A) = \sqrt{n}\left(\mu_t^{(n)}(A) - \lambda_t(A)\right).$$

Then under certain conditions the random variables

$$\int \varphi(z)\nu_t^{(n)}(dz)$$

converge as $n \to \infty$ to a Gaussian distribution, i.e., $\nu_t^{(n)}(\cdot)$ converges weakly to some generalized Gaussian field $\nu_t(dz)$. For the case $a(z) = A(z, z') = 0$, this field satisfies

$$d \int \varphi(z)\nu_t(dz) = \iiint [\varphi(z + f(\theta, z, z')) - \varphi(z)]m(d\theta)\lambda_t(dz)\nu_t(dz') \\ + \iiint [\varphi(z + f(\theta, z, z')) - \varphi(z)]\gamma(d\theta \times dz \times dz' \times dt)$$

where $\gamma(d\theta \times dz \times dz' \times dt)$ is a zero-mean Gaussian measure with independent values for which $E(\gamma^2(d\theta \times dz \times dz' \times dt)) = m(d\theta)\lambda_t(dz)\lambda_t(dz')dt$.

Recently Mitoma **[12]** has established a stronger central limit theory for fluctuations from the law of large numbers limit for a system of interacting diffusions. In this case he constructs a nuclear space Φ' and establishes weak convergence of the probability measures on $C_{\Phi'}$.

The author points out two basic restrictions to his work on limit theorems—first the total empirical measures must be finite and second the interaction forces are assumed to be bounded. Similar assumptions have sometimes been assumed by other authors. However the law of large numbers has been proved for interacting diffusions with unbounded coefficients by a number of authors including Léonard, Gärtner and Oelschläger. Moreover in the recent paper of Mitoma **[12]** the fluctuation results are obtained for unbounded coefficients. There is also a series of papers on branching particle systems beginning with the paper of Holley and Stroock **[6]** in which fluctuations are studied for spatially homogeneous (infinite) systems.

As a further application of the ideas developed above Skorohod also discusses a problem that goes back to a series of papers written by Albert Einstein **[3]** between 1905 and 1908. In these papers Einstein derived the diffusion equation for the probability distribution at time t of a particle undergoing Brownian motion. However the derivation of Brownian motion can be carried out at many levels and the problem is still of interest. In his introduction, Skorohod writes "Finally we must point out that the possibility of obtaining a probabilistic Brownian motion from the equations of motion of a system of particles has interested me for a long time. This interest also stimulated to a significant degree all the investigations carried out in this book."

Assume that the motion of a tagged particle is continuous and its velocity can vary in a jumplike manner. Moreover there exists a braking force (viscosity) which is directed opposite to the velocity and is approximately proportional to it. If the viscosity is given by $(1/\varepsilon)$ where ε is a small parameter, then the position and velocity of the particle given by $x_\varepsilon, v_\varepsilon$, respectively, satisfy

$$x_\varepsilon(t) = x_0 + \int_0^t v_\varepsilon(s)\, ds$$

$$dv_\varepsilon(t) = (1/\varepsilon)(-M(t, x_\varepsilon)v_\varepsilon + a(t, x_\varepsilon) + \alpha_\varepsilon(t, x_\varepsilon, v_e))\, dt \\ + \int f_\varepsilon(\theta, x_\varepsilon, v_\varepsilon)q_\varepsilon(d\theta \times dt)$$

where $M(t, x)$ is a strictly positive operator, and

$$q_\varepsilon(d\theta, dt) = p_\varepsilon(d\theta \times dt) - m_\varepsilon(t, d\theta)\, dt,$$
$$m_\varepsilon(t, d\theta)\, dt = Ep_\varepsilon(d\theta \times dt).$$

Under the assumption $x_\varepsilon(0) \to x(0)$, $|v_\varepsilon(0)| = O(1/\varepsilon^{1/2})$ and certain assumptions on $\varepsilon f_\varepsilon(\cdot, \cdot, \cdot)$, as $\varepsilon \to 0$ x_ε converges in the sense of weak convergence of probability measures on D_E to a diffusion process which is characterized as the solution of the Itô equation of the form

$$dx(t) = \overline{a}(t, x(t))\, dt + \overline{B}(t, x(t))\, dw(t).$$

To conclude, just a few more comments on the nature of the book under review. Gihman and Skorohod [**5**] have written a three volume systematic exposition of the modern theory of stochastic processes. The present book is quite different—it is more of a research level monograph examining some basic problems of current research interest and presenting some approaches to these problems. Unfortunately, the references to the growing body of literature in this field are rather incomplete. Nevertheless, this book will be of considerable interest to researchers working on interacting particle systems and related topics.

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Transformation groups, by Tammo tom Dieck. Studies in Mathematics, vol. 8, Walter de Gruyter, Berlin, New York, 1987, x + 311 pp., $71.00. ISBN 0-89925-029-7

In 1981 **[9]**, I reviewed a book with a similar title **[5]** by the same author. There has been considerable progress in this general area since then. Both books focus on topics in equivariant topology, the study of spaces with group actions, primarily actions by compact Lie groups.

In geometric topology, the study of high dimensional manifolds has more and more come to focus on problems concerning smooth, *PL*, and topological group actions. In algebraic topology, classical homotopy theory has moved more and more in the direction of equivariant theory, although there is still a little gap between those who approach problems from an equivariant point of view and those who approach problems from a more classical point of view.

Some of the most important work in algebraic topology since 1981 has concerned the Segal conjecture, the Sullivan conjecture, and various generalizations and applications of those results. Much of this work is intrinsically equivariant in nature. Perhaps a little discussion of these results will illuminate the difference in points of view one can take on these matters.

The Sullivan conjecture, in its generalized form, starts with a finite p-group G, a contractible space EG with a free action by G, and a G-space X. One defines the "homotopy fixed point space of X," denoted X^{hG}, to be the space of G-maps $f\colon EG \to X$. To say that f is a G-map just means that $f(gy) = gf(y)$ for $g \in G$ and $y \in EG$. If x is a fixed point of X, so that $gx = x$ for all $g \in G$, then we have the constant G-map f_x specified by $f_x(y) = x$ for all $y \in EG$. There results an inclusion $\iota\colon X^G \to X^{hG}$. A special case of the "homotopy limit problem" **[12]** asks how near this map is to being a homotopy equivalence. Roughly speaking, the generalized Sullivan conjecture asserts that this map becomes an equivalence after p-adic completion when X is finite dimensional. The conjecture has been proven independently by Haynes Miller, Jean Lannes, and Gunnar Carlsson **[4, 7, 10]**, and numerous authors have obtained interesting applications. While the statement may seem technical and unintuitive, the fact is that the result opens the way to a variety of concrete calculations in homotopy theory of a sort unimaginable just a few years ago.

There is a slightly different, more equivariant, way of thinking about the generalized Sullivan conjecture. One can consider the space Map(EG, X) of