BOOK REVIEWS

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Weak convergence methods for nonlinear partial differential equations, by Lawrence C. Evans. CBMS Regional Conf. Ser. in Math., vol. 74, Amer. Math. Soc. Providence, RI, 1990, 80 pp., $19.00. ISBN 0-8218-0724-2

As working analysts we do not have to be reminded about the significance of understanding weak convergence. The routine of our work, the pattern of our daily lives, consists in searching for appropriate weak topologies or proving estimates suitable for them. Our motto, "Whatever is bounded converges," asks of us only to decide what "bounded" means and what "converges" means. Nonetheless, we should be mindful of what weak convergence means to our colleagues in other sciences. In this context, it is often associated with persistent oscillatory behavior, even as amplitudes decay, or the coagulation or concentration of singular behavior or defect behavior. This occurs in the treatment of almost all physical problems. The importance of understanding the role of weak convergence in these areas is fundamental to understanding the underlying physical phenomena. The primary difficulty is that there are so few weakly continuous functions. Knowing the limit of a sequence does not help us to know the limit of its square, for example.

What can be new in weak convergence? Here we have a slim volume of some of the most useful and fruitful methods developed over the last fifteen years applied to questions in partial differential equations. It is so clear and well organized, we have bought copies of it for our engineer friends. We expect our students to master it. It is not a reference work, but contains most of the methods every working analyst in this field wants to know in a concise form, often explained by a careful choice of example. It is, in its own way, a sort of hight brow Cliff Notes that we can roll up and stick in our pockets. An AMS paperback, it is a rave bargain (along with all the other AMS/CBMS paperbacks).

To understand the organization of the book, let us recall that one encounters a weakly convergent sequence when, for example, attempting to solve a nonlinear partial differential equation by approximation or when solving a variational principle by extracting a minimizing sequence. The weak convergence inhibits us from deciding immediately that the limit function is the solution of the limit problem. The methods for overcoming this difficulty are the subject of the monograph. Most commonly, there are three possible scenarios, all of which take advantage of the structure inherent in the particular problem. We may seek to show that the sequence converges strongly after all. Secondly, concentrations may arise. Finally, oscillations may develop. These last two must be controlled in a suitable manner or the result of the errant behavior must be incorporated into the final limit system of equations.

The first chapter is a review of basic notions with some refinements. Several methods of concentration are introduced, and the Young measure, useful in the study of oscillatory behavior, is described. The second chapter is a brief review of convexity. Quasiconvexity in the sense of Morrey, lower semicontinuity of variational integrals, and related issues are discussed in Chapter 3. Chapter 4 is devoted to concentration compactness with applications given to quasilinear elliptic systems and vorticity bounds for the Euler Equations. Chapter 5 concerns compensated compactness. The many applications include monotonicity methods and the div-curl lemma. Hyperbolic conservation laws are treated as well. Finally, Chapter 6 contains a discussion of the maximum principle for fully nonlinear equations and the homogenization of nondivergence structure equations. In explaining these contents, we have striven for extreme brevity. The ample citations and many historical references outline the contributions of many many scientists and offer avenues for further study.

The volume is dedicated to our beloved colleague Ron DiPerna.

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Finsler geometry and applications, by Aurel Bejancu. Ellis Horwood Limited, Chichester, 1990, 198 pp., $59.50. ISBN 0-13-317975-3

Let M denote an n-dimensional differentiable manifold with coordinate functions $\{x^j : j = 1, \dots, n\}$ on a coordinate neighborhood U. The tangent vector of a smooth arc $\gamma : [t_0, t_1] \to U$, referred to a parameter t, has components $y^j = dx^j/dt$. If there is given a smooth nonnegative function F on the tangent bundle TM of M, an arc-length may be assigned to γ, namely

$$s = \int_{t_0}^{t_1} F(x^1, \dots, x^n; y^1, \dots, y^n)\, dt. \tag{1}$$

In order to ensure that this integral be independent of the choice of the coordinates on U and of the parameter t, it is assumed that F is a scalar function on M that is homogeneous of the first degree in the directional arguments $\{y^j\}$. Under these circumstances F is called a metric function; for instance, if F^2 is an invariant positive definite quadratic form in $y^1, \dots, y^n$, a Riemannian metric is thus defined. However, if F merely satisfies the aforementioned invariance conditions, it is said to give rise to a Finsler metric. It is remarkable that when Riemann introduced his quadratic metric he also suggested the possibility of other metrics, such as that defined by the fourth root of a homogeneous polynomial of order four [8, p. 278]. Thus it is fair to say that the concept of a Finsler metric had been anticipated by Riemann as early as 1854.