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BOOK REVIEW

Induced modules over group algebras, by Gregory Karpilovsky. North-Holland Mathematics Studies, vol. 161, North-Holland, Amsterdam, 1990, 520 pp., \$120.50. ISBN 0-444-88414-9

Frobenius introduced the idea of induced characters as a method of obtaining complex characters of a finite group from those of a subgroup. Frobenius Reciprocity shows that induction is in some sense dual to restriction of characters. Induction then became one of the main techniques in character theory, culminating in Brauer's Induction Theorem, which states that a character of a finite group is an integral linear combination of characters induced from subgroups that are the direct products of p-groups and cyclic groups. This has numerous important consequences; to name just two, Brauer showed that Artin L-functions are meromorphic, and that a splitting field for a group of exponent n can be obtained by adjoining an nth root of unity of the rationals.

In the 1950s D. G. Higman introduced the idea of an induced module. If H is a subgroup of a finite group G and M is a module for the group ring RH where R is any ring, then the induced module denoted M^G is defined as $RG \otimes_{RH} M$. Frobenius's theory can be recovered by taking R to be complex numbers. If R is a field of characteristic $p > 0$ there are numerous difficulties and open questions in representation theory, and most of this book is concerned with induced RG-modules where R is such a field or a p-adic ring (the integral closure of the p-adic integers $\mathbf{Z}_p$ in a finite extension field of the p-adic rationals.)

Some of the most useful work is due to J. A. Green. Since RG-modules in general need not be completely reducible, indecomposability becomes an important issue. Green's Indecomposability Theorem states that an absolutely indecomposable RH-module remains absolutely indecomposable upon induction to G if H is a normal subgroup of G of index p; here a module is termed absolutely indecomposable if it remains indecomposable upon extension of scalars R to a bigger field of characteristic p or a bigger p-adic ring. Green also introduced the useful concepts of vertices and sources. The vertex of an indecomposable RG-module is a certain p-subgroup of G that turns out to be defined up to conjugacy in G. There is a one-to-one correspondence, called the Green Correspondence, between isomorphism types of indecomposable RG-modules of vertex V and isomorphism types of indecomposable modules for the normalizer of V in G with vertex V. The theory has come full circle, as Alperin [1] has used the Green Correspondence to prove Brauer's Induction Theorem.

In recent years permutation modules have become important; these are free R-modules that have bases whose elements are permuted by G. A transitive permutation module with H the stabilizer of one of the basis vectors is isomorphic to the trivial module R for H induced up to G. Its endomorphism ring has a basis related to double cosets HgH. This endomorphism ring, commonly called a Hecke algebra, is very important in the study of representations of finite groups of Lie type. The Hecke algebra, in the case that R is a field of characteristic p and H is a Sylow p-subgroup, has been used by Robinson [6] to study fusion and to give a proof of Frobenius's Theorem on the existence of normal p-complements.

Weiss [8] has used permutation modules to study finite subgroups H of the p-adic group ring $\mathbf{Z}_pG$ of a finite p-group G. In the case that all the elements of H have augmentation 1, Weiss considers the $\mathbf{Z}_p(H \times G)$-module M, whose underlying space is $\mathbf{Z}_pG$, and where (h, g) acts on $x \in \mathbf{Z}_pG$ by sending x to hxg^{-1}. Then M is a permutation module for $H \times G$ precisely if $u^{-1}Hu \subseteq G$ for some unit u of $\mathbf{Z}_pG$. Setting $N = 1 \times G$, we see that the restriction of M to N is a free $\mathbf{Z}_pN$-module and the N-fixed points of M form a permutation module for G/N. Weiss uses this to show that M is a permutation module for $H \times G$, and hence H is indeed conjugate to a subgroup of G. From this, one obtains the result of Roggenkamp and Scott [7] that the isomorphism problem for integral group rings is solved for finite nilpotent groups.

This book is written as a research monograph on induced modules. Basic concepts of representation theory such as the orthogonality relations are assumed to be known. However the author does give a fair amount of preliminary material, such as a complete proof of the Artin-Wedderburn theorem on simple artinian rings. He gives detailed accounts of Clifford Theory, Mackey's theorem on the restriction of an induced module to a subgroup, Green's Indecomposability Theorem, and the Green Correspondence. He gives the rudiments of block theory, including the Brauer morphism but not Brauer's First Main Theorem. All this has become standard material in representation theory and can be found in numerous books, such as [2–5].

Much of the material in the book consists of fairly recent results that have not appeared in textbooks. The author gives complete accounts of the results of Alperin, Robinson, and Weiss mentioned above, as well as many other interesting results. In particular, permutation modules are treated at length. (Applications to groups of Lie type are not mentioned; a good account of this is given in [3, vol. II].) The presentation is often quite close to the original sources, with little motivation given. I would question the inclusion of so much standard material, which is well covered in other texts, including several of the author's books.

The book is printed from camera-ready copy produced in TEX. I find the price somewhat high for work which is reproduced in this way.

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