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BOOK REVIEW

Symmetric and G-algebras (*with applications to group representations*), by Gregory Karpilovsky. Kluwer Academic Publishers, Dordrecht, 1990, 366 pp., $99.00. ISBN 0-7923-0761-5

The theory of finite-dimensional algebras over a field and the connections with finite group representation theory have been quite active for the last several years. Recent important developments have invigorated research in quasi-Frobenius, Frobenius, and symmetric algebras, group graded algebras, and G-algebras, interior G-algebras (a G-algebra consists of a group homomorphism $\alpha\colon G \to \mathrm{Aut}_F(A)$, where G is a finite group, A is a finite-dimensional algebra over a field F, and an interior G-algebra is a G-algebra with a homomorphism $\beta\colon G \to U(A)$, the group units of A, such that $\alpha = c \circ \beta$ where conjugation induces the group homomorphisms $c\colon U(A) \to \mathrm{Aut}_F(A)$), and points and pointed groups (cf. [6]). The connections with finite group representation theory arise from the fact that $F[G]$ is a symmetric algebra and a G-algebra, that if V is a finite-dimensional $F[G]$-module then the group action induces the interior G-algebra $\beta\colon G \to \mathrm{GL}(V/F)$ where $A = \mathrm{End}(V/F)$, and that Clifford Theory (a study of the connections between the representation theories of a finite group, of a normal subgroup, and of the quotient group) leads naturally (cf. [3]) to G-graded algebras and G-crossed product algebras. Thus the work of R. Brauer and J. A. Green on blocks, defect groups, vertices, sources, trace maps, Brauer morphisms, algebras over complete discrete valuation rings, etc., and the more recent group representation theoretic work of such authors as J. L. Alperin, M. Broué, L. Puig, and E. C. Dade on subpairs, nilpotent blocks, and Clifford Theory have stimulated a broad spectrum of research in these related areas.

The book is a compendium of basic theory and some important recent advances in various topics in ring theory and above mentioned topics.

As is evident from the table of contents, the book includes (beside the basic theory) developments in quasi-Frobenius, Frobenius, and symmetric algebras, crossed product algebras, uniserial algebras, symmetric local algebras, G-algebras, permutation G-algebras, algebras over complete noetherian local rings, defect groups in G-algebras, vertices as defect groups, the G-algebra $\mathrm{End}_R((I_H)^G)$, the Brauer morphism, points and pointed groups, interior G-algebras, and bilinear forms on G-algebras.

Frequently, the book follows very closely a (referenced) paper in the literature to obtain major results of the paper (for example, this holds for §§2.2 and 2.3 and the reviewer's paper [5]). Thus the book calls attention to several important recent results and currently developing theories that are closely related to finite group

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representation theory. The reader can then use the references to delve deeply into a topic of interest.

I shall now discuss some shortcomings of the book. Chapter 1 presents classical basic ring theory, however, one finds many results that are part of the basic theory and seem to belong to Chapter 1 scattered inefficiently and misleadingly throughout the book (Proposition 2.1.10, §2.5, Propositions 2.7.7–2.7.9, Lemmas 2.8.1–2.8.2, Lemmas 2.12.7–2.12.10, Lemma 4.4.1, Lemma 4.10.2, Lemma 4.12.5, etc.).

More significantly, categorical equivalence and Morita Theory (appropriate for Chapter 1) are not mentioned at all. Thus, instead of the elegant treatment of Clifford Theory of Dade in [3], a clumsy treatment of Clifford Theory is presented in §2.10 and motivation for G-graded algebras and crossed products in this context (in [3]) is entirely omitted.

The book would have been greatly improved by giving many more examples. Chapter 3 (50 pages) seems to be too specialized for inclusion. There also seem to be several misstatements and gaps in proofs (e.g., Theorem 2.7.6, Lemma 2.12.21, Theorem 4.3.5, Proposition 4.7.8, and Lemma 4.12.7). The exposition is uneven: some trivialities are proved at length and difficult points are frequently glossed over.

In summary, the book presents several important recent advances in several topics in symmetric and G-algebras without any sort of overview of how these results fit into a larger framework or how these results have been used. Hopefully, this book will stimulate interest in these areas, in the referenced original research papers, in more focused books such as [1, 6], and in basic books such as [2, 4, 7].

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