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BOOK REVIEW

Patterns and waves: *The theory and applications of reaction-diffusion equations*, by Peter Grinrod. Clarendon Press, Oxford, 1991, ix+237 pp., $39.95. ISBN 0-19-859692-8

Patterns and waves by Peter Grinrod deals with the mathematical techniques that can be applied to nonlinear parabolic partial differential equations. Much of the treatment deals with the semilinear parabolic equation

$$U_t = \Delta u + f(u, \nabla u, x, t)$$

where $x \in \Omega \subset R^n$, Ω is an open subset, and $f\colon R \to R$ is some smooth function and represents the reaction term. Additional boundary conditions are imposed on $\partial\Omega$, the boundary of Ω. Also $u(x, 0) = u_0(x)$ is specified. The boundary conditions considered are of (i) Dirichlet type, (ii) Neumann or no-flux type, and (iii) Robin or mixed type. For example, in the dispersive behavior of populations or concentrations, u represents the density function, Δu represents the diffusion term, and f represents the net creation or destruction rate of particles at $x \in \Omega$ at time t. While the continuous case is described in the main part of the text, the stochastic process is described in boxes that give ample directions to the interested reader for further study. The text is suitable for well-prepared senior college level or beginning graduate students. The boxes describe material that is at the current research level and will be suitable to researchers in the field. There is an adequate number of examples that are clearly solved using current techniques and the solutions are well illustrated through phase-plane plots, bifurcation analysis, and excellent computer graphics. I found this text to be an excellent introduction to the current research topics of semilinear parabolic partial differential equations or reaction-diffusion systems.

Some comments about the flow of topics in this treatment is in order. The first three chapters deal with the basic topics of conservation laws, equilibria and linear stability, traveling waves, local existence and blowup, local bifurcation analysis, transition layers, and plane waves. Chapters 4 and 5 give excellent introduction to current research topics such as geometric theory for spirals, scrolls, stationary spiral waves, toroidal scroll waves, chemotaxis, and the physical problems in physiology, biology, and chemistry. The treatment is clear, concise, and takes the reader from the beginning material to research topics in a well-illustrated, skillful manner. It was a joy for me to spend a few weeks reviewing this text. I hope to dig deeper into its contents very soon. I recommend this text to everyone, who is interested in the subject of semilinear parabolic partial differential equations. I would like to

thank Professor Peter Grinrod for treating such a difficult subject in a very clearly illustrated and interesting manner.

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