

BOOK REVIEW

Cycles and bridges in graphs, by Heinz-Jürgen Voss. Kluwer Academic Publishers, Dordrecht, Boston, London, 1991, xii + 271 pp., \$112.00. ISBN 0-7923-0899-9

It would be hard to find a graph theorist who has not written at least a paper or two on some question involving cycles. Not that this should come as a great surprise, since only very special graphs (forests) contain no cycles; the fact that a graph contains cycles leads naturally to many specific questions. What is the shortest cycle? What is the longest cycle? Is there a cycle containing all of the vertices? In what ways do various graph parameters, for example, the minimum degree, influence the existence of cycles of specified length? What conditions ensure cycles with many diagonals? Graph theory has developed an array of cycle-related properties (girth, circumference, hamiltonian graph, etc.) and presents the researcher with the perpetual challenge of relating these properties to such graphical features as minimum degree, neighborhood unions, forbidden subgraphs, connectivity, planarity, etc.

Proof techniques for problems involving cycles vary in sophistication. Early results of Dirac and Ore have inspired many similar approaches. These arguments often involve high levels of creativity and technical skill but may leave something to be desired in the way of “structure”. For those who crave a somewhat higher level of sophistication, the idea of a bridge (with respect to a subgraph), introduced by W. T. Tutte and explored at length in *Cycles and bridges in graphs*, may be just what the doctor ordered. Given a graph G and one of its subgraphs H , a *bridge in G with respect to H* (or simply H -bridge) is the subgraph of $G - E(H)$ induced by an equivalence class of edges under the following equivalence relation: $e_1 \sim e_2$ if there is a path in G in which the first edge is e_1 , the last is e_2 , and none of the internal vertices of the path belong to H . If B is a bridge in G with respect to H , the vertices of $B \cap H$ are the *vertices of attachment* of B . The special case of a bridge with respect to a cycle was introduced in Tutte’s 1956 paper in which the fact that every 4-connected planar graph is hamiltonian [2] is proved. Indeed, the main theorem of this paper is one about bridges. It asserts that in a 2-connected plane graph in which distinct edges e and e' belong to a facial cycle C_1 , there exists a cycle C containing e and e' such that each C -bridge has at most three vertices of attachment and each C -bridge containing an edge of $C_1 \cup C_2$, where C_2 is the other facial cycle containing e , has at most two vertices of attachment. From this result, it follows that given any 4-connected plane graph and distinct edges e, e' belonging to a facial cycle, there is a hamiltonian cycle through e and e' . Strengthening Tutte’s basic result on bridges, C. Thomassen later proved that every 4-connected planar graph is hamiltonian connected [1].

At first, bridges were only used as devices for studying cycles in planar graphs. Now the bridge concept is applied in the nonplanar case and bridges are studied in their own right. In fact, there is a flourishing industry in which properties of bridges are explored and the “bridge method” is developed as a proof technique with widespread application. The present status of this industry is authoritatively presented in *Cycles and bridges in graphs*. Among the topics dealt with are separating and nonseparating cycles, lengths of bridges, isomorphic bridges, long cycles in graphs with given minimum degree, cycles with diagonals in graphs with minimum degree r and girth t , extremal problems related to long cycles with many diagonals, and longest cycles in graphs with given minimum degree. *Cycles and bridges in graphs* is intended for researchers in graph theory who are interested in problems having to do with cycles and want to know more about some of the most powerful proof techniques for dealing with such problems. As remarked at the beginning, this may be just about every graph theorist.

REFERENCES

1. C. Thomassen, *A theorem on paths in planar graphs*, J. Graph Theory **7** (1983), 129–141.
2. W. T. Tutte, *A theorem on planar graphs*, Trans. Amer. Math. Soc. **82** (1956), 99–116.

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