In conclusion, the author has done us a service by preparing a digestible exposition of the current state of affairs in an interesting area at the confluence of function theory and operator theory. The book does not leave us with the impression that the theory is in a finished state, but rather that the area is an active and inviting research field with a lot of rough edges and surfaces begging to be smoothed out. Enjoy!

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Combinatorial matrix theory, by Richard A. Brualdi and Herbert J. Ryser. Cambridge Univ. Press, Cambridge, 1991, ix+367 pp. $49.50. ISBN 0-521-32265-0

This book explores the interrelations between the theories of graphs and matrices.

Graph theory is embedded in a not very well-defined region of mathematics called "combinatorics", which would seem to include the theory of sets of subsets of a given finite set. For example, many theorems about graphs can be stated in terms of circuits and edges only, the circuits being treated as subsets of the edge set satisfying some set-theoretical axioms. Generalized in a natural way, this variation becomes the theory of matroids. Matroids get only a passing mention in this book, but general families of subsets are important, for they have incidence matrices, defined and discussed in the first chapter.

In the early days of my acquaintance with matroids I supposed that by constructing their theory one superseded the theory of graphs and would no longer have to use it, so it was with a shock of surprise that I found myself forced to use what I considered a graph-theoretical argument in a proof of a theorem about matroids. I had a similar experience more recently in an attempt to clarify the Birkhoff-Lewis theory of "free" and "constrained" chromatic polynomials. Having transformed that theory into one about partitions of a cyclically ordered sequence, I claimed to have replaced all the graph theory in that problem by algebra. But then I needed to evaluate the determinant of a matrix defined by the relevant partitions. To my surprise I was unable to do this without going back to graph theory.

So I approached this book confident that it could not absorb graph theory into matrix theory. Conversely, I did not expect matrix theory to be shown as only an aspect of graph theory. To be sure though, I have heard it contended that all mathematics, properly presented, is graph theory; meaning, I suppose, that graph theory is a style of writing rather than a restricted region of mathematics.

Wherever diagrams without metrical attributes are found to be helpful, there you have graph theory. The book justified my confidence. The authors maintain an easy parity between the two theories, making it natural to use either theory to prove results in the other.

I met many old friends in this book, often appearing in a new aspect. On page 2, for example, there is a diagram of a rectangle dissected into rectangles. I was reminded that it was from such dissections, preferably of rectangles into unequal squares, that I got my own introduction to combinatorial research. Here the dissection is transformed directly into a matrix of 0's and 1's, whereas my colleagues and I represented it as a graph and went on to the Laplacian matrix of that graph (p. 30).

From a graph or digraph we get its incidence matrix showing the relations between edges and vertices, its adjacency matrix showing how pairs of vertices are joined or not joined, and a diagonal matrix showing the valencies or out-valencies of the vertices. The Laplacian matrix is obtained by subtracting the adjacency matrix from the diagonal one or by postmultiplying the incidence matrix by its transpose. There is also a special kind of matrix for bipartite graphs, in which the rows correspond to the vertices of one part of a bipartition, the columns correspond to the vertices of the other part, and the elements show which vertex pairs are joined by how many edges. Thus there are several routes whereby results about graphs can be transformed into theorems about matrices, and conversely. They are explored in Chapters 2–5.

There is sometimes a choice for the entries in one of these matrices. Sometimes integers are called for, sometimes indeterminates. In the latter case we may with advantage bring in the theory of polynomials to help us with determinants, permanents, or pfaffians (Chapter 9).

I once thought there was little to be done with matrices except multiply them together, evaluate their determinants, and calculate or at least theorize about their eigenvalues. All these things can be done in a combinatorial context, and graph theorists often concern themselves with the eigenvalues of adjacency matrices, but this book tells us more. Matrices are combinatorial objects whose basic parts are rows and columns, with incidences defined by the elements.

For many of the combinatorial properties of a matrix the elements need only be distinguished as zero and nonzero; hence, the prominence of matrices of 0's and 1's in this work. With this restriction the matrix is also a bipartite graph. A matrix has a combinatorial or "term" rank that is not necessarily the same as the conventional rank. The standard transforming operations are permutations of rows or columns. There are theorems that say under what graph-theoretical or other conditions these permutations can transform a matrix into some convenient standard form, and others that say when a matrix can be represented as a sum of matrices of specified simple forms. There are also existence theorems for matrices satisfying given conditions, such as being symmetrical and having given row sums (Chapter 6).

There is an entire Chapter (7) on the permanent of a matrix, how to cope with it, and how to use it. There is another (Chapter 8) devoted to Latin squares, which are of course matrices.

I was pleased with the treatment of some more old friends, transportation theory in Chapter 6 and the similar but more difficult theory of 1-factors, with its pfaffians, in Chapter 9.

I welcome this book as a major addition to the literature of combinatorics. How sad it is that Dr. Ryser did not live to see it completed! Dr. Brualdi warns us that it covers only a part of combinatorial matrix theory, but he promises us a sequel.

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Error analysis in numerical processes, by Solomon G. Mikhlin. Wiley, Chichester, New York, Brisbane, Toronto, and Singapore, 1991, 283 pp., $87.95. ISBN 0-471-92133-5

The book deals with classical discretization methods for solving linear and nonlinear problems in mathematical physics, mechanics, and numerical analysis, like Ritz, Bubnov-Galerkin, finite element, and collocation methods. In the book, the methods are mainly applied to two-point boundary value problems in ordinary differential equations, to the standard example of a Fredholm integral equation, and to nonlinear variational problems in elasticity. Discretization methods approximate the given problem by systems of algebraic equations that can be solved numerically both by direct and iterative methods. Accordingly, the book contains material on Gaussian elimination, the Cholesky method, QR-factorization, and, very briefly, some simple classical iterative methods.

The main interest of the author is an analysis of the different kinds of errors that occur in applications of the numerical methods. Part I explains the meaning of the term *error analysis* in a very general setting. The given *problem* is described by

$$\{x \in X : xrf\}, \tag{1}$$

where f is a given object and x the solution in a given class X satisfying a certain relation r to the object f. Problem (1) is approximated by a sequence of problems

$$\{v^{(n)} \in X_n : v^{(n)} r_n f^{(n)}\}, \qquad n = 1, 2, \ldots. \tag{2}$$

Then, using an operator $p_n : X_n \to X$, $x^{(n)} = p_n v^{(n)}$, is taken as an approximate solution of (1). The author considers four types of errors:

1) *Approximation error.* $\rho_n = \|x - p_n v^{(n)}\|$.

2) *Perturbation error.* The element $f^{(n)}$ must be computed, giving some perturbed element $\tilde{f}^{(n)}$; likewise, the relation r_n is obtained as a perturbed one $\tilde{r}_n$. Instead of (2), one obtains perturbed approximating problems

$$\{z^{(n)} \in X_n : z^{(n)} \tilde{r}_n \tilde{f}^{(n)}\}, \qquad n = 1, 2, \ldots. \tag{3}$$

This leads to the perturbation errors

$$\|z_n - v^{(n)}\|_{X_n}, \qquad \|p_n z_n - p_n v^{(n)}\|_X.$$