I welcome this book as a major addition to the literature of combinatorics. How sad it is that Dr. Ryser did not live to see it completed! Dr. Brualdi warns us that it covers only a part of combinatorial matrix theory, but he promises us a sequel.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 28, Number 1, January 1993
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0273-0979/93 $1.00 + $.25 per page

Error analysis in numerical processes, by Solomon G. Mikhlin. Wiley, Chichester, New York, Brisbane, Toronto, and Singapore, 1991, 283 pp., $87.95. ISBN 0-471-92133-5

The book deals with classical discretization methods for solving linear and nonlinear problems in mathematical physics, mechanics, and numerical analysis, like Ritz, Bubnov-Galerkin, finite element, and collocation methods. In the book, the methods are mainly applied to two-point boundary value problems in ordinary differential equations, to the standard example of a Fredholm integral equation, and to nonlinear variational problems in elasticity. Discretization methods approximate the given problem by systems of algebraic equations that can be solved numerically both by direct and iterative methods. Accordingly, the book contains material on Gaussian elimination, the Cholesky method, QR-factorization, and, very briefly, some simple classical iterative methods.

The main interest of the author is an analysis of the different kinds of errors that occur in applications of the numerical methods. Part I explains the meaning of the term *error analysis* in a very general setting. The given *problem* is described by

$$\{x \in X : xrf\}, \tag{1}$$

where f is a given object and x the solution in a given class X satisfying a certain relation r to the object f. Problem (1) is approximated by a sequence of problems

$$\{v^{(n)} \in X_n : v^{(n)} r_n f^{(n)}\}, \qquad n = 1, 2, \ldots. \tag{2}$$

Then, using an operator $p_n : X_n \to X$, $x^{(n)} = p_n v^{(n)}$, is taken as an approximate solution of (1). The author considers four types of errors:

1) *Approximation error.* $\rho_n = \|x - p_n v^{(n)}\|$.

2) *Perturbation error.* The element $f^{(n)}$ must be computed, giving some perturbed element $\tilde{f}^{(n)}$; likewise, the relation r_n is obtained as a perturbed one $\tilde{r}_n$. Instead of (2), one obtains perturbed approximating problems

$$\{z^{(n)} \in X_n : z^{(n)} \tilde{r}_n \tilde{f}^{(n)}\}, \qquad n = 1, 2, \ldots. \tag{3}$$

This leads to the perturbation errors

$$\|z_n - v^{(n)}\|_{X_n}, \qquad \|p_n z_n - p_n v^{(n)}\|_X.$$

3) *Algorithm error.* The perturbed problem (3) must be solved by some algorithm that yields the result $w^{(n)} \in X_n$. The associated algorithm errors are defined by

$$\|w_n - z^{(n)}\|_{X_n}, \qquad \|p_n w_n - p_n z^{(n)}\|_X.$$

4) *Rounding error.* Due to rounding errors in floating point operations, instead of $w^{(n)}$ approximations, $\tilde{w}^{(n)}$ are computed with errors

$$\|\tilde{w}_n - w^{(n)}\|_{X_n}, \qquad \|p_n \tilde{w}_n - p_n w^{(n)}\|_X.$$

Part II of the book illustrates the general concepts of approximation, perturbation, algorithm, and rounding error in the context of Ritz, Bubnov-Galerkin, collocation, and finite difference methods, as well as in a few direct and iterative methods for solving systems of algebraic equations. In addition, the perturbation of linear recurrent numerical processes is analyzed. Also, discretization error estimates and stability results for the methods are given in terms of the classical functional analysis of Sobolev function spaces or of abstract Hilbert or Banach spaces in the style of Kantorovitch-Akilov's book.

The author places a particular emphasis on his treatment of rounding errors in the numerical solution of systems of linear algebraic equations; however, the rounding error analysis of the book is strongly simplified by the assumption that a double word floating point arithmetic with a final roundoff (after all computations are performed) to single precision is used and by the fact that higher order terms are neglected. This means, for instance, that the product of s matrices, $A = A_1 \cdots A_s$, can be computed with an error $\delta(A) = (\delta(a_{ij}))$, where the errors of the computed elements $\tilde{a}_{ij}$ are bounded by

$$|\delta(a_{ij})| = |\tilde{a}_{ij} - a_{ij}| \leq \varepsilon_1 |a_{ij}|$$

and ε_1 is the greatest floating point number ε such that $1 + \varepsilon = 1$ in machine arithmetic. As a result, the error analysis of the book is not a genuine rounding error analysis as the well-known backward error analysis or a corresponding forward error analysis. Part III describes and to some extent analyzes the solution of linear algebraic equations by Gaussian elimination and some other direct and iterative methods such as Progonka, Cholesky, bordering, conjugate direction, steepest descent, and Richardson method. Most results in this part are concerned with norm estimates of perturbation errors, algorithm errors of the iteration processes, and the influence of the special kind of rounding errors described above.

Part IV starts with the error analysis of a one-dimensional finite element approximation for an ordinary differential equation of order $2s$. The class of basis functions is defined by three basic properties. Estimates for the approximation error, the perturbation error, and the condition number of the FEM-matrix are derived. Further results concern the algorithm and the rounding error. The rest of Part IV deals with methods for solving Fredholm integral equations. In particular, the convergence of the quadrature method is proved. Several methods for reducing a regular or singular integral equation to an algebraic system of equations are surveyed. Then the classical resolvent method for Fredhom integral equations is analyzed in detail.

The final Part V is devoted to the functional analysis of nonlinear problems, especially variational problems, including unilateral variational problems and variational inequalities, and to applications in elasticity. This class of problems is approximated by Ritz and FEM methods. An associated approximation error estimate is given, and the stability of the methods is analyzed. In this context, stability results for the solution of general nonlinear recurrences and the Newton-Kantorovich method under perturbation and rounding errors are proved.

The monograph under review is basically a collection of work of Mikhlin, his students, and other Russian authors on numerical methods. The bibliography lists a great number of their publications since 1949. There are also references to work of non-Russian mathematicians, mostly in introductory remarks. It seems that this work had only a limited influence on the style and content of most of the book.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 28, Number 1, January 1993
©1993 American Mathematical Society
0273-0979/93 $1.00 + $.25 per page

Two-dimensional geometric variational problems, by Jürgen Jost. Wiley, New York, 1991, x+234 pp. $87.95, ISBN 0-471-92839-9

In the first chapter the author presents the most important and fascinating geometric variational problem, namely, to span a disc-type surface of minimal area into a given spatial Jordan curve Γ. This mathematical problem can be realized by soap films spanning into a given wire, initiated by experiments of the Belgian physicist Plateau. Then the general two-dimensional conformally invariant variational problem is considered, where the function $u \in H^{1,2}(\Omega, \mathbb{R}^d)$ with $d \geq 2$ renders the integral

$$(1) \qquad I(u) := \frac{1}{2}\int_\Omega \{g_{ik}(u)\nabla u^i \nabla u^k + b_{ik}(u)\det(\nabla u^i, \nabla u^k)\}\,dx\,dy$$

stationary. Here $(g_{ik})_{ik}$ is a positive definite and $(b_{ik})_{ik}$ a skew-symmetric matrix. The Euler-Lagrange equations of I are given by

$$(2) \qquad \Delta u^i + \Gamma^i_{kl}\nabla u^k \nabla u^l = g^{im}(b_{mk,l} + b_{kl,m} + b_{lm,k})\det(\nabla u^k, \nabla u^l)$$

with $(g^{ik})_{ik} = (g_{ik})^{-1}_{ik}$, $g_{kl,m} = \partial g_{kl}/\partial u_m$, and the Christoffel symbols Γ^i_{kl}. One can interpret (2) as the equation for a surface of prescribed mean curvature in a Riemannian manifold. The two-dimensional conformally invariant variational problems especially give rise to conformal maps between surfaces, parametric minimal surfaces in Riemannian manifolds, and harmonic maps from a surface into a Riemannian manifold.

For surfaces of prescribed mean curvature in $\mathbb{R}^3$ one considers the following functional of Heinz and Hildebrandt

$$(3) \qquad I(u) = \frac{1}{2}\int_\Omega \{|\nabla u|^2 + Q(u)\cdot u_x \wedge u_y\}\,dx\,dy$$